

ASSESSED COURSEWORK 2

Mathematics Tutorial I

Nagoya University

G30 Program, Fall 2012

Deadline: November 27, 14:45

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 5 points. Hand in at the start of the tutorial class on November 27.

Exercise 1. Find d^2y/dx^2 in terms of x and y , if $2x^3 - 3y^2 = 8$.

Exercise 2. Find all θ such that $0 \leq \theta < 2\pi$ and

$$\cos^2 \theta = \sin^2 \theta.$$

Exercise 3.

(a) Find the derivative of $(\cos x)^4$.

(b) Find the derivative of $\sin(x^2 + 1)$.

(c) Find all real numbers a such that the function $f(x) = \cos(ax)$ satisfies

$$f''(x) + 4f(x) = 0$$

for all $x \in \mathbb{R}$.

Exercise 4. Solve the equation

$$\ln(x) - \ln(x - 1) = 1.$$

Exercise 5. Using L'Hospital's rule, prove that

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e.$$