ASSESSED COURSEWORK 1
Mathematics Tutorial I
Nagoya University
G30 Program, Fall 2012
Deadline: November 6, 14:45

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 5 points. Hand in at the start of the tutorial class on November 6.

Exercise 1.
   (a) Find the domain of $y = \frac{x+3}{4-\sqrt{x^2-9}}$.
   (b) Find the range of $y = 2 + \frac{x^2}{x^2+4}$.

Exercise 2. Let $g(x) = [[x/3]]$, where $[[x]]$ is the largest integer that is less than or equal to x.
   (a) Sketch the graph of g.
   (b) Evaluate each of the following limits if it exists and if does not, explain why: $\lim_{x\to 1} g(x)$; $\lim_{x\to 2} g(x)$; $\lim_{x\to 3} g(x)$.
   (c) For what values of a does $\lim_{x\to a} g(x)$ exist?

Exercise 3. Write down all elements in the following sets.
   (a) $\{1, 2, 3, 4, 5\} \cap \{x \in \mathbb{Z} \mid x^2 \geq 9\}$
   (b) $\{r \in \mathbb{Q} \mid 3r \in \mathbb{Z} \text{ and } 1 < r < 3\}$
   (c) $\{n \in \mathbb{Z} \mid n = k^2 \text{ for some } k \in \{0, 1, 2, 3\}\}$
   (d) $\{y \in \mathbb{Z} \mid (y-3)^2 \leq 4\}$
   (e) $\{(x, y) \mid x, y \in \mathbb{Z} \text{ and } 1 \leq x \leq y \leq 4\}$

Exercise 4. Find the point on the plane given by the equation
$$x + 2y + z = 1,$$
which is closest to the point $(5, 2, 4)$.

Exercise 5. Find the following limits
   (a) $\lim_{x\to\infty} \sqrt{x^2 + 1} - \sqrt{x^2 - 1}$
   (b) $\lim_{x\to\infty} \frac{1-\sqrt{x}}{1+\sqrt{x}}$
   (c) Using the ϵ, δ-definition of a limit, prove that
$$\lim_{x\to-5}\left(4 - \frac{3x}{5}\right) = 7$$
   .