Problem set 4

Topics in Representation Theory I

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 5 points. Hand in before or during the lecture on July 31.

Problem 1. Let Q be the quiver

$$1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$$

Use the Nakayama functor to compute the Auslander-Reiten translation of the following indecomposable K-linear representations of Q:

$$K \xrightarrow{0} 0 \xrightarrow{0} 0, \quad 0 \xrightarrow{0} K \xrightarrow{0} 0, \quad 0 \xrightarrow{0} 0 \xrightarrow{0} K,$$
$$K \xrightarrow{1} K \xrightarrow{0} 0, \quad 0 \xrightarrow{0} K \xrightarrow{1} K, \quad K \xrightarrow{1} K \xrightarrow{1} K.$$

Problem 2. Let

$$0 \longrightarrow L \xrightarrow{f} M \xrightarrow{g} N \longrightarrow 0$$

$$0 \longrightarrow L' \xrightarrow{f'} M' \xrightarrow{g'} N' \longrightarrow 0$$

be two almost split sequences in $\operatorname{mod} \Lambda$. Show that if $L \simeq L'$ or $N \simeq N'$, then the sequences are isomorphic, i.e., there is a commutative diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & L & \xrightarrow{f} & M & \xrightarrow{g} & N & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle u} & & \downarrow{\scriptstyle v} & & \downarrow{\scriptstyle w} & & \\ 0 & \longrightarrow & L' & \xrightarrow{f'} & M' & \xrightarrow{g'} & N' & \longrightarrow & 0 \end{array}$$

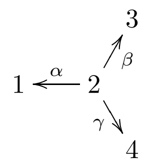
where u, v and w are isomorphisms.

Problem 3. (From Assem-Simson-Skowronski) Let X be a non-zero Λ-module. Show that, up to isomorphism, there are at most finitely many almost split sequences

$$0 \longrightarrow L \xrightarrow{f} M \xrightarrow{g} N \longrightarrow 0$$

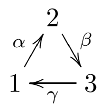
in $\operatorname{mod} A$ such that X is a direct summand of M.

Problem 4. Let Q be the following quiver



Find the Auslaner-Reiten quiver of KQ.

Problem 5. Let Q be the following quiver



and I the ideal in KQ generated by $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$. Find the Auslaner-Reiten quiver of KQ/I.