Problem set 3

Topics in Representation Theory I

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 5 points. Hand in before or during the lecture on July 3.

Problem 1. Let Λ be a finite dimensional K-algebra. Recall that rad_Λ is the ideal in the K-category $\mathrm{mod}\,\Lambda$ defined by

$$\mathrm{rad}_\Lambda(M, N) = \{f : M \to N \mid 1_N - f \circ g \in \mathrm{Aut}_\Lambda(N) \text{ for all } g \in \mathrm{Hom}_\Lambda(N, M)\}$$

for all $M, N \in \mathrm{mod}\,\Lambda$.

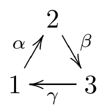
Let $M, N \in \mathrm{mod}\,\Lambda$ be indecomposable and not isomorphic. Show that

$$\mathrm{rad}_\Lambda(M, M) = \mathrm{rad}(\mathrm{End}_\Lambda(M))$$

and

$$\mathrm{rad}_\Lambda(M, N) = \mathrm{Hom}_\Lambda(M, N).$$

Problem 2. Let Q be the following quiver



and I the ideal in KQ generated by $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$. Describe explicitly all simples, indecomposable projectives and indecomposable injectives in $\mathrm{rep}_K(Q, I)$.

Problem 3. (From Assem-Simson-Skowronski) Let A be a K-algebra and $f : M \to N$ a morphism in $\mathrm{mod}\,A$. Show that the following conditions are equivalent.

(a) For any $L \in \mathrm{mod}\,A$ and epimorphism $h : L \to N$, there is $g : M \to L$ such that $f = hg$.

(b) For any projective $P \in \mathrm{mod}\,A$ and epimorphism $h : P \to N$, there is $g : M \to P$ such that $f = hg$.

(c) There is a projective $P \in \mathrm{mod}\,A$ and morphisms $g : M \to P$ and $h : P \to N$ such that $f = hg$.

Problem 4. Let Λ be a finite dimensional K-algebra and $f : P \to M$ an epimorphism in $\operatorname{mod} \Lambda$ with $P \in \operatorname{mod} \Lambda$ projective. Show that $f : P \to M$ is a projective cover of M if and only if f is right minimal.

Problem 5. Let $\Lambda = K[T]/(T^2)$ and S be the simple Λ-module $S = \Lambda / \operatorname{rad} \Lambda \simeq K$. Show that Λ and S classify all indecomposable Λ-modules. Show that there is an almost split sequence

$$0 \to S \to \Lambda \to S \to 0.$$