# Problem set 2

## Topics in Representation Theory I

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 5 points. Hand in before or during the lecture on June 12.

**Problem 1.** Let $M$ and $N$ be modules over a $K$-algebra $\Lambda$. Show that $M \oplus N$ is projective if and only if $M$ and $N$ are projective.

**Problem 2.** Consider the $K$-algebra

$$\Lambda = \begin{bmatrix} K & K & K \\ K & K & K \\ 0 & 0 & K \end{bmatrix}.$$

Decompose $\Lambda_\Lambda$ into a direct sum of indecomposable $\Lambda$-modules and determine which indecomposable summands are isomorphic. Describe the basic algebra associated to $\Lambda$.

**Problem 3.** Let $Q$ be an acyclic quiver and $\Lambda = KQ$. Show that there exists a simple projective $\Lambda$-module.

**Problem 4.** (From Assem-Simson-Skowronski) Let $K$ be an algebraically closed field. Describe, up to isomorphism, all basic three-dimensional $K$-algebras.

**Problem 5.** Let $\Lambda$ be the $\mathbb{R}$-algebra

$$\begin{bmatrix} \mathbb{C} & \mathbb{C} \\ 0 & \mathbb{R} \end{bmatrix}.$$

Show that $\Lambda$ is a basic $\mathbb{R}$-algebra, but there is no quiver $Q$ such that $\Lambda \simeq \mathbb{R}Q/I$ for some admissible ideal $I$.