Problem set 1

Topics in Representation Theory I

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 5 points. Hand in before or during the lecture on May 22.

Problem 1. Find finite dimensional K-algebras A, B and a K-algebra homomorphism $f : A \to B$ such that $f(\operatorname{rad} A) \not\subseteq \operatorname{rad} B$. Hint: As B, you can take the algebra of 2×2-matrices over K.

Problem 2. Let A be a finite dimensional K-algebra with a unique maximal right ideal. Show that every element in A is either invertible or nilpotent.

Problem 3. Let A be the path algebra of the quiver

$$\begin{tikzcd}1 \arrow[r, "\alpha"] & 2 \arrow[r, "\gamma"] \arrow[loop below, "\beta"] & 3.\end{tikzcd}$$

Find $\operatorname{rad} A$.

Problem 4. Let Q be the quiver

$$\begin{tikzcd}1 \arrow[r, "\alpha"] & 2 \arrow[r, "\beta"] & 3\end{tikzcd}$$

and consider the following K-linear representations of Q:

$$\begin{tikzcd}
K \arrow[r, "0"] & 0 \arrow[r, "0"] & 0, & 0 \arrow[r, "0"] & K \arrow[r, "0"] & 0, & 0 \arrow[r, "0"] & 0 \arrow[r, "0"] & K, \\
K \arrow[r, "1"] & K \arrow[r, "0"] & 0, & 0 \arrow[r, "0"] & K \arrow[r, "1"] & K, & K \arrow[r, "1"] & K \arrow[r, "1"] & K.
\end{tikzcd}$$

(a) Compute the endomorphism algebras of the above representations.

(b) Determine which of them are indecomposable.

Problem 5. Let Q be the quiver

$$\begin{tikzcd}1 \arrow[r, "\alpha"] & 2 \arrow[r, "\beta"] & 3.\end{tikzcd}$$

Show that every finite dimensional K-linear representation of Q is isomorphic to a direct sum of representations taken from the list in Problem 4.