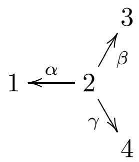
Problem set 4

Representation Theory of Associative Algebras

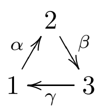
Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 25 points. Hand in before or during the lecture on December 13.

Problem 1. Let Q be the following quiver



Describe explicitly all simples, indecomposable projectives and indecomposable injectives in $\mathrm{rep}_K(Q)$.

Problem 2. Let Q be the following quiver



and I the ideal in KQ generated by $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$. Describe explicitly all simples, indecomposable projectives and indecomposable injectives in $\mathrm{rep}_K(Q, I)$.

Problem 3. (From Assem-Simson-Skowronski) Let A be a K-algebra and $f : M \to N$ a morphism in $\mathrm{mod}\, A$. Show that the following conditions are equivalent.

(a) For any $L \in \mathrm{mod}\, A$ and epimorphism $h : L \to N$, there is $g : M \to L$ such that $f = hg$.

(b) For any projective $P \in \mathrm{mod}\, A$ and epimorphism $h : P \to N$, there is $g : M \to P$ such that $f = hg$.

(c) There is a projective $P \in \mathrm{mod}\, A$ and morphisms $g : M \to P$ and $h : P \to N$ such that $f = hg$.

Problem 4. Let $A = K[T]/(T^2)$ and S be the simple A-module $S = A/\operatorname{rad} A \simeq K$. Show that A and S classify all indecomposable A-modules. Show that there is an almost split sequence

$$0 \to S \to A \to S \to 0.$$