Problem set 3

Representation Theory of Associative Algebras

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 25 points. Hand in before or during the lecture on November 29.

Problem 1. Let Q be the quiver

$$1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$$

and consider the following K-linear representations of Q:

$$\begin{array}{ccc} K \xrightarrow{0} 0 \xrightarrow{0} 0, & 0 \xrightarrow{0} K \xrightarrow{0} 0, & 0 \xrightarrow{0} 0 \xrightarrow{0} K, \\ K \xrightarrow{1} K \xrightarrow{0} 0, & 0 \xrightarrow{0} K \xrightarrow{1} K, & K \xrightarrow{1} K \xrightarrow{1} K. \end{array}$$

(a) Compute the endomorphism algebras of the above representations.

(b) Determine which of them are indecomposable.

Problem 2. Let Q be the quiver

$$1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3.$$

Show that every finite dimensional K-linear representation of Q is isomorphic to a direct sum of representations taken from the list in Problem 1.

Problem 3. (From Assem-Simson-Skowronski) Let K be an algebraically closed field. Describe, up to isomorphism, all basic three-dimensional K-algebras.

Problem 4. Let A be the $\mathbb{R}$-algebra

$$\begin{bmatrix} \mathbb{C} & \mathbb{C} \\ 0 & \mathbb{R} \end{bmatrix}.$$

Show that A is a basic $\mathbb{R}$-algebra, but there is no quiver Q such that $A \simeq \mathbb{R}Q/I$ for some admissible ideal I.