Problem set 2

Representation Theory of Associative Algebras

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 25 points. Hand in before or during the lecture on November 15.

Problem 1. Let M and N be modules over a K-algebra A. Show that $M \oplus N$ is projective if and only if M and N are projective.

Problem 2. Consider the K-algebra

$$A = \begin{bmatrix} K & K & K \\ K & K & K \\ 0 & 0 & K \end{bmatrix}.$$

Decompose A_A into a direct sum of indecomposable A-modules and determine which indecomposable summands are isomorphic. Describe the basic algebra associated to A.

Problem 3. Let Q be an acyclic quiver and $A = KQ$. Show that there exists a simple projective A-module.

Problem 4. Let A be the path algebra of the quiver

$$\begin{tikzcd} 1 \arrow[r, "\alpha"] & 2 \arrow[r, "\gamma"] \arrow[loop below, "\beta"] & 3. \end{tikzcd}$$

Find $\operatorname{rad} A$.