Problem set 1

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 25 points. Hand in before or during the lecture on November 1.

Problem 1. Let R be a commutative ring (with identity element) and I an ideal in R. Show that I is maximal if and only if R/I is a field.

Problem 2. Find finite dimensional K-algebras A, B and an algebra K-homomorphism $f : A \to B$ such that $f(\operatorname{rad} A) \not\subseteq \operatorname{rad} B$. Hint: As B, you can take the algebra of 2×2-matrices over K.

Problem 3. Let A be a finite dimensional K-algebra with a unique maximal right ideal. Show that every element in A is either invertible or nilpotent.

Problem 4. Find a complete set of primitive idempotents for the $\mathbb{C}$-algebra A, where

(a) $A = \mathbb{C}[x]/(x^2 - 1)$,

(b) $A = \mathbb{C}[x]/(x^4 - 1)$.