PRACTICE PROBLEMS FOR FINAL EXAM

Linear Algebra II Nagoya University G30 Program, Spring 2013

The following problems should be used as practice for the final exam. The actual exam problems will be different but these can be considered as examples of what might appear. The time limit for the final exam will be 90 minutes. No tools will be allowed except pen and paper.

Problem 1. Let

$$A = \begin{bmatrix} 1 & -2 & 1 & 2 \\ -2 & 4 & -2 & -4 \end{bmatrix}.$$

- (a) Find an orthonormal basis in the kernel of A.
- (b) Find an orthonormal basis in the image of A.

Problem 2. Let

$$\vec{u}_1 = \frac{1}{3} \begin{bmatrix} 2\\2\\1 \end{bmatrix}, \quad \vec{u}_2 = \frac{1}{3} \begin{bmatrix} -2\\1\\2 \end{bmatrix}$$

and $V = \operatorname{span}(\vec{u}_1, \vec{u}_2).$

- (a) Show that (\vec{u}_1, \vec{u}_2) is an orthonormal basis of V.
- (b) Find the matrix A of the orthogonal projection onto V.

Problem 3. Let

$$A = \begin{bmatrix} 10 & -20 & -20 \\ -1 & 0 & 2 \\ 7 & -12 & -14 \end{bmatrix}.$$

- (a) Find all eigenvalues of A.
- (b) Find all eigenvectors of A.
- (c) Is the matrix A diagonalizable?

Problem 4. Let V be the subspace of \mathbb{R}^4 spanned by

$$\vec{v}_1 = \begin{bmatrix} 2\\0\\0\\0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1\\1\\2\\0 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1\\3\\1\\2 \end{bmatrix}$$

Find an orthonormal basis in V.

Problem 5. Find all eigenvalues of the matrix

$$A = \begin{bmatrix} -6 & -9 & 2\\ 4 & 6 & -1\\ 0 & 0 & 2 \end{bmatrix}$$

and determine their algebraic and geometric multiplicities. Is A diagonalizable?

Problem 6. Let

$$A = \begin{bmatrix} 11 & -6\\ 20 & -11 \end{bmatrix}.$$

(a) Find an invertible matrix S and a diagonal matrix D such that

$$S^{-1}AS = D.$$

(b) Caclulate A^n for each positive integer n.

Problem 7. Let A be the matrix of the reflection in the plane given by the equation 2x + 2y + z = 0 in the standard basis.

(a) Find an invertible matrix S and a diagonal matrix D such that

$$S^{-1}AS = D.$$

(b) Find the matrix A.

Problem 8. Let

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & 2 \\ 0 & 0 & 4 \end{bmatrix}.$$

- (a) Find all eigenvalues of A.
- (b) Find all eigenvectors of A.
- (c) Find an invertible matrix S and a diagonal matrix D such that $S^{-1}AS = D$.