PRACTICE PROBLEMS FOR MIDTERM EXAM
Linear Algebra I
Nagoya University
G30 Program, Fall 2012

The following problems should be used as practice for the midterm exam. The actual exam will contain fewer problems, which will be different but these can be considered as examples of what might appear. The time limit for the midterm exam will be 90 minutes. No tools will be allowed except pen and paper.

Problem 1.

(a) Calculate $3\begin{bmatrix}1\\2\end{bmatrix}+2\begin{bmatrix}0\\-1\end{bmatrix}$.

(b) Calculate $\begin{bmatrix}1&2\\3&4\end{bmatrix}+\begin{bmatrix}0&1\\-1&0\end{bmatrix}$.

(c) Calculate $\begin{bmatrix}1&3\\-2&1\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix}$.

(d) Is the vector $\vec{u}=\begin{bmatrix}1\\-2\\4\end{bmatrix}$ orthogonal to $\vec{v}=\begin{bmatrix}-1\\4\\3\end{bmatrix}$ or $\vec{w}=\begin{bmatrix}2\\3\\1\end{bmatrix}$.

(e) What is the length of the vector $\vec{x}=\begin{bmatrix}3\\4\end{bmatrix}$?

Problem 2. Let

$$\vec{v}=\begin{bmatrix}5\\4\\2\end{bmatrix} \text{ and } \vec{w}=\begin{bmatrix}1\\-2\\3\end{bmatrix}.$$

Calculate the vectors

(a) $3\vec{v}$,

(b) $2\vec{w}$,

(c) $3\vec{v}-2\vec{w}$,

(d) $\vec{v}\cdot\vec{w}$.

Problem 3.

(a) Solve the following system of linear equations.

$$\begin{cases} x_1 \phantom{{}+3x_2} + x_3 + 2x_4 = 3 \\ 2x_1 + 3x_2 - x_3 + x_4 = 9 \\ 2x_1 - 3x_2 + 5x_3 + 7x_4 = 3 \end{cases}$$

(b) What is the rank of the matrix

$$\begin{bmatrix}1&0&1&2\\2&3&-1&1\\2&-3&5&7\end{bmatrix}.$$

(c) What is the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 3 & -1 & 1 \end{bmatrix}.$$

Problem 4. Three lines in the plane are given by the equations

$$x - 2y = 1, \quad 3x + 2y = 4 \text{ and } -2x + 4y = 4$$

respectively. Sketch the three lines. Are any of them parallel? Do any of them intersect each other? In that case find the intersection.

Problem 5. Find the intersection of the three planes given by the following equations:

$$\begin{array}{rcrcrcr} 2x & - & 5y & + & 6z & = & 5, \\ x & - & 3y & + & 2z & = & 0, \\ -x & + & 6y & + & 2z & = & 11. \end{array}$$

Interpret your result geometrically.

Problem 6. Let $A = \begin{bmatrix} 3 & -2 & 5 \\ -1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -2 & 2 \end{bmatrix}$. Find all vectors $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that

$$A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

What is the rank of A?

Problem 7. Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 13 & 4 \\ 4 & -2 \end{bmatrix}.$$

Find all real numbers x, y and z such that

$$xA + yB + zC = D.$$

Problem 8. Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

and $S : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by $S(\vec{x}) = A\vec{x}$.

(a) Show that the function $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(\vec{x}) = S(S(\vec{x}))$ is a linear transformation.

(b) Find the matrix of T.