PRACTICE PROBLEMS FOR MIDTERM EXAM
Linear Algebra I
Nagoya University
G30 Program, Fall 2011

The following problems should be used as practice for the midterm exam. The actual exam problems will be different but these can be considered as examples of what might appear. The time limit for the midterm exam will be 90 minutes. No tools will be allowed except pen and paper.

Problem 1. Let

$$\vec{v} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix} \text{ and } \vec{w} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}.$$

Calculate the vectors

(a) $3\vec{v}$,

(b) $2\vec{w}$,

(c) $3\vec{v} - 2\vec{w}$,

(d) $\vec{v} \cdot \vec{w}$.

Problem 2.

(a) Solve the following system of linear equations.

$$\begin{cases} x_1 & + & x_3 + 2x_4 = 3 \\ 2x_1 + 3x_2 - & x_3 + & x_4 = 9 \\ 2x_1 - 3x_2 + 5x_3 + 7x_4 = 3 \end{cases}$$

(b) What is the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 3 & -1 & 1 \\ 2 & -3 & 5 & 7 \end{bmatrix}.$$

(c) What is the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 3 & -1 & 1 \end{bmatrix}.$$

Problem 3. Three lines in the plane are given by the equations

$$x - 2y = 1, \quad 3x + 2y = 4 \text{ and } -2x + 4y = 4$$

respectively. Sketch the three lines. Are any of them parallel? Do any of them intersect each other? In that case find the intersection.

Problem 4. Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ C = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 13 & 4 \\ 4 & -2 \end{bmatrix}.$$

Find all real numbers x, y and z such that

$$xA + yB + zC = D.$$