

Ricci Flow on 3-dimensional Lie groups and 4-dimensional Ricci-flat manifolds

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Abstract

Hamilton によって導入された Ricci flow は、Perelman の結果に代表されるようにリーマン幾何学の多くの問題への効果的な研究手法の一つである。今回は、3次元リー群上の Ricci flow の振る舞いを復習し、その後、4次元余等質1多様体に Ricci flow を用いてリッチ平坦計量を構成する。ここで構成された計量の例の一つは、Taub-NUT 計量である。Taub-NUT 計量がハイパーケーラー構造を持つように、今回構成したリッチ平坦計量もハイパーケーラー構造を持つことを示す。

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