Some microlocal regularity theorem and its application to propagation of regularities to solutions of boundary value problems

Kazuhiro Yamamoto (Nagoya Institute of Technology)

In this talk first we shall show a microlocal regularity theorem to solutions of some boundary value problem for a second order differential operator. Next we shall apply the theorem to solutions of the boundary value problem for elastic equation and show theorems on propagation of regularities to the solutions.

First let us consider the solution of the following;

$$D_n^2 u + R(x, D_{x'})u = f \quad in\ R^n \cap \{x_n > 0\}, \tag{1}$$

where f is a smooth function and $R(x, D_{x'})$ is a second order differential operator of $x' = (x_1, \ldots, x_{n-1})$ with the real principal symbol $r_2(x, \xi')$. We assume the following:
A.1. $r_2(0, \bar{\xi'}) = 0, \quad \nabla_{\xi'} r_2(0, \bar{\xi'}) \neq 0$.
A.2. $u(x', 0)$ and $\partial_n u(x', 0)$ belong to microlocal Sobolev space $H^s_{mc}(\bar{p'})$ and $H^{s-1}_{mc}(\bar{p'})$, respectively, where $\bar{p'} = (0, \bar{\xi'})$.
A.3. Let $\gamma(t : \rho_0)$ be the bicharacteristics of $\xi_n^2 + r_2(x, \xi')$ starting at $\rho_0 = (0, (\bar{\xi'}, 0)) \in T^*(R^n) \setminus 0$ and u^c be an extended distribution to $U_0 \cap \{x_n < 0\}$ as $u = 0$ in $x_n < 0$. Then for any small neighbourhood $\tilde{\Gamma}_0$ of ρ_0, there exists a closed interval $I \subset R \setminus 0$ such that $\{\gamma(t; \rho_0); t \in I\} \subset \tilde{\Gamma}_0$ and u^c belongs to H_s on $\{\gamma(t; \rho_0); t \in I\}$.
Then we have the following theorem:
Theorem. If the solution $u(x)$ satisfies the A.1 to A.3, then u belongs to H_s at $(0, \bar{\xi'})$.

Next we shall consider solution of the boundary value problem for elastic equation;

$$\partial^2 u/\partial t^2 - (\lambda + \mu) grad(div u) - \mu \Delta u = f \quad in\ R \times \Omega, \tag{2}$$

$$Bu = g \qquad on\ R \times \partial\Omega, \tag{3}$$

where $u =^t (u_1, \ldots, u_n)$ is the displacement, λ and μ are Lamé constants such that $\lambda + 2\mu > 0, \mu > 0$ and $\lambda \neq 0$. Here Ω is an open set in R^n $(n \geq 2)$ with the smooth boundary $\partial\Omega$. The boundary condition is the Dirichlet condition $Bu = u$ or the free boundary condition

$$(Bu)_j = \sum_{i=1}^{n} \nu_i \{\lambda(div u)\delta_{ij} + \mu(\partial u_i/\partial x_j + \partial u_j/\partial x_i)\},$$

where $j = 1, \cdots, n$ and $\nu = (\nu_1, \ldots, \nu_n)$ is the unit normal vector to $\partial\Omega$.
Making use of the theorem stated above, we shall show the theorem on propagation of regularities for the solution of the above boundary value problem to elastic equation.