Mathematical Analysis of SKT Model in Population Biology

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My talk is concerned with some quasilinear parabolic systems of the form

$$
\text{(P)} \quad \begin{cases}
u_t = \Delta[(1+\alpha v+\gamma v)u] + u(a_1 - b_1 u - c_1 v) & \text{in } \Omega \times (0,\infty), \\
v_t = \Delta[(1+\beta u+\delta v)v] + v(a_2 - b_2 u - c_2 v) & \text{in } \Omega \times (0,\infty), \\
\dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega \times (0,\infty), \\
u(\cdot,0) = u_0,\ v(\cdot,0) = v_0 & \text{in } \Omega,
\end{cases}
$$

where $\alpha, \beta, \gamma, \delta$ are nonnegative constants, a_i, b_i, c_i $(i = 1, 2)$ are positive constants and u_0, v_0 are nonnegative functions. In (P), u and v denote the population densities of two competing species which are migrating and interacting in the same region $\Omega \subset R^N$. The above system takes account of nonlinear diffusion effects. Such a model was first proposed by Shigesada-Kawasaki-Teramoto in 1979 to describe the habitat segregation phenomena for competing species. Numerical simulations for (P) exhibit us interesting information; so this system has been studied by a lot of mathematicians.

Mathematically, there are two important problems for (P):

(1) Show the existence of global solutions without any restrictions on initial functions,

(2) Give complete information on the structure of the set of steady-state solutions corresponding to (P).

I will mainly discuss the non-stationary part for (P) and give some existence results of global solutions to (P) and related systems.