Maillet type theorem for a general system of partial differential equations in complex domain

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Abstract

For a single partial differential operator P with holomorphic coefficients in $\Omega \Subset \mathbb{C}^{1+n}_{t,x}$, the Cauchy problem at a point $(t_0, x_0) \in \Omega$ is formulated by

$$(P;(t_0,x_0)) \qquad \begin{cases} Pu \equiv \left\{\partial_t^m - \sum_{j=1}^{m} a_j(t,x;\partial_x)\partial_t^{m-j}\right\} u(t,x) = f(t,x) \in \mathcal{O}_{t_0,x_0}, \\ \partial_t^k u(0,x) = \varphi_k(x) \in \mathcal{O}_{x_0}, \quad 0 \le k < m, \end{cases}$$

where $\mathcal{O}_*$ denotes the set of holomorphic functions at $*$.

If a unique solution $u(t,x)$ of $(P;(t_0,x_0))$ exists in $\mathcal{O}_{t_0,x_0}$ for any data, we say that the Cauchy-Kowalevski theorem for P at a point (t_0,x_0) does hold. It was proved by myself and S. Mizohata that the Cauchy-Kowalevski theorem for P does hold at any fixed point in Ω if and only if the operator P is Kowalevskian, that is, $\mathrm{order}(a_j) \le j$ $(1 \le j \le m)$. Therefore for any non Kowalevskian operator P, the Cauchy problem $(P;(t_0,x_0))$ has a divergent solution by a suitable choice of data. The Maillet type theorem is to find a formal Gevrey index $\sigma = \sigma_P(t_0,x_0) > 1$ for the formal solution $u(t,x) = \sum_{k=0}^{\infty} u_k(x)\frac{(t-t_0)^k}{k!}$, that is, $\sum_{k=0}^{\infty} u_k(x)\frac{(t-t_0)^k}{(k!)^\sigma} \in \mathcal{O}_{t_0,x_0}$. To this problem we can easily prove that

$$(1) \qquad \sigma_P(\Omega) := \max_{(t_0,x_0)\in\mathcal{O}} \sigma_P(t_0,x_0) = \max_{1\le j\le m} \mathrm{order}(a_j)/j.$$

This number is obtained by drawing a Newton polygon of P. To obtain a precise estimate for $\sigma_P(t_0,x_0)$, it is convenient to draw a Newton polygon by taking account of the degeneracy of coefficients at $t = t_0$. Such observations for the Maillet type theorem for single equations were extensively studied by many authors for various kind of operators.

We want to extend the results for single equations to system $L \equiv \partial_t I_N - A(t,x;\partial_x)$ $(A = (a_{ij}(t,x;\partial_x)))$. To this system S. Mizahata obtained a necessary condition to hold the Cauchy-Kowalevski theorem for L at a point $(t_0,x_0) \in \Omega$. Moreover in case $\dim x = 1$, I obtained a necessary and sufficient condition to hold the Cauchy-Kowalevski theorem for L at every point in Ω. But we did not have Maillet type theorem for a system L. In this talk, I will show how we approach to this problem, and the corresponding result with (1) will be obtained in case $\dim x = 1$. Moreover we develop an argument to obtain $\sigma_L(t_0,x_0)$ by taking account of the degeneracy of coefficients of a_{ij} at $t = t_0$.