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Title: **Multisummability of divergent solutions of linear partial differential equations with constant coefficients**

Abstract: We consider the Cauchy problem for the general linear partial differential equations with constant coefficients

$$(1) \qquad P(\partial_t, \partial_z)u(t,z) = 0, \quad \partial_t^n u(0,z) = \varphi_n(z) \in \mathcal{O}(D) \quad n = 0, ..., m-1,$$

where D is a complex neighbourhood of origin and a polynomial $P(\lambda, \xi)$ satisfies

$$P(\lambda, \xi) = \lambda^m P(\xi) - \sum_{j=1}^{m} \lambda^{m-j} P_j(\xi) = P(\xi)(\lambda - \lambda_1(\xi))^{m_1} ... (\lambda - \lambda_l(\xi))^{m_l}.$$

W. Balser [1] has found the sufficient condition for the multisummability of normalized solution of (1) in terms of analytic continuation property and growth estimates of the Cauchy data $\varphi_n(z)$. We prove that this sufficient condition is also necessary.

To this end we use the modified Borel transform. Since after appropriate change of variables this transform converts the usually derivative to fractional, it is convenient to replace (1) by more general fractional equation

$$(2) \qquad P(\partial_t^{1/p}, \partial_z^{1/p})u(t,z) = 0, \quad (\partial_t^{1/p})^n u(0,z) = \varphi_n(z) \quad n = 0, ..., m-1,$$

where $p \in \mathbb{N}$, $\varphi_n(z)$ is $1/p$-analytic on D and $1/p$-derivative is given by

$$\partial_t^{1/p}\Big(\sum_{n=0}^{\infty} \frac{a_n}{\Gamma(1+n/p)} t^{n/p}\Big) = \sum_{n=0}^{\infty} \frac{a_{n+1}}{\Gamma(1+n/p)} t^{n/p}.$$

We construct the integral representation of normalized formal solution $\hat{u}(t,z)$ of (2), which is based upon the idea of W. Balser and M. Miyake [2]. Moreover, we show that $\hat{u}(t,z) = \hat{u}_1(t,z) + ... + \hat{u}_l(t,z)$, where $\hat{u}_j(t,z)$ satisfies the pseudodifferential equation

$$(\partial_t^{1/p} - \lambda_j(\partial_z^{1/p}))^{m_j} u_j(t,z) = 0 \quad \text{for} \quad j = 1, ..., l.$$

Next, we describe the behaviour of $\hat{u}_j(t,z)$ depending on a pole order $q_j \in \mathbb{Q}$ at infinity and a leading term $\lambda_j \in \mathbb{C} \setminus \{0\}$ of $\lambda_j(\xi)$. As a consequence, we obtain the sufficient and necessary condition for the multisummability of normalized formal solution of (2).

References:

[1] W. Balser, *Multisummability of formal power series solutions of partial differential equations with constant coefficients*, J. Differential Equations 201 (2004), 63–74.
[2] W. Balser and M. Miyake, *Summability of formal solutions of certain partial differential equations*, Acta Sci. Math. (Szeged) 65 (1999), 543–551.