Viscous Fluid Flow in Domains with Moving Boundaries

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We consider the Navier-Stokes system modelling the flow of a viscous incompressible fluid in a domain with moving boundary $\partial\Omega(t)$ and Dirichlet boundary conditions. Fixing a reference domain Ω_0, we reduce the problem via a coordinate transform $x = \phi(t,\xi) : \Omega_0 \to \Omega(t)$ to a modified non-autonomous Navier-Stokes system

$$\partial_t u(t) + A(t)u(t) = P(t)F - P(t)u \cdot \nabla^{\phi(t)} u, \quad u(0) = u_0 \quad \text{in } \ \Omega_0.$$

Here $A(t)$ is a t-dependent modified Stokes operator on $L^q_\sigma(\Omega_0)$, $P(t)$ a modified Helmholtz projection with range $L^q_\sigma(\Omega_0)$, and $\nabla^{\phi(t)}$ denotes a $\phi(t)$-dependent gradient.

To solve the initial-boundary value problem or find time-periodic solutions the construction of the fundamental operator $\{U(t,s)\}_{t>s}$ of the linear non-autonomous system

$$\partial_t u(t) + A(t)u(t) = 0,\ t > s, \quad u(s) = u_0$$

poses new problems in unbounded domains since then the operators $A(t)$ are not boundedly invertible. Another important property are t-independent estimates of $A(t)$, e.g. Sobolev embeddings for fractional Stokes operators $A(t)^\theta$ with t-independent bounds. The adjoint operators $A(t)^*$ will be analyzed similarly. The final aim is to get global-in-time estimates of $\{U(t,s)\}$ and to establish sufficiently fast decay rates.

The focus of the talk is put on the half space $\mathbb{R}^n_+$ with compact perturbations. The results are based on joint papers with K. Tsuda (Kyushu Sangyo University, Fukuoka).