Abstracts of Talks

Harmonic Analysis and Wave Phenomena
August 10 – 11, 2021

- **Matania Ben-Artzi (Hebrew University of Jerusalem)**

  Splines, biharmonic operator and approximate eigenvalues

  The biharmonic operator plays a central role in a wide array of physical models, such as elasticity theory and the streamfunction formulation of the Navier-Stokes equations. Its spectral theory has been extensively studied. In particular the one-dimensional case (over an interval) constitutes the basic model of a high order Sturm-Liouville problem. The need for corresponding numerical simulations has led to numerous works. In this talk we present a discrete biharmonic calculus. The primary object of this calculus is a high-order compact discrete biharmonic operator (DBO). The DBO is constructed in terms of the discrete Hermitian derivative. However, the underlying reason for its accuracy has surprising aspects. We discuss the strong connection between cubic spline functions (on an interval) and the DBO. The first observation is that the (scaled) fourth-order distributional derivative of the cubic spline is identical to the action of the DBO on grid functions. It is shown that the kernel of the inverse of the discrete operator is (up to scaling) equal to the grid evaluation of the kernel of $\left[\left(\frac{d}{dx}\right)^4\right]^{-1}$, and explicit expressions are presented for both kernels. As an important application, the relation between the (infinite) set of eigenvalues of the fourth-order Sturm-Liouville problem and the finite set of eigenvalues of the discrete biharmonic operator is studied. The discrete eigenvalues are proved to converge (at an "optimal" $O(h^4)$ rate) to the continuous ones. Another consequence is the validity of a *comparison principle.* It is well known that there is no maximum principle for the fourth-order equation. However, a positivity result is derived, both for the continuous and the discrete biharmonic equation, showing that in both cases the kernels are order preserving.

  Based on joint work with GUY KATRIEL.

- **Neal Bez (Saitama University)**

  Dispersive estimates for the wave equation

  Weighted oscillatory integrals associated with hypersurfaces are ubiquitous in analysis. If the hypersurface is given as the graph of ϕ, often the weight is taken to be a power of the determinant of the hessian matrix of ϕ. Such a choice of damping weight mitigates bad behaviour near degeneracies and can allow one to obtain optimal rates of decay. In this talk we consider the case $\phi(\xi) = |\xi|$ and weights of the form $|\xi|^{-\lambda}$ in a neighbourhood of the origin. Despite the determinant of the hessian vanishing everywhere and the singularity at the origin, we show how one can establish the optimal decay rate of decay. Applications of such estimates to Strichartz estimates for orthonormal families of initial data for the wave equation will be discussed too. The talk is based on joint work with Sanghyuk Lee and Shohei Nakamura.

- **Tomoya Kato (Gunma University)**

  Boundedness of bilinear pseudo-differential operators of $S_{0,0}$-type in Wiener amalgam spaces and in Lebesgue spaces

  We extend several known results about the boundedness of the bilinear pseudo-differential operators. First, we consider symbol classes of $S_{0,0}$-type that generalize the bilinear Hörmander class $S_{0,0}$ and characterize those classes for which the corresponding bilinear operators are bounded in Wiener amalgam spaces. Secondly, using the results for Wiener amalgam spaces, we prove sharp results that certain L^q or $L^{q,\infty}$ integrability of the symbol and its derivatives with respect to the frequency variables implies boundedness of the corresponding bilinear operator in Lebesgue spaces. This is a joint work with Prof. Akihiko Miyachi and Prof. Naohito Tomita.

- **Shinya Kinoshita (Saitama University)**

  Well-posedness of fractional NLS and semi-relativistic equations with Hartree type nonlinearity

  We consider the Cauchy problem of fractional NLS and semi-relativistic equations with Hartree type nonlinearity. Under some angular regularity assumption, we show the small data global well-posedness and scattering. The key tools are the U^p, V^p spaces introduced by Koch-Tataru and the classical L^2-bilinear transversal estimates.

- **Haruya Mizutani (Osaka University)**

  Kato–Yajima and Strichartz estimates for fractional and higher-order Schrödinger equations with Hardy potential

  The Kato–Yajima estimate is a kind of smoothing effects for the free Schrödinger equation and has been extended to various dispersive equations. We discuss recent progress on the Kato–Yajima estimate and its application to the Strichartz estimates for a class of generalized Schrödinger equations with Hardy potentials. This particularly extends a seminal result by Burq et al. for the Schrödinger equation with an inverse-square potential to the fractional and higher-order cases. This talk is based on joint work with Xiaohua Yao (CCNU, China).

- **Michael Reissig (Technische Universität Bergakademie Freiberg)**

  Global (in time) existence versus blow-up phenomena

  In this talk we will discuss the Cauchy problem for semi-linear evolution models with source nonlinearities of power type. The source nonlinearity allows, in general, the proof of global (in time) existence results for small data only. We discuss the question for critical exponents dividing the range of admissible exponents into a subset allows global (in time) existence results and a subset which implies blow-up phenomena (even) for small data. In the lecture we introduce recent results and explain different methods to attack different evolution models. At the end of the talk we propose some open problems.

- **Michael Ruzhansky (Universiteit Gent / Queen Mary University of London)**

  Nonharmonic operator analysis

  In this talk we will give an overview of older and note recent results on the nonharmonic analysis of operators.

- **Joachim Toft (Linnæus University)**

  Schatten-von Neumann properties of Weyl operators of Hörmander type

  Let $t \in \mathbf{R}$ be fixed and consider the *pseudo-differential operators* $\mathrm{Op}_t(a)$ with *symbol* a which is defined by the formula:

$$\mathrm{Op}_t(a)f(x) \equiv (2\pi)^{-n} \iint_{\mathbf{R}^n \times \mathbf{R}^n} a((1-t)x + ty, \xi) f(y) e^{i\langle x-y,\xi\rangle}\, dyd\xi$$

  It is well-known that if $0 \le \delta < \rho \le 1$ and $r \in \mathbf{R}$, then each $\mathrm{Op}_t(a)$ with $a \in S^r_{\rho,\delta}(\mathbf{R}^{2n})$ is L^2-continuous, if and only if $S^r_{\rho,\delta} \subseteq L^\infty$ (i.e. $r \le 0$). Here $S^r_{\rho,\delta}(\mathbf{R}^{2n})$ consists of all $a \in C^\infty(\mathbf{R}^{2n})$ such that

$$|\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| \le C_{\alpha,\beta}(1+|\xi|)^{r-\rho|\beta|+\delta|\alpha|}.$$

  Recently, results which are focused on "individual symbols" instead of whole symbol classes can be found in e.g. [1], from which it follows that if $a \in S^r_{\rho,\delta}$ for some r, then $\mathrm{Op}_t(a)$ is L^2-continuous, if and only if $a \in L^\infty$.

  The general theory involving these results, is formulated within the Hörmander-Weyl calculus, where the symbol classes $S(m,g)$ are parameterized with weight functions m and Riemannian metrics g. The continuity investigations also involve Schatten properties. Especially, the following general result is deduced: Let $p \in [1,\infty]$, and that the g-Planck's function h_g satisfies $h_g^N m \in L^p$, for some $N \ge 0$. Then $\mathrm{Op}_t(a)$ is a Schatten-p operator, if and only if $a \in L^p$.

  Recently, a related result were obtained also when $p \le 1$. More precisely, in [2] it is proved that if $p \in (0,1]$, $m \in L^p$ and $a \in S(m,g)$, then $\mathrm{Op}_t(a)$ is a Schatten-von Neumann operator of order p.

  In the talk we explain these results with explicit examples, and present some ideas of some proofs.

  [1] E. Buzano, J. Toft *Schatten-von Neumann properties in the Weyl calculus*, J. Funct. Anal. **259** (2010), 3080–3114.

  [2] J. Toft *Continuity and compactness for pseudo-differential operators with symbols in quasi-Banach spaces or Hörmander classes*, Anal. Appl. **15** (2017), 353–389.

- **Baoxiang Wang (Peking University)**

  Nonlinear evolution equations in super-critical spaces

  We consider the Cauchy problem for the Navier-Stokes, semi-linear heat, nonlocal NLS and KdV equations and obtain some global existence and uniqueness results in certain supercritical spaces. For instance, we get some global existence and uniqueness results for the semilinear heat equation when the nonlinear powers are less than or equal to Fujita's critical indices in one and two spatial dimensions. This is a joint work with Dr. Jie Chen.