Global Weak Solutions of the Navier-Stokes Equations with Nonhomogeneous Data

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The instationary Navier-Stokes system possesses for any bounded domain $\Omega \subset \mathbb{R}^3$, every solenoidal initial value $u_0 \in L^2(\Omega)$ and external force f in L^2 at least one global in time weak solution with vanishing boundary data in the sense of J. Leray and E. Hopf. This solution satisfies an energy inequality and a global energy estimate leading to a bounded kinetic energy and a bounded dissipation energy in time.

In the talk we will address the problem of existence of weak Leray-Hopf type solutions when the boundary data are nonzero and even the divergence $k = \operatorname{div} u$ may be prescribed. We show the existence of at least one global in time weak solution satisfying a modified energy inequality and energy estimate leading - in the worst case - to an exponential blow up in time. Under certain assumptions on the data related to the topology of the domain an exponential blow up may be avoided. This latter problem is related to the famous open question on the existence of stationary solutions with nonhomogeneous boundary data in multiply connected domains.