

# Program

## 14 February (Saturday)

13:30 – 14:20 Yuuki Tadokoro (Kisarazu National College of Technology)  
The period matrix of the hyperelliptic curve  $w^2 = z^{2g+1} - 1$

14:30 – 15:20 Yuichi Kabaya (Kyoto University)  
Exotic components in linear slices of quasi-Fuchsian groups

15:50 – 16:40 Masakazu Shiba (Hiroshima University)  
Some new problems in the theory of conformal mappings of an open Riemann surface of finite genus

## 15 February (Sunday)

9:00 – 9:50 Hirokazu Shimauchi (Tohoku University)  
Numerical quasiconformal mappings by certain linear systems

10:00 – 10:50 Lijie Sun (Tohoku University)  
Notes on complex hyperbolic triangle groups of type  $(m, n, \infty)$

11:10 – 12:00 Daisuke Yamaki (Tokyo Institute of Technology)  
Holomorphic 1-cochains and combinatorial periods

Lunch

13:30 – 14:20 Yohei Komori (Waseda University)  
Projective embeddings of the Teichmüller spaces

14:30 – 15:20 Dariusz Partyka (The John Paul II Catholic University of Lublin)  
The Schwarz type inequalities for harmonic mappings in the unit disc with boundary normalization

15:50 – 16:40 Ken-ichi Sakan (Osaka City University)  
On quasiconformality and some properties of harmonic mappings in the unit disk

Banquet

## 16 February (Monday)

9:00 – 9:50 Masahiro Yanagishita (Waseda University)  
Complex analytic structure on the  $p$ -integrable Teichmüller space

10:00 – 10:50 Katsuhiko Matsuzaki (Waseda University)  
The barycentric extension of circle diffeomorphisms

11:10 – 12:00 Yi Huang (The University of Melbourne)  
Flipping numbers and curves

Lunch

13:30 – 14:20 Ryuji Abe (Tokyo Polytechnic University)  
Diophantine approximation via Gaussian integers

14:30 – 15:20 Sachiko Hamano (Fukushima University)  
On the reproducing kernel for the space of semi-exact analytic differentials

15:50 – 16:40 Yukiitaka Abe (Toyama University)  
Analytic study of singular curves

# Abstract

**Yuuki Tadokoro** (Kisarazu National College of Technology)

**The period matrix of the hyperelliptic curve  $w^2 = z^{2g+1} - 1$**

Our talk consists of two parts. First, we explicitly obtain the period matrix of the hyperelliptic curve defined by the affine equation  $w^2 = z^{2g+1} - 1$ , its entries being elements of the  $(2g + 1)$ -st cyclotomic field. Second, we introduce an algorithm for obtaining the period matrix for a compact Riemann surface, which is a  $p$ -cyclic covering of  $\mathbb{C}P^1$  branched over 3 points.

**Yuichi Kabaya** (Kyoto University)

**Exotic components in linear slices of quasi-Fuchsian groups**

The linear slice of quasi-Fuchsian punctured torus groups is defined by fixing the length of some simple closed curve to be a fixed positive real number. It is known that the linear slice is a union of disks, and it has one standard component containing Fuchsian groups. Komori-Yamashita proved that there exist non-standard components if the length is sufficiently large. In this talk, I give another proof based on the theory of complex projective structures.

**Masakazu Shiba** (Hiroshima University)

**Some new problems in the theory of conformal mappings of an open Riemann surface of finite genus**

Let  $R$  be an open (=noncompact) Riemann surface of finite genus  $g$ . If a closed (=compact) Riemann surface  $R'$  of genus  $g$  contains  $R$  as a subregion,  $R'$  is historically called a “compact continuation of the same genus” of  $R$ , but we prefer to use a shorter term a “closing.” We give a precise definition in modern terminology and construct a closing of  $R$  with a remarkable hydrodynamic property. These closings are used to comprehend the totality  $\mathcal{C}$  of the closings of  $R$ ; if  $g = 1$  in particular, we use the modulus of a torus to describe  $\mathcal{C}$  as a closed disk  $M$  in  $\mathbb{H}$ . We generalize this result to  $g > 1$ . The hyperbolic diameter  $\sigma_H(R)$  of  $M$  is called the hyperbolic span of  $R$ . If  $R = R_t$  moves holomorphically so that the set  $\{(R_t, t) \mid t \in \mathbb{D}\}$  is pseudoconvex,  $\sigma_H(R_t)$  is a subharmonic function.

**Hirokazu Shimauchi** (Tohoku University)

**Numerical quasiconformal mappings by certain linear systems**

In this talk, we propose a numerical method for quasiconformal self mappings of the unit disk. The unit disk is triangulated in a simple way and the quasiconformal mappings are approximated by piecewise linear mappings. The images of the vertices of the triangles are defined by an overdetermined system of linear equations. Further the sequence of the approximation converges to the true solution, at least in the case where the Beltrami coefficients are in  $C^1$ . We will also present several numerical experiments. This talk is based on a joint work with R. Michael Porter (CINVESTAV).

**Lijie Sun** (Tohoku University)

**Notes on complex hyperbolic triangle groups of type  $(m, n, \infty)$**

The triangle groups are not necessarily discrete in complex hyperbolic space which is different from the real hyperbolic case. Many authors investigated the discreteness of ideal triangle groups and the triangle groups of type  $(n, n, \infty)$ . The difficult point for giving discrete cases is that there are no totally geodesic real hypersurfaces in  $\mathbb{H}_{\mathbb{C}}^2$ . In this talk we mainly consider the complex hyperbolic triangle groups of type  $(m, n, \infty)$  and give some discrete cases using the complex hyperbolic version of Klein's combination theorem. From the results more explicit conclusions about non-discrete triangle groups of type  $(m, \infty, \infty)$  will also be given.

**Daisuke Yamaki** (Tokyo Institute of Technology)

**Holomorphic 1-cochains and combinatorial periods**

We discuss holomorphic 1-cochains and periods of holomorphic 1-cochains. Holomorphic 1-cochains are defined on Riemann surfaces with triangulations and satisfy Riemann's bi-linear relation. Using holomorphic 1-cochains, Wilson defined combinatorial period matrices and showed that for a triangulated Riemann surface, the combinatorial period matrix converges to the (conformal) period matrix as the mesh of the triangulation tends to zero. In this talk, we give another relation between combinatorial period matrices and (conformal) period matrices and study its applications.

**Yohei Komori** (Waseda University)

**Projective embeddings of the Teichmüller spaces**

Let  $X$  be an orientable hyperbolic surface of genus  $g$  with  $n$  punctures and  $r$  holes. Then the Teichmüller space  $\mathcal{T}(X)$  of  $X$  is homeomorphic to the real affine space  $V$  of  $\dim V = 6g - 6 + 2n + 3r$ . I have been considering the following question:

Can we find  $\dim V + 1$ -number of simple closed geodesics whose hyperbolic lengths embed  $\mathcal{T}(X)$  into the finite dimensional real projective space  $P(V)$ ?

Because of the PL-Structure of the Thurston boundary, we might expect that the image of  $\mathcal{T}(X)$  should be the interior of some convex polyhedron in  $P(V)$ .

In this talk I will answer this question for surfaces having at least one hole, with few exceptional cases.

**Dariusz Partyka** (The John Paul II Catholic University of Lublin)

**The Schwarz type inequalities for harmonic mappings in the unit disc with boundary normalization**

This talk is intended to give an exposition of the Schwarz type inequalities for harmonic self-mappings of the unit disc with certain additional properties. However this time the classical normalization condition, with the origin as a fixed point, is replaced by certain boundary conditions. In particular, the case is considered, where a harmonic mapping is injective and has a continuous extension to the closed unit disk which keeps the cube roots of unity fixed. Some other cases of this type are also discussed, especially in the context of quasiconformal mappings.

**Ken-ichi Sakan** (Osaka City University)

### **On quasiconformality and some properties of harmonic mappings in the unit disk**

In this talk we give a summary of our results on quasiconformality and some properties of harmonic mappings in the unit disk which have been obtained jointly with D. Partyka. To begin with we first give brief explanations of Lewy's theorem and Radó-Kneser-Choquet theorem. Next we state (A) (primitive) Schwarz's lemma for harmonic mappings. Moreover, for sense-preserving injective harmonic mappings of the unit disk onto itself, we state (B) (primitive) Heinz's inequality and (C) a theorem by Pavlović on quasiconformality of such mappings. We then explain that under appropriate assumptions we could obtain many improved or modified forms of the results (A), (B) and (C), respectively.

**Masahiro Yanagishita** (Waseda University)

### **Complex analytic structure on the $p$ -integrable Teichmüller space**

The  $p$ -integrable Teichmüller space is a metric subspace of the Teichmüller space of Teichmüller equivalence classes containing Beltrami coefficients with finite hyperbolic  $L^p$ -norm. If a Riemann surface  $R$  is analytically finite, then the  $p$ -integrable Teichmüller space of  $R$  coincides with the Teichmüller space of  $R$ . Hence, this study has a significance for Riemann surfaces of analytically infinite type. Cui, Takhtajan-Teo and Tang considered the complex analytic structure on the  $p$ -integrable Teichmüller space of the unit disk for  $p \geq 2$ . In this talk, we extend their results to the case of hyperbolic Riemann surfaces.

**Katsuhiko Matsuzaki** (Waseda University)

### **The barycentric extension of circle diffeomorphisms**

The barycentric extension due to Douady and Earle gives a conformally natural extension of a quasimetric automorphism of the circle to a quasiconformal automorphism of the unit disk. In this talk, we consider such extensions for circle diffeomorphisms of Hölder continuous derivatives and show that this operation is continuous with respect to an appropriate topology for the space of corresponding Beltrami coefficients.

**Yi Huang** (The University of Melbourne)

### **Flipping numbers and curves**

Solutions to the equation  $x^2 + y^2 + z^2 = xyz$  satisfy the following property: given one solution  $(x, y, z)$ , we can easily write down a new "flipped" solution given by  $(x, y, xy - z)$ . In particular, this means that an integer solution is flipped to another integer solution. These integer solutions are well-known as Markoff triples, and arise in beautiful results in geometry and number theory.

In recent work with Paul Norbury, we discover similar phenomena for solutions to the equation  $(a+b+c+d)^2 = abcd$  — called Markoff quads. We begin with a gentle motivating survey of several famous results related to Markoff triples, before introducing their Markoff quad analogues.

**Ryuji Abe** (Tokyo Polytechnic University)

**Diophantine approximation via Gaussian integers**

The Markoff spectrum for the rational number field  $\mathbb{Q}$  is defined by means of the minimum of binary indefinite quadratic forms with real coefficients and the Lagrange spectrum is defined with respect to approximation of real numbers by rational ones. It is well-known that the discrete parts of them coincide.

In this talk, we show that there exists an analogy between the Markoff spectrum for the imaginary quadratic number field  $\mathbb{Q}(i)$  and the Lagrange spectrum by rational numbers of Gaussian integers, using a geometric characterization of the Markoff spectrum for  $\mathbb{Q}(i)$  by simple closed geodesics in an immersed totally geodesic twice punctured torus in the Borromean rings complement.

**Sachiko Hamano** (Fukushima University)

**On the reproducing kernel for the space of semi-exact analytic differentials**

We shall discuss the reproducing kernel for the Hilbert space  $S(R)$  of all semi-exact  $L^2$ -analytic differentials on a finite bordered Riemann surface  $R$ . We show that the Bergman kernel restricted to  $S(R)$  has a close relation to the  $L_1$ -constant with two logarithmic poles, and then discuss a problem related to a conjecture of Suita type for  $S(R)$ .

**Yukitaka Abe** (Toyama University)

**Analytic study of singular curves**

We study singular curves from analytic point of view. The classical theory of compact Riemann surfaces and their Jacobi varieties was generalized to singular curves and generalized Jacobi varieties by algebraic way. It seems to us that there is no analytic study of them. We treat singular curves and generalized Jacobi varieties completely analytically. We give analytic proofs of the Serre duality and generalized Abel's theorem without any help from algebra. Generalized Jacobi varieties are considered as complex Lie groups. We investigate their properties.