Program

9 November (Saturday)

14:00–14:50 Lizhen Ji (University of Michigan)
Spines of Teichmuller spaces and symmetric spaces

15:00–15:50 Yoshihiko Shinomiya (Tokyo Institute of Technology)
Periodic points on Veech surfaces

16:00–16:50 Chikako Mese (Johns Hopkins University)
Harmonic maps in rigidity problems

Banquet

10 November (Sunday)

10:00–10:50 Makoto Masumoto (Yamaguchi University)
On the existence of holomorphic mappings of once-holed tori

11:00–11:50 Hideki Miyachi (Osaka University)
Rigidity of isometries on Teichmueller space at infinity

Lunch

14:00–14:50 Hiroshige Shiga (Tokyo Institute of Technology)
On deformations spaces of Kleinian groups

15:00–15:50 Yu Kawakami (Yamaguchi University)
On function-theoretic properties for Gauss maps of several classes of surfaces

16:00–16:50 Yuriko Umemoto (Osaka City University)
Growth rates of cocompact hyperbolic Coxeter groups and 2–Salem numbers

11 November (Monday)

10:00–10:50 Masanori Amano (Tokyo Institute of Technology)
On behavior of pairs of Teichmüller geodesic rays

11:00–11:50 Tanran Zhang (Tohoku University)
Uniformisation and description of a once-punctured annulus

Lunch

14:00–14:50 Ryosuke Mineyama (Osaka University)
Limit sets of Coxeter groups of type (n-1,1)

15:00–15:50 Ken'ichi Ohshika (Osaka University)
Primitive stable closed hyperbolic 3-manifolds

Abstract

Lizhen Ji (University of Michigan)
Spines of Teichmuller spaces and symmetric spaces
Abstract: Let T_g be the Teichmuller space of a compact surface S_g of genus g, and Mod_g the mapping class group of S_g. Then Mod_g acts properly on T_g, and the quotient $Mod_g\ T_g$ is the moduli space of compact Riemann surfaces of genus g. This action of Mod_g on T_g is an analogue of the action of an arithmetic subgroup Γ of a semisimple Lie group G on the associated symmetric space $X = G/K$, where K is a maximal compact subgroup of G.

A longstanding open problem concerns spines of T_g, i.e., equivariant deformation retracts of T_g with compact quotient by Mod_g and of dimension equal to the virtual cohomological dimension of Mod_g. Similarly, when Γ is a nonuniform arithmetic subgroup, existence of spines of X is also open in general.

In this talk, I will describe the history of these problems (for example, Thurston's attempt) and some recent results on them.

Yoshihiko Shinomiya (Tokyo Institute of Technology)
Periodic points on Veech surfaces
Abstract: We will discuss periodic points on Veech surfaces. A periodic point on a Veech surface is a point whose orbit under the affine group is finite. It is known that the number of periodic points on a non-arithmetic Veech surface is finite. We will give upper bounds of the numbers of periodic points depending only on the types of Veech surfaces and signatures of the Veech groups.

Chikako Mese (Johns Hopkins University)
Harmonic maps in rigidity problems
Abstract: We discuss harmonic maps into non-positively curved metric spaces (NPC spaces). Of particular interest is the regularity for these maps into special classes of spaces that include the Euclidean and Hyperbolic buildings and Weil-Petersson completion of Teichmuller space. As an application of the regularity theory, we study rigidity questions.

Makoto Masumoto (Yamaguchi University)
On the existence of holomorphic mappings of once-holed tori
Abstract: We address the existence problem of handle-preserving holomorphic mappings of once-holed tori into a given Riemann surface of positive genus. The once-holed tori allowing such mappings form a subset of the Teichmüller space of a once-holed torus. We are particularly interested in geometric properties of the set.

By a *once-holed torus* we mean a noncompact Riemann surface of genus one with exactly one (Kerékjártó-Stoïlow) boundary component. For example, the Riemann surface obtained from a compact Riemann surface of genus one, or a *torus*, by removing one point is a once-holed torus, which will be referred to as a *once-punctured torus*.

Let R be a Riemann surface of positive genus; it may be compact or the genus may be infinite. A *mark of handle* of R means an ordered pair $\chi = \{a, b\}$ of simple loops a and b on R whose intersection number $a \times b$ is equal to one. The pair $Y = (R, \chi)$ is said to be a Riemann surface *with marked handle*. Since the genus of R is positive, the surface has one or more handles. We choose just one of them and mark it with a pair of simple loops.

Let $Y' = (R', \chi')$, where $\chi' = \{a', b'\}$, be another Riemann surface with marked handle. If $f : R \to R'$ is continuous and maps a and b onto loops freely homotopic to a' and b' on R', respectively, then we say that f is a continuous mapping of Y into Y' and use the notation $f : Y \to Y'$. If $f : R \to R'$ possesses some additional properties, then $f : Y \to Y'$ is said to have the same properties. For example, if $f : R \to R'$ is conformal, that is, if $f : R \to R'$ is holomorphic and injective, then f is called a conformal mapping of Y into Y'.

A once-holed torus (resp. torus, once-punctured torus) with marked handle is usually called a marked once-holed torus (resp. marked torus, marked once-punctured torus). Let $\mathfrak{T}$ be the set of marked once-holed tori, where two marked once-holed tori are identified with each other if there is a conformal mapping of one *onto* the other.

We introduce a global coordinate system on $\mathfrak{T}$ as follows. For a marked once-holed torus $X = (T, \chi)$, where $\chi = \{a, b\}$, set $\Lambda(X) = (\lambda_1, \lambda_2, \lambda_3)$, where λ_1, λ_2 and λ_3 are the extremal lengths of the free homotopy classes of a, b and ab^{-1}, respectively. Then Λ defines an injective mapping of $\mathfrak{T}$ into $\mathbb{R}^3_+$, whose image is

$$\Lambda(\mathfrak{T}) = \{(\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3_+ \mid \xi_1^2 + \xi_2^2 + \xi_3^2 - 2(\xi_1\xi_2 + \xi_2\xi_3 + \xi_3\xi_1) + 4 \leqq 0\}.$$

Identifying $\mathfrak{T}$ with $\Lambda(\mathfrak{T})$, we consider $\mathfrak{T}$ as a 3-dimensional real analytic manifold with boundary. A marked once-holed torus lies on the boundary if and only if it is a marked once-punctured torus.

As a set, $\mathfrak{T}$ is the union of the Teichmüller space of a once-punctured torus and the reduced Teichmüller space of a once-holed torus which is not a once-punctured torus. The real analytic structure on $\mathfrak{T}$ is compatible with the real analytic structures on those Teichmüller spaces. We will call $\mathfrak{T}$ the *Teichmüller space of a once-holed torus.*

Now, fix a Riemann surface Y_0 with marked handle. We are interested in the set $\mathfrak{T}_a[Y_0]$ (resp. $\mathfrak{T}_c[Y_0]$) of marked once-holed tori $X \in \mathfrak{T}$ for which there is a holomorphic (resp. conformal) mapping of X into Y_0. Clearly, $\mathfrak{T}_c[Y_0]$ is nonempty and included in $\mathfrak{T}_a[Y_0]$.

THEOREM 1. *The sets $\mathfrak{T}_a[Y_0]$ and $\mathfrak{T}_c[Y_0]$ are noncompact closed domains with Lipschitz boundary.*

Our next result is expressed in terms of another global coordinate system on $\mathfrak{T}$. Every marked once-holed torus is realized as a horizontal slit domain of a marked torus. To be more specific let $\mathbb{H}$ denote the upper half-plane. For any $\tau \in \mathbb{H}$ let G_τ be the additive group generated by 1 and τ, and set $T_\tau = \mathbb{C}/G_\tau$, which is a torus. The oriented segments $[0, 1]$ and $[0, \tau]$ are projected onto simple loops a_τ and b_τ on T_τ, respectively, which make a mark χ_τ of handle of T_τ. We set $X_\tau = (T_\tau, \chi_\tau)$. Let $\pi_\tau : \mathbb{C} \to T_\tau$ be the natural projection. Cutting T_τ along the image $\pi_\tau([0, s])$ of the segment $[0, s]$, where $0 \leqq s < 1$, we obtain a once-holed torus $T_\tau^{(s)} := T_\tau \setminus \pi_\tau([0, s])$. It is a horizontal slit domain of the torus T_τ. Note that $T_\tau^{(0)}$ is a once-punctured torus. Choose a mark $\chi_\tau^{(s)} = \{a_\tau^{(s)}, b_\tau^{(s)}\}$ of handle of $T_\tau^{(s)}$ so that the inclusion mapping $T_\tau^{(s)} \hookrightarrow T_\tau$ is a conformal mapping of $X_\tau^{(s)} := (T_\tau^{(s)}, \chi_\tau^{(s)})$ into X_τ. Then the correspondence $(\tau, s) \mapsto X_\tau^{(s)}$ is a homeomorphism of $\mathbb{H} \times [0, 1)$ onto $\mathfrak{T}$, whose restrictions to $\mathbb{H} \times (0, 1)$ and to $\mathbb{H} \times \{0\}$ are real analytic. Note that $1/\operatorname{Im}\tau$ is exactly the extremal length of the free homotopy class of $a_\tau^{(s)}$.

THEOREM 2_a. *There is a nonnegative real number $\lambda_a[Y_0]$ such that* (i$_a$) *if* $\operatorname{Im}\tau \geqq 1/\lambda_a[Y_0]$, *then there are no holomorphic mappings of* $X_\tau^{(s)}$ *into* Y_0 *for any* $s \in [0,1)$, *while*

(ii$_a$) *if* $\operatorname{Im}\tau < 1/\lambda_a[Y_0]$, *then there are holomorphic mappings of* $X_\tau^{(s)}$ *into* Y_0 *for some* $s \in [0,1)$,

where $1/0 = +\infty$.

For the existence of conformal mappings of marked once-holed tori, we have the following theorem. It is quite similar to the previous theorem though the sign of equality does not appear in (i$_c$).

THEOREM 2$_c$. *There is a positive real number* $\lambda_c[Y_0]$ *such that*

(i$_c$) *if* $\operatorname{Im}\tau > 1/\lambda_c[Y_0]$, *then there are no conformal mappings of* $X_\tau^{(s)}$ *into* Y_0 *for any* $s \in [0,1)$, *while*

(ii$_c$) *if* $\operatorname{Im}\tau < 1/\lambda_c[Y_0]$, *then there are conformal mappings of* $X_\tau^{(s)}$ *into* Y_0 *for some* $s \in [0,1)$.

Finally, we evaluate the critical extremal lengths $\lambda_a[Y_0]$ and $\lambda_c[Y_0]$. Let $Y_0 = (R_0, \chi_0)$, where $\chi_0 = \{a_0, b_0\}$. Let $\lambda[Y_0]$ stand for the extremal length of the free homotopy class of a_0. If R_0 is not a torus, then it carries a hyperbolic metric. We denote by $l[Y_0]$ the length of the geodesic freely homotopic to a_0, where the curvature is normalized to be -1. If R_0 is a torus, then we define $l[Y_0] = 0$.

THEOREM 3. *It holds that* $\lambda_a[Y_0] = \dfrac{1}{\pi} l[Y_0]$ *and* $\lambda_c[Y_0] = \lambda[Y_0]$.

It follows that $\lambda_a[Y_0] < \lambda_c[Y_0]$ for any Y_0. Also, $\lambda_a[Y_0]$ is strictly positive unless Y_0 is a marked torus.

Hideki Miyachi (Osaka University)
Rigidity of isometries on Teichmueller space at infinity
Abstract: In this talk, I will give a rigidity result for isometries with respect to the Teichmueller distance on Teichmueller space of Riemann surfaces of analytically finite type. Indeed, we will provide mappings acting on Teichmueller space which are close to isometries at infinity, and discuss properties of the mappings. If time permits, we will re-prove Ivanov's theorem, which says that except for few cases, the isometry group of Teichmuller space is isomorphic to the extended mapping class group.

Hiroshige Shiga (Tokyo Institute of Technology)
On deformations spaces of Kleinian groups
Abstract: Let G be a non-elementary Kleinian group. We consider the space of quasiconformal deformations of G. The space has a natural complex structure and it is finite dimensional if G is finitely generated. In this talk, we consider complex analytic properties of the spaces, which are related to some results by Bers, Kra-Maskit and McMullen.

Yu Kawakami (Yamaguchi University)
On function-theoretic properties for Gauss maps of several classes of surfaces
Abstract: The aim of this talk is to reveal the geometric background of function-theoretic properties for Gauss maps of several classes of immersed surfaces in space forms (e.g. minimal surfaces in the Euclidean 3-space, flat surfaces in the hyperbolic 3-space etc.). For the purpose, we give an optimal curvature bound for a specified conformal metric on an open Riemann surface and give some applications.

Yuriko Umemoto (Osaka City University)
Growth rates of cocompact hyperbolic Coxeter groups and 2–Salem numbers
Abstract: The group generated by reflections with respect to facets of a Coxeter polytope in n–dimensional hyperbolic space $\mathbb{H}^n$ is called a hyperboric Coxeter group. By the results of Cannon, Wagreich and Parry, it is known that the growth rate of a cocompact Coxeter group in $\mathbb{H}^2$ and $\mathbb{H}^3$ is a Salem number. On the other hand, Kerada defined a j–Salem number, which is a generalization of a Salem number. In this talk, I will present that we realize infinitely many 2–Salem numbers as the growth rates of cocompact Coxeter groups in $\mathbb{H}^4$. Our Coxeter polytopes are constructed by successive gluing of Coxeter polytopes which we call Coxeter dominoes.

Masanori Amano (Tokyo Institute of Technology)
On behavior of pairs of Teichmüller geodesic rays
Abstruct: In this talk, we obtain the explicit limit value of the Teichmüller distance between two Teichmüller geodesic rays which are determined by Jenkins-Strebel differentials having a common end point in the augmented Teichmüller space. Furthermore, we also obtain a condition under which these two rays are asymptotic. This is the Teichmüller space varsion of a result of Farb and Masur for the moduli space.

Tanran Zhang (Tohoku University)
Uniformisation and description of a once-punctured annulus
Abstract: The Uniformisation Theorem shows that the universal covering space $\widetilde{X}$ of an arbitrary Riemann surface X is homeomorphic, by a conformal map $\mathfrak{m}$, to either the Riemann sphere $\hat{\mathbb{C}}$, the complex plane $\mathbb{C}$ or the unit disk $\mathbb{D}$. And then the fundamental group $\Pi_1(X)$ has a representation as a group G of conformal homeomorphisms of $\mathfrak{m}(\widetilde{X})$. This theorem also indicates that if $\widetilde{X}$ is homeomorphic to a proper subset of $\mathbb{C}$ with at least three boundary points, then $\widetilde{X}$ is conformally equivalent to a quotient space $\mathbb{D}/G$, where G is a torsion-free Fuchsian group that acts (discontinuously) on $\mathbb{D}$ (or $\mathbb{H}$). The group G is isomorphic to $\Pi_1(X)$. Hempel and Smith studied the hyperbolic Riemann surface model of the twice-punctured disk $\mathbb{D}\backslash\{p_1, p_2\}$ in 1980s. They estimated the hyperbolic density on it near aone puncture and considered the coalescing of the two punctures. Later on Beardon gave five different ways to uniformize $\mathbb{D}\backslash\{p_1, p_2\}$ in 2012. He investigated several conformal invariants to characterize $\mathbb{D}\backslash\{p_1, p_2\}$ considering the fundamental domain, symmetric collars and extremal length. We extend his work to the once-punctured annulus $A := \{z : 1/R < |z| < R\}\backslash\{a\}$, $R > 1$, $1/R < a < R$. We provide several parameter pairs to uniformize and characterize it. The main tools we use are Möbius transformations, covering space, homotopy classes and elliptic integrals.

References

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2. A.F. Beardon, *The uniformisation of a twice-punctured disc*, Comput. Methods Funct. Theory **12** (2012), no. 2, 585–596.

3. J.A. Hempel and S.J. Smith, *Uniformization of the twice-punctured disc - problems of confluence*, Bull. Australian Math. Soc. **39** (1989), 369–387.

Ryosuke Mineyama (Osaka University)
Limit sets of Coxeter groups of type (n-1,1)
Abstract: Recentry Hohlweg, Labbe, Ripoll introduced a non-linear action of Coxeter groups to investigate asymptotic behavior of their roots. This turns out to be a discrete action on a CAT(0) space in the case that associating bilinear form of the Coxeter group has singnature (n-1,1). I am interested in how geometric aspects of Coxeter groups are mirrored on their limit sets. In this talk we discuss the existence of Cannon-Thurston maps from Gromov boundaries of Coxeter groups to their limit sets. If we have the time left, we observe a relationship between limit sets and sets of accumulation points of roots. This talk partially based on the joint work with Akihiro Higashitani and Norihiro Nakashima.

Ken'ichi Ohshika (Osaka University)
Primitive stable closed hyperbolic 3-manifolds
Abstract: This is joint work with Cyril Lecuire and Inkang Kim. We show that every Heegaard splitting with large Hempel distance and bounded combinatorics induces a primitive stable representation of a free group. This implies that every point on the boundary of the Schottky space can be approximated by unfaithful primitive stable representations corresponding to closed hyperbolic 3-manifolds.