# **RIEMANN SURFACES AND DISCONTINUOUS GROUPS 2013**

Osaka Univeristy, January 12-14, 2013 Organaizers: Hiroshige Shiga (Tokyo Institute of Technology) Tamas Kalman (Tokyo Institute of Technology GE) Kentaro Ito (Nagoya University) Hideki Miyachi (Osaka University)

This is an annual conference on topics of Riemann surfaces and discontinuous groups, including geometric function theory, potential theory, Teichmuller theory and hyperbolic geometry. This conference is partially supported by the Global Edge Institute at Tokyo Tech and the "Program to Promote the Tenure Track System" (テニュアトラック普及・定着事業) of the Ministry of Education, Culture, Sports, Science & Technology (文部科学省), and Grant-in-Aid for Scientific Research (A) 22244005.

**Venue** : Graduate school of Science, Building E, Room E301 (Third floor) Osaka University, Machikaneyama 1-1, Toyonaka, Osaka

### INVITED SPEAKERS

Ege Fujikawa (Chiba University) Yuki Iguchi (Tokyo Institute of Technology) Yuichi Kabaya (Osaka University) David Kalaj (University of Montenegro) Eiko Kin (Osaka University) Erina Kinjo (Tokyo Institute of Technology) Hidetoshi Masai (Tokyo Institute of Technology) Katsuhiko Matsuzaki (Waseda University) Toshihiro Nogi (Osaka City University) Kasra Rafi (University of Toronto) Ken-ichi Sakan (Osaka City University) Masaharu Tanabe (Tokyo Institute of Technology) Masahiro Yanagishita (Waseda University)

#### Program

# January 12 (Saturday).

13:30–14:20 Ege Fujikawa (Chiba University)

The order of periodic elements of the asymptotic Teichmuller modular group

14:30–15:20 Kasra Rafi (University of Toronto) Geometry of Teichmüller space (part I)

15:40–16:30 Toshihiro Nogi (Osaka City University) On extendibility of a map induced by Bers isomorphism

16:40–17:30 Eiko Kin (Osaka University) Minimal dilatations of pseudo-Anosovs generated by the magic 3-manifold and their asymptotic behavior

### January 13 (Sunday).

10:00–10:50 Yuichi Kabaya (Osaka University) Parametrization of  ${\rm PGL}(n,\mathbb{C})\text{-representations of surface groups}$ 

11:00–11:50 Hidetoshi Masai (Tokyo Institute of Technology) On commensurability of fibrations on a hyperbolic 3-manifold

Lunch Break

13:30–14:20 Ken-ichi Sakan (Osaka City University) Quasiconformal and Lipschitz harmonic mappings of the unit disk onto bounded convex domains

14:30–15:20 David Kalaj (University of Montenegro) Energy-minimal diffeomorphisms between doubly connected Riemann surfaces

15:40–16:30 Erina Kinjo (Tokyo Institute of Technology) On the length spectrum metric in infinite-dimensional Teichmuller spaces

16:40–17:30 Kasra Rafi (University of Toronto) Geometry of Teichmüller space (part II)

# January 14 (Monday).

10:00–10:50 Yuki Iguchi (Tokyo Institute of Technology) On accumulation points of geodesics in Thurston's boundary of Teichmüller spaces

11:00–11:50 Masaharu Tanabe (Tokyo Institute of Technology) On the combinatorial Hodge star operator and holomorphic cochains

# Lunch Break

13:30–14:20 Masahiro Yanagishita (Waseda University) Teichmüller distance and Kobayashi distance on subspaces of the universal Teichmüller space

14:30–15:20 Katsuhiko Matsuzaki (Waseda University) Circle diffeomorphisms and Teichmüller spaces

#### Abstruct

# Ege Fujikawa.

Title. The order of periodic elements of the asymptotic Teichmuller modular group

*Abstruct.* We give a sufficient condition for an asymptotic Teichmuller modular transformation to be of finite order. Furthermore, we estimate the order by using hyperbolic geometry.

### Yuki Iguchi.

*Title.* On accumulation points of geodesics in Thurston's boundary of Teichmüller spaces

Abstract. In this talk, we detect accumulation points of arbitrary Teichmüller geodesic rays in Thurston's compactification of a Teichmüller space. We showed that accumulation points of a ray are written as a sum of measured foliations supported on the partitions of minimal decomposition of the vertical foliation associated with the ray. We also showed that there exists a boundary point to which no ray accumulates.

### Yuichi Kabaya.

*Title.* Parametrization of  $PGL(n, \mathbb{C})$ -representations of surface groups

Abstruct. Let S be a surface of genus g with n boundary components. The interior of S is ideally triangulated into 2(2g-2+n) ideal triangles. Fock and Goncharov gave a parametrization of (framed) PGL $(n, \mathbb{C})$ -representations of the fundamental group of S by (n-1)(n-2)/2 parameters for each ideal triangle and (n-1) parameters for each edge of the triangulation. In this talk, we will give a parametrization of PGL $(n, \mathbb{C})$ -representations as an analogue of the Fenchel-Nielsen coordinates using the Fock-Goncharov coordinates. This is joint work with Xin Nie.

### David Kalaj.

 $\it Title.$  Energy-minimal diffeomorphisms between doubly connected Riemann surfaces

Abstruct. Let  $N = (\Omega, \sigma)$  and  $M = (\Omega^*, \rho)$  be doubly connected Riemann surfaces and assume that  $\rho$  is a smooth metric with bounded Gauss curvature  $\mathcal{K}$  and finite area. We establishes the existence of homeomorphisms between  $\Omega$  and  $\Omega^*$  that minimize the Dirichlet energy. Among all homeomorphisms  $f: \Omega \to \Omega^*$  between doubly connected domains such that  $\operatorname{Mod}\Omega \leq \operatorname{Mod}\Omega^*$  there exists, unique up to conformal authomorphisms of  $\Omega$ , an energy-minimal diffeomorphism which is a harmonic diffeomorphism. The results improve and extend some recent results of Iwaniec, Koh, Kovalev and Onninen (Inven. Math. (2011)). Further, the case of radial metrics is discussed in details.

#### Eiko Kin.

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Title. Minimal dilatations of pseudo-Anosovs generated by the magic 3-manifold and their asymptotic behavior

Abstract. Let  $\delta_g > 1$  be the minimal dilatation of pseudo-Anosovs defined on a closed surface of genus g. Penner proved that  $\log \delta_g$  behaves like  $\frac{1}{g}$ . We are interested in McMullen's question: Does  $\lim_{g \to \infty} g \log \delta_g$  exist? What is its value? We examine his question in the large set  $\widehat{\mathcal{M}}$  of pseudo-Anosovs on closed surfaces "generated by" the magic manifold N which is homeomorphic to the 3-chain link exterior. Let  $\widehat{\delta}_g$  be the minimum among dilatations of elements in  $\widehat{\mathcal{M}}$  defined on a closed surface of genus g. We prove that  $\lim_{g\to\infty} g \log \widehat{\delta}_g = \log(\frac{3+\sqrt{5}}{2})$ . Moreover for large g,  $\widehat{\delta}_g$  is achieved by the monodromy of some  $\Sigma_g$ -bundle over the circle obtained from either  $N(\frac{3}{-2})$  ( $\simeq$  (-2, 3, 8)-pretzel link exterior) or  $N(\frac{1}{-2})$  ( $\simeq$   $6^2_2$  link exterior) by Dehn filling both cusps, where N(r) is the manifold obtained from N by Dehn filling one cusp along the slope  $r \in \mathbb{Q}$ . This is a joint work with Sadayoshi Kojima and Mitsuhiko Takasawa.

### Erina Kinjo.

Title. On the length spectrum metric in infinite-dimensional Teichmuller spaces

Abstruct. We consider Teichmuller metric and the length spectrum metric in Teichmuller spaces. It is known that these metrics define the same topology in finitedimensional Teichmuller spaces. In this talk, we study infinite-dimensional Teichmuller spaces where they define the same topology.

### Hidetoshi Masai.

Title. On commensurability of fibrations on a hyperbolic 3-manifold

*Abstruct.* We discuss the fibered commensurability of fibrations on a hyperbolic 3manifold. The notion of fibered commensurability is defined by Calegari, Sun and Wang (2010). Calegari, Sun and Wang asked in their paper if there is a manifold with a pair of commensurable fibrations whose fiberes are of different topology. In this talk we will construct an infinite sequence of manifolds with such pairs of fibrations. We further show that two fibrations of each pair belong to the same fibered face.

### Katsuhiko Matsuzaki.

Title. Circle diffeomorphisms and Teichmüller spaces

*Abstruct.* By characterizing a diffeomorphism of the circle with Hölder continuous derivative in terms of the quasiconformal Teichmüller theory, we show certain rigidity of groups of circle diffeomorphisms.

#### Toshihiro Nogi.

Title. On extendibility of a map induced by Bers isomorphism

Abstruct. Let T(S) be the Teichmüller space of a closed Riemann surface S of genus g(> 1). Denote by U the universal covering surface of S, that is, the upper half-plane and denote by  $\dot{S}$  the surface obtained by removing a point from S. By the Bers isomorphism theorem, we have a homeomorphism of  $T(S) \times U$  onto  $T(\dot{S})$ . The Bers embedding shows that the spaces  $T(S) \times U$  and  $T(\dot{S})$  are embedded in (3g-2)-dimensional complex vector space. Thus the boundaries of both spaces are naturally defined.

Let A be a subset of the boundary  $\partial U$  of U consisting of all points filling S. In this talk, we show that the homeomorphism of  $T(S) \times U$  onto  $T(\dot{S})$  has a continuous extension to  $T(S) \times (U \cup A)$ . This is a joint work with Hideki Miyachi (Osaka University).

### Kasra Rafi.

Title. Geometry of Teichmüller space (part I and II).

*Abstruct.* We review recent results about the Teichmüller space equipped with the Teichmüller metric. We give an inductive description of a Teichmüller geodesic using the Teichmüller geodesics of surfaces with lower complexity. We use these results to compare how the Teichmüller space is similar or different from the hyperbolic space.

### Ken-ichi Sakan.

Title. Quasiconformal and Lipschitz harmonic mappings of the unit disk onto bounded convex domains

Abstract. For a sense-preserving univalent harmonic self-mapping F of the unit disk, Pavlović showed that F is quasiconformal iff F is bi-Lipschitz. He gave another characterization, too, for the quasiconformality of F by means of some properties of the boundary-valued mapping of F. If the target of F is a bounded convex domain, then this result does not hold in general as it stands. If the Lipschitz property of F is pre-assumed, however, then we could obtain a variant of the result by Pavlović. In other words, in this talk we show some characterizations of a quasiconformal and Lipschitz harmonic mapping of the unit disk onto a bounded convex domain. This is a joint work with Dariusz Partyka.

#### Masaharu Tanabe.

*Title.* On the combinatorial Hodge star operator and holomorphic cochains

Abstruct. For cochains equipped with an inner product of a triangulated manifold, S.O. Wilson defined the combinatorial Hodge star operator  $\bigstar$  in his paper of 2007 and showed that for a certain cochain inner product which he named the Whitney inner product, this operator converges to the smooth Hodge star operator if the manifold is Riemannian. He also stated that  $\bigstar \bigstar \neq \pm Id$  in general and raised a question if  $\bigstar \bigstar$  approaches  $\pm Id$  as the mesh of the triangulation tends to zero. In this talk, we solve this problem affirmatively.

# Masahiro Yanagishita.

 $\it Title.$  Teichmüller distance and Kobayashi distance on subspaces of the universal Teichmüller space

Abstruct. It is known that the Teichmüller distance on the universal Teichmüller space T coincides with the Kobayashi distance. For a metric subspace of T having a comparable complex structure with that of T, we can similarly consider whether or not the Teichmüller distance on the subspace coincides with the Kobayashi distance. In this talk, we give a sufficient condition for metric subspaces under which the problem above has a affimative answer. Moreover, we introduce an example of such subspaces.

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