Conference on

Infinite-Dimensional Harmonic Analysis

September 10 – 14, 2007

held at

Graduate School of Mathematical Sciences,
The University of Tokyo

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Organizers :

H. Arai (Tokyo),
J. Hilgert (Paderborn),
A. Hora (Nagoya),
T. Kawazoe (Keio),
K. Nishiyama (Kyoto),
M. Voit (Dortmund)

▸*September 10 (Monday)*

9:15 – 9:20 Opening
9:20 – 10:20 **H. Heyer** (Tübingen):
Where group representations meet Lévy processes
—
10:30 – 11:10 **R. Lasser** (München):
Amenability and weak amenability of Banach algebras on commutative hypergroups
—
11:20 – 12:00 **S. Kawakami** (Nara):
Extensions of hypergroups
—— (Lunch break)
13:30 – 14:10 **H. Yamashita** (Hokkaido):
Isotropy representations and Howe duality correspondence
14:10 – 14:50 **H. Glöckner** (Darmstadt):
Continuity and differentiability properties of functions on direct limits of infinite-dimensional Lie groups
—— (Coffee break)
15:20 – 16:00 **B. Kümmerer** (Darmstadt):
Asymptotic behaviour of quantum Markov processes
16:00 – 16:40 **T. Kondo** (Kyoto):
Fixed-point property of random groups
—
16:50 – 17:30 **N. Obata** (Tohoku):
Asymptotic spectral analysis and applications to complex networks

The talk of M. Voit has moved to Wednesday morning.

▸*September 11 (Tuesday)*

9:20 – 10:20 C.F. Dunkl (Virginia):
Transforms, polynomials and integrable models associated with reflection groups

—

10:30 – 11:10 M. Rösler (Clausthal):
Convolution structures associated with multivariate hypergeometric functions

—

11:20 – 12:00 H.-P. Scheffler (Siegen):
On the limit distribution of coupled continuous time random walks

—— (Lunch break)

13:30 – 14:10 P. Eichelsbacher (Bochum):
Ordered random walks

—

14:20 – 15:10 H. Arai (Tokyo):
A nonlinear model of visual information processing and visual illusions

—— (Coffee break)

15:40 – 16:30 H. Fujiwara (Kinki):
Intertwining integral for exponential Lie groups

The talk of K.-H. Neeb has been canceled.

▸*September 12 (Wednesday)*

9:20 – 10:20 **M. Voit** (Dortmund):
Limit theorems for radial random walks of high dimensions
—
10:30 – 11:20 **T. Hirai** (Kyoto):
Spin characters of infinite complex reflexion groups
—— (Lunch break)

12:40 – **Excursion**

▸*September 13 (Thursday)*

9:20 – 10:20 J.-P. Anker (Orléans):
The Schrödinger equation on symmetric spaces
—
10:30 – 11:10 A. Püttmann (Bochum):
An infinite-dimensional representation of a Lie supergroup and applications to autocorrelations
—
11:20 – 12:00 H. Shimomura (Kochi):
Unitary representations and quasi-invariant measures on infinite-dimensional groups
—— (Lunch break)
13:30 – 14:10 S. Naito (Tsukuba):
An explicit description of the crystal structure on the set of MV polytopes of type B or C
14:10 – 14:50 P. Becker-Kern (Dortmund):
Semi-selfsimilar additive processes on simply connected nilpotent Lie groups
—— (Coffee break)
15:20 – 16:00 P. Ressel (Eichstaett):
Exchangeable probability measures and positive definite functions
16:00 – 16:40 S. Matsumoto (Kyushu):
Moments of characteristic polynomials of a random matrix associated with compact symmetric spaces
—
16:50 – 17:30 M. Stolz (Bochum):
Limit theorems for random matrix ensembles associated to symmetric spaces

►*September 14 (Friday)*

9:20 – 10:00 T. Nomura (Kyushu):
Tube domains and basic relative invariants
—
10:10 – 10:50 T. Kawazoe (Keio):
Real Hardy spaces for Jacobi analysis and its applications
—
11:00 – 11:40 U. Franz (Franche-Comté, Tohoku):
On idempotents on quantum groups

—— (Lunch break)

13:00 – 13:40 P. Ramacher (Göttingen):
Equivariant spectral asymptotics and compact group actions
13:40 – 14:20 M. Olbrich (Luxembourg):
Limit sets of Kleinian groups and harmonic analysis
—
14:30 – 15:10 A. Alldridge (Paderborn):
Index theory for Wiener–Hopf operators on arbitrary cones

—— (Coffee break)

15:40 – 16:20 K. Nishiyama (Kyoto):
Capelli identities for Hermitian symmetric spaces
16:20 – 17:00 J. Hilgert (Paderborn):
Symbolic dynamics for geodesic flows on locally symmetric spaces
——

18:00 – 20:00 Farewell Dinner

Index Theory for Wiener–Hopf Operators on Arbitrary Cones

Alexander Alldridge

University of Paderborn

We study Wiener–Hopf operators W_f ; these are the bounded operators on $\mathrm{L}^2(\Omega, dx)$, where $\Omega \subset \mathbb{R}^n$ is a closed convex cone, given by

$$W_f\xi(x) = \int_\Omega f(x-y)\xi(y)\,dy \quad \text{for all } f \in \mathrm{L}^1(\mathbb{R}^n, dx)\,,\ \xi \in \mathrm{L}^2(\Omega, dx)\,,\ x \in \Omega\,.$$

Classically, $n = 1$ and $\Omega = \mathbb{R}_+$; then the operators W_f correspond to Toeplitz operators on the unit circle with continuous symbol vanishing at the identity. It is then well known exactly when $1 + W_f$ is a Fredholm operator, and in this case, its index can be easily computed. (In this particular case, this characterises the invertibility of $1 + W_f$.)

In general, the Fredholmness of $1+W_f$ is related to the ideal structure of the C*-algebra A generated by the operators W_f , $f \in \mathrm{L}^1(\mathbb{R}^n, dx)$, and the computation of the index is related to the K-theory of certain subquotients of A .

So far, a complete theory for the index problem of Wiener–Hopf operators was only known for the case of a symmetric cone Ω , from the work of Upmeier. The Fredholmness question was solved for polyhedral and for symmetric cones by Muhly and Renault, using the technique of groupoid C*-algebras.

We have extended their approach and thereby completely solved the Fredholmness question and the computation of the index for a large class of cones which contains, in particular, all polyhedral and all homogeneous (not necessarily symmetric) cones. We will explain our results and discuss some special classes of cones.

Part of this work was conducted jointly with Troels R. Johansen (U Paderborn).

The Schrödinger equation on symmetric spaces

Jean–Philippe Anker, Université d'Orléans, France

Abstract

Nonlinear heat, Schrödinger or wave equations have been extensively studied during the past 25 years in the Euclidean setting. In this talk, we shall mainly consider the Schrödinger equation

$$\begin{cases} i\,\partial_t u(t,x) + \Delta_x u(t,x) = \omega(u(t,x)) \\ u(0,x) = f(x) \end{cases}$$

on real hyperbolic spaces and review recent results (dispersive inequalities, Strichartz type estimates, scattering, ...), including joint work in progress with Vittoria Pierfelice. As expected, small scale results are similar to the Euclidean setting, while large scale results are stronger in negative curvature.

A Nonlinear Model of Visual Information Processing and Visual Illusions

Hitoshi ARAI
The University of Tokyo
Department of Mathematical Sciences

1 How can you see?

Recent development of neurophysiology and fMRI has been revealing the functional map of human's brain. Thanks to this, we have been able to know considerably correspondence between areas of the brain and functions of vision; for example, if one recognizes motion of an object, then his MT area is activated; if one perceives color of an object, then his V4 area is working, and so on. However, little is known about how the brain computes there visual information. This means that we know "where", but not "how". The main purpose of my research is to comprehend "how". For this aim I have studied computational algorithms and mathematical models of the visual system. Now a question will come to the mind; why do we need mathematical study, although real visual information processing is executed biochemically by neurons in the brain?

2 Visual perception, computer vision and illusions

I think the study of computational models of the visual system is very closely related to understanding how the brain processes visual information. A cue to combine these is visual illusions. Indeed if computational algorithms which we obtain as analogies of functions of subareas of the brain are appropriate, implementing them into a computer machine will get it to possess analogous functions of human's vision. Consequently, the computer machine must "see" illusions which are produced in those areas. On the other hand, by seeking algorithms in order to reproduce visual illusions whose causes are unknown, we can presume neural algorithms for vision in the brain. I think the study of computational models of the visual system will become a powerful method in vision science.

3 Plan of this talk

As a preliminary I will give a brief review of neuroscience and psychophysics related to vision. Then I will propose a mathematical model of a part of V1, the visual area 1. This model can produce good computer simulations of visual illusions such as the Hermann grid illusion, the Chevreul illusion, the cafe wall illusion and some illusions related to color vision. After demonstrating these simulations, I will talk what they mean, and then discuss a psychological question related to the color perception by using the mathematical model. Furthermore I will propose our new wavelet frames constructed by taking recent neuroscientific opinions into account, and explain their efficiency.

The author is supported partly by the Grant-in-Aid for Scientific Research (B), Japan Society for the Promotion of Science.

Semi-selfsimilar additive processes on simply connected nilpotent Lie groups

Peter Becker-Kern, Universität Dortmund

Identifying an additive process $\{\xi_t\}_{t\geq 0}$ on a simply connected nilpotent Lie group $\mathbb{G}$ with a process on the finite-dimensional Lie algebra $\mathcal{G} \cong \mathbb{R}^d$, under a (rather strong) differentiability condition in [Ku] the process is shown to be the solution of a stochastic differential equation driven by an additive process $\{X_t\}_{t\geq 0}$ on $\mathbb{R}^d$. Assuming selfsimilarity of $\{\xi_t\}_{t\geq 0}$ with respect to a contractive one-parameter subgroup of automorphisms, the differentiability condition is fulfilled and, equivalently, $\{X_t\}_{t\geq 0}$ is operator-selfsimilar with an exponent Q, which is quite well understood. The process increments correspond to a stable hemigroup of measures and it is known that $\{X_t\}_{t\geq 0}$ has an integral representation driven by a Lévy process with finite logarithmic moment. On the other hand, for the distribution μ of X_1 we have operator-selfdecomposability $\mu = e^{-tQ}\mu * \nu(t)$ for a Mehler semigroup $e^{-tQ}\nu(s) * \nu(t) = \nu(s+t)$. Conversely, given a Mehler semigroup we can reconstruct a selfsimilar additive process, its background driving Lévy processes, and the corresponding Ornstein-Uhlenbeck type process via Lamperti's transformation, a concept which by [Ha] also extends to processes on $\mathbb{G}$.

In case of the weaker scaling property of operator-semi-selfsimilarity, the corresponding background driving process of $\{X_t\}_{t\geq 0}$ is known by [MS], [BK] to have only periodically stationary increments. We show that certain discrete parameter Mehler semigroups, corresponding to the operator-semi-selfdecomposability, can be embedded into a (differentiable) semistable hemigroup, the increment distributions of an additive operator-semi-selfsimilar process. This is the starting point for studying corresponding objects on $\mathbb{G}$ and their interrelation.

References

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[MS] Maejima, M.; and Sato, K.I. (2003) Semi-Lévy processes, semi-selfsimilar additive processes, and semi-stationary Ornstein-Uhlenbeck type processes. *J. Math. Kyoto Univ.* **43** 609–639.

Transforms, Polynomials and Integrable Models Associated with Reflection Groups

Charles F. Dunkl

Department of Mathematics, University of Virginia

Abstract:

The parametrized commutative algebra of differential-difference (”Dunkl”) operators associated with a finite reflection group can be studied both algebraically and analytically. On the side of analysis one usually imposes the requirement that the parameters be positive. There is a generalization of the Fourier transform which provides a spectral decomposition of the operators, and there is also an intertwining operator, which has been shown to be positive in general by Rösler, but for which only the $(Z_2)^N$ version has been exhibited by an explicitly positive formula. We discuss recent work regarding an integral for the B_2 case which is positive when restricted to B_2-invariant functions. Positivity is always an important property; for example there are Markov processes associated to the operators, which have been studied by Rösler, Voit, Gallardo and Yor. The algebra of operators also allows exact solution of Calgero-Sutherland quantum mechanical systems of identical particles with inverse-square interactions, on a circle, or on a line subject to harmonic confinement. The Macdonald-Mehta-Selberg integral associated to the algebra of operators involves an expression in gamma functions whose poles have algebraic interpretations. In particular for the symmetric and hyperoctahedral groups (types A and B, respectively) the singularities can be explained in terms of certain nonsymmetric Jack polynomials and the representation theory of the groups.

Ordered random walks

Peter Eichelsbacher

Fakultät für Mathematik, Ruhr-Universität Bochum

Abstract:

We construct the conditional version of k independent and identically distributed random walks on $\mathbb{R}$ given that they stay in strict order at all times. This is a generalisation of so-called non-colliding or non-intersecting random walks, the discrete variant of Dyson's Brownian motions, which have been considered yet only for nearest-neighbor walks on the lattice. Our only assumptions are moment conditions on the steps and the validity of the local central limit theorem. The conditional process is constructed as a Doob h-transform with some positive regular function V that is strongly related with the Vandermonde determinant and reduces to that function for simple random walk. Furthermore, we prove an invariance principle, i.e., a functional limit theorem towards Dyson's Brownian motions, the continuous analogue.

On idempotents on quantum groups

Uwe Franz

Université de Franche-Comté, Tohoku University

The idempotent measures on a locally compact group G are exactly the Haar measures on compact subgroups of G. This fact has many applications, e.g., in ergodic theory and probability. Pal showed in 1996 that the analogous statement for quantum groups is false. In my talk I will show that idempotents states on finite quantum groups can be characterized by quantum hypergroups. I will then discuss the extension of this result to compact quantum groups and algebraic quantum groups (in the sense of van Daele). This talk is based on joint work with Adam Skalski.

Intertwining integral for exponential Lie groups

Hidenori Fujiwara (Kinki University)

Let $G = \exp \mathfrak{g}$ be an exponential solvable Lie group with Lie algebra $\mathfrak{g}$. Let's introduce some notions used in the orbit method. We note $\mathfrak{g}^*$ the dual vector space of $\mathfrak{g}$ and $S(f, \mathfrak{g})$ for $f \in \mathfrak{g}^*$ the set of subalgebras $\mathfrak{h}$ of $\mathfrak{g}$ satisfying $f([\mathfrak{h}, \mathfrak{h}]) = \{0\}$. In this situation we consider the unitary character χ_f of the analytic subgroup $H = \exp \mathfrak{h}$ defined by $\chi_f(\exp X) = e^{if(X)}$ $(X \in \mathfrak{h})$, and construct the induced representation $\rho(f, \mathfrak{h}, G) = \mathrm{ind}_H^G \chi_f$ of G. We note $I(f, \mathfrak{g})$ the set of $\mathfrak{h} \in S(f, \mathfrak{g})$ such that the representation $\rho(f, \mathfrak{h}, G)$ is irreducible. It is well known that $I(f, \mathfrak{g}) \neq \emptyset$ and that

$$\rho_1 = \rho(f, \mathfrak{h}_1, G) \simeq \rho_2 = \rho(f, \mathfrak{h}_2, G)$$

for any $\mathfrak{h}_1$, $\mathfrak{h}_2$ in $I(f, \mathfrak{g})$. We are interested how to construct an explicit intertwining operator between these two realizations.

Put $H_j = \exp \mathfrak{h}_j$ $(j = 1, 2)$ and note Δ_j the modular function of H_j. Let $\mathcal{K}(H_2, H_1 \cap H_2)$ the space of $\mathbb{C}$-valued continuous functions F on H_2 with compact support modulo $H_1 \cap H_2$ and satisfying

$$F(hk) = \Delta_{H_1 \cap H_2, H_2}(k) F(h) \ (h \in H_2,\ k \in H_1 \cap H_2)$$

with $\Delta_{H_1 \cap H_2, H_2}(k) = \frac{\Delta_{H_1 \cap H_2}(k)}{\Delta_{H_2}(k)}$. Then, H_2 acts on this space $\mathcal{K}(H_2, H_1 \cap H_2)$ by left translation and there exists a unique, up to scalar multiplication, H_2-invariant positive linear form $\nu = \nu_{H_2, H_1 \cap H_2}$. We write this form as

$$\nu(F) = \oint_{H_2/(H_1 \cap H_2)} F(h) d\nu(h),\ F \in \mathcal{K}(H_2, H_1 \cap H_2).$$

We now have a formal candidate T_{21} of intertwining operator from ρ_1 to ρ_2:

$$T_{21}(\varphi)(g) = \oint_{H_2/(H_1 \cap H_2)} \varphi(gh) \Delta_{H_2, G}^{-1/2}(h) \chi_f(h) d\nu(h) \quad (g \in G).$$

This intertwining integral also appears in some studies of monomial representations for nilpotent Lie groups as Penney's distributions or eigen distributions of invariant differential operators.

Continuity and differentiability properties of functions on direct limits of infinite-dimensional Lie groups

Helge Glöckner

Technische Universität Darmstadt

Summary:
It frequently happens that an infinite-dimensional Lie group G of interest can be expressed as the union of an ascending sequence of Lie groups G_n. Given a mapping f on G, it is then natural to ask whether continuity (or smoothness) of $f|_{G_n}$ for each n entails continuity (or smoothness) of f. For general f, the answer is, usually, negative. But the situation improves if f is a homomorphism to a Lie group, a unitary representation, or a positive-definite function.

In the talk, I'll describe the exact answers for the main examples of direct limits groups (notably, for test function groups and groups of compactly supported diffeomorphisms of non-compact manifolds).

Reference:

Glöckner, H., Direct limits of infinite-dimensional Lie groups compared to direct limits in related categories, J. Funct. Anal. **245** (2007), 19–61.

WHERE GROUP REPRESENTATIONS MEET LEVY PROCESSES

by Herbert Heyer, Tuebingen

A significant point of contact between representations of locally compact groups and stochastic processes with independent increments in such groups is the concept of Fourier transform.

Since the appearance of the speaker's monograph of 1977 in which the Fourier transform has been efficiently applied to establishing Levy-Khintchine decompositions and central limit results, interesting new developments lead to potential-theoretic and asymptotic properties of random walks and diffusions on groups, to progress in random matrix theory, and to the solution of a martingale problem for Levy processes.

In his expository lecture the speaker intends to describe the method of Fourier transform within the framework of groups and hypergroups and to report on martingale representations for nonstationary Levy processes in these structures.

The main tool in dealing with the problem is the notion of an integrating function attached to an evolution family of continuous finite variation.

Symbolic Dynamics for Geodesic Flows on Locally Symmetrc Spaces

Joachim Hilgert

Universität Paderborn

Geodesic flows on locally symmetric spaces are model systems for the study of chaos in quantum systems. More precisely, one tries to relate properties like ergodicity of the geodesic flows to properties of its quantizations. Typically this would be statistical properties of the spectrum of the Hamilton operator. For the case of hyperbolic surfaces a classical object that turns out to carry a lot of information related to the quantum system is the transfer operator. Transfer operators come from statistical mechanics and so far it is not possible to construct them directly from the geodesic flow. It is necessary to replace it by a symbolic dynamics which is then a special type of shift. In order to construct such a symbolic dynamics, one needs to have precise information on the geometry of the locally symmetric space and its fundamental group. We will talk about a unified way to construct symbolic dynamics for rank one spaces.

Spin characters of infinite complex reflexion groups

Takeshi HIRAI (Kyoto)

Abstract:

Let T be a finite abelian group, and put $D_n = \prod_{1 \le i \le n} T_i, T_i = T\ (\forall i)$. Then the symmetric group $\mathfrak{S}_n$ acts on D_n naturally as

$$D_n \ni d = (t_i)_{1 \le i \le n} \xrightarrow{\sigma} \sigma(d) = (t_{\sigma^{-1}(i)})_{1 \le i \le n} \in D_n \quad (t_i \in T, \sigma \in \mathfrak{S}_n).$$

The semidirect product $D_n \rtimes \mathfrak{S}_n$ is called the wreath product of T with the symmetric group $\mathfrak{S}_n$ and is denoted by $\mathfrak{S}_n(T)$. For a subgroup S of T, we define a canonical normal subgroup of $\mathfrak{S}_n(T)$ as

$$\mathfrak{S}_n(T)^S := \big\{ g = (d,\sigma) \in \mathfrak{S}_n(T);\ P(d) := t_1 t_2 \cdots t_n \in S \big\}, \qquad d = (t_i) \in D_n, \sigma \in \mathfrak{S}_n.$$

When $n \to \infty$, we get $\mathfrak{S}_\infty(T)^S$ as their inductive limits. We have given a unified character formula for factor representations of finite type of $\mathfrak{S}_\infty(T)^S$.

This time, we take $T = \boldsymbol{Z}_m$, and take a representation group $R(G)$ of $G = \mathfrak{S}_\infty(T)^S$ (we may call it as *universal* covering group of G) and study similar problems, that is, to determine all the characters of factor representations of finite type and so on.

For the case $T = \boldsymbol{Z}_m$, any subgroup S of T is of the form, for a positive integer $p|m$,

$$S = S(p) := \big\{ t^p\,;\ t \in T \big\} \cong \boldsymbol{Z}_{m/p},$$

and the following groups are called *generalized symmetric groups* by M. Oshima (1954), J.W. Davies - A.O. Morris (1974):

$$G(m,p,n) := \mathfrak{S}_n(\boldsymbol{Z}_m)^{S(p)},$$

and consists of three infinite families of complex reflexion groups. When $n \to \infty$, we get $G(m,p,\infty)$ as thier inductive limits, which are main objects of our study.

In the case where $m = 2$, $G(2,1,n) = W_{B_n}$ are Weyl groups of type B and their normal subgroups $G(2,2,n) = W_{D_n}$ are Weyl groups of type D.

Every projective representation of $G = G(m,p,n), 1 \le n \le \infty$, can be linealized by going up to the level of representation group $R(G)$ (*universal* covering group of G) resolving its obstruction as in the case of compact Lie groups.

In this talk, we discuss spin (= projective) representations and spin characters of these groups, in particular for the case $n = \infty$. Our final goal is to obtain general character formulas for spin factor representations of finite type.

(jointly with A. Hora and E. Hirai)

Extensions of Hypergroups

Satoshi Kawakami (Nara University of Education)

This is a joint work with Professor Heyer and with my colleagues listed in the references as coauthors. We will report some results for the extension problem in the category of commutative hypergroups, which is to determine all extensions K of L by H when two hypergroups H and L are given. We think that the extension problem is important in order to understand full structures of hypergroups. The hypergroup K is called an extension of L by H if the hypergroup H is imbedded in K as a closed subhypergroup and there exists a continuous homomorphism φ from K onto L such that $\mathrm{Ker}\varphi = H$.

We will introduce some models of extensions K of L by H.

[i] Fundamental models.
(1) hypergroup product $H \times L$, (2) hypergroup join $H \vee L$,
(3) hypergroup substitutions ([7]).

[ii] Hypergroups constructed from group actions ([1], [5]).

[iii] Splitting extensions constructed by applying a notion of a field of compact subhypergroups of H based on L ([2], [4])

[iv] Non-splitting models of extensions of hypergroups L by locally compact abelian groups H ([6], [8]).

[v] Extensions and cohomology of hypergroups ([3]).

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Real Hardy spaces for Jacobi analysis and its applications

Takeshi KAWAZOE
Keio University

Let $\alpha, \beta, \lambda \in \mathbb{C}$ and $x \in \mathbb{R}_+ = [0, \infty)$. Let $\phi_\lambda^{\alpha,\beta}(x)$ denote the Jacobi function of order (α, β) and $\hat{f}(\lambda)$ the Jacobi transform of $f \in C_{c,e}^\infty(\mathbb{R})$. Then the map $f \to \hat{f}$ extends to an isometry between $L^2(\Delta) = L^2(\mathbb{R}_+, \Delta(x)dx)$ and $L^2(\mathbb{R}_+, |C(\lambda)|^{-2}d\lambda)$, where $\Delta(x) = (2\mathrm{sh}x)^{2\alpha+1}(2\mathrm{ch}x)^{2\beta+1}$ and $C(\lambda)$ is the Harish-Chandra C-function. We define a dilation ϕ_t, $t > 0$, of $\phi \in C_{c,e}^\infty(\mathbb{R})$ as preserving the $L^1(\Delta)$-norm of ϕ and put $M_\phi = \sup_{t>0} f * \phi_t$. We introduce a real Hardy space $H^1(\Delta)$ as

$$H^1(\Delta) = \{f \in L^1_{\mathrm{loc}}(\Delta) \; ; \; M_\phi(f) \in L^1(\Delta)\}.$$

This space can be characterized by using weighted H^1 Hardy spaces on $\mathbb{R}$. Let $F(x) = e^{\rho x}F_f(x)$, where $\rho = \alpha + \beta + 1$ and F_f the Abel transform of f, and put $w_\gamma(x) = |\mathrm{th}x|^\gamma$. Then

$$\|f\|_{H^1(\Delta)} = \|M_\phi(f)\|_{L^1(\Delta)} \sim \sum_{\gamma\in\Gamma} \|W_{-\gamma}^{\mathbb{R}}(F)\|_{H^1_{w_\gamma}(\mathbb{R})},$$

where Γ is a finite set in $[0, \alpha + 1/2]$ and $W_{-\gamma}^{\mathbb{R}}$ is a Weyl type fractional derivative.

Let h_t and p_t denote the heat and the Poisson kernels for the Jacobi analysis, that is, even functions on $\mathbb{R}$ whose Jacobi transforms $\hat{h}_t$ and $\hat{p}_t$ are respectively given by $\hat{h}_t(\lambda) = e^{-t(\lambda^2+\rho^2)}$ and $\hat{p}_t(\lambda) = e^{-t\sqrt{\lambda^2+\rho^2}}$. Then, as in the case of the Euclidean space, we can introduce the heat maximal operator M_h, the Poisson maximal operator M_P, the Littelwood-Paley g-function $g(f)$, and the Lusin area function $S(f)$ on $\mathbb{R}_+$. These operators are bounded on $L^p(\Delta)$, $p > 1$, and satisfies the weak type $L^1(\Delta)$ estimate. As an application of the real Hardy space $H^1(\Delta)$, we can prove that M_P and g-operator are bounded from $H^1(\Delta)$ to $L^1(\Delta)$. As for M_h and S, we need some modifications to obtain strong boundedness on $H^1(\Delta)$. In the talk we shall consider an interpolation between $H^1(\Delta)$ and $L^2(\Delta)$.

Fixed-pooint property of random groups

Takefumi Kondo (Kyoto University)

This talk is due to a joint work with Izeki and Nayatani [1].

Żuk showed in [3] that random groups of triangular model with density greater than 1/3 have Property(T). To prove this theorem, Żuk defined a certain finite graph L for a presentation of a group and showed that Property(T) can be deduced from some spectral condition of L. Property(T) is known to be equivalent to the fixed-point property for Hilbert spaces.

On the other hand, Izeki and Nayatani formulated in [2] a sufficient condition for a given discrete group to have the fixed-point property for CAT(0) spaces.

Comnining their result with the result of Żuk, we conclude that random groups with density greater than 1/3 have a strong fixed-point property.

References

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Asymptotic Behaviour of Quantum Markov Processes

Burkhard Kümmerer

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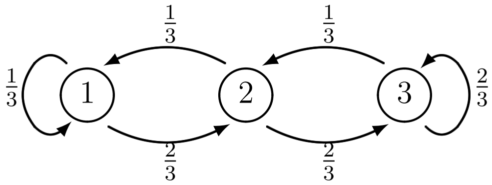
The diagram



represents a classical Markov process on the three states $\{1, 2, 3\}$. When considered as a labelled graph this diagram can be viewed as a road-coloured graph which, in addition, allows a synchronizing word. In this talk we discuss quantum versions of these notions.

The notion of a road-coloured graph generalizes to the non-commutative or quantum context, where it becomes an injective *-homomorphism $j : \mathcal{A} \to \mathcal{A} \otimes \mathcal{C}$, where $\mathcal{A}$ and $\mathcal{C}$ are C*-algebras. Again, we can construct a quantum Markov process from such a *-homomorphism. In the case of a classical Markov process the existence of a synchronizing word turns out to be equivalent to the asymptotic completeness of this Markov process. As a first step this allows to extend the discussion of synchronizing words to classical Markov processes with a countable state space. In the non-commutative context asymptotic completeness can be shown to be equivalent to the possibility of preparing quantum states on $\mathcal{A}$ in a such way that we can recognize the idea of a synchronizing word also in this context.

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Amenability and weak amenability of Banach algebras on commutative hypergroups

Rupert Lasser, München

Abstract:

The L^1-algebras of hypergroups have properties which are very different from the L^1-algebras of groups. We present new results on the amenability and weak amenability of the L^1-algebras of polynomial hypergroups, see [2]. Among other results we show that $l^1(h)$ is not amenable if $h(n) \to \infty$. Moreover we investigate the existence of nonzero bounded point derivations (which is a very weak form of amenability). We derive sufficient and necessary conditions for the existence of such derivations, and study examples in the class of polynomial hypergroups. Finally we present new characterizations of the α-amenability of the L^1-algebras extending results of [1].

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Moments of characteristic polynomials of a random matrix associated with compact symmetric spaces

Sho Matsumoto (Kyushu University)

We deal with the compact symmetric spaces G/K classified by Cartan, where G is a compact subgroup in $GL(N,\mathbb{C})$ for some positive integer N, and K is a closed subgroup of G. Assume G/K is realized as a subspace S in G, i.e., $S \simeq G/K$, and the probability measure $\mathrm{d}M$ on S is then induced from G/K. We call the probability space $(S, \mathrm{d}M)$ the random matrix ensemble associated with G/K.

For example, $U(n)/O(n)$ is the symmetric space with a restricted root system of type A, and is realized by $S = \{M \in U(n) \mid M = M^{\mathrm{T}}\}$. Here M^{T} stands for the transposed matrix of M. The induced measure $\mathrm{d}M$ on S satisfies the invariance $\mathrm{d}(HMH^{\mathrm{T}}) = \mathrm{d}M$ for any $H \in U(n)$. This random matrix ensemble $(S, \mathrm{d}M)$ is well known as the circular orthogonal ensemble (COE for short).

Given the random matrix ensembles $(S, \mathrm{d}M)$, we consider the averages of the products of characteristic polynomials:

$$\langle \det(I + x_1 M) \det(I + x_2 M) \cdots \det(I + x_k M) \rangle_S .$$

Here the symbol $\langle \cdot \rangle_S$ stands for the average with respect to the measure $\mathrm{d}M$. In particular, we are interested in the moment $\left\langle \det(I+M)^k \right\rangle_S$ (or $\left\langle |\det(I+M)|^{2k} \right\rangle_S$) and the asymptotic behavior of this in the limit as the matrix size goes to infinity.

We will express the characteristic polynomial averages in terms of Jack polynomials or Heckman and Opdam's Jacobi polynomials depending on the root system of the space.

Reference: **arXiv:math/0608751** (version 3).

An explicit description of the crystal structure on the set of MV polytopes of type B or C

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Abstract

I. Mirković and K. Vilonen introduced Mirković-Vilonen (MV for short) cycles, which are certain (irreducible) algebraic subvarieties of the affine Grassmannian $\mathcal{G} = G(\mathcal{K})/G(\mathcal{O})$ associated to the loop group $G(\mathbb{C}[t, t^{-1}])$, where G is a simply-connected simple algebraic group over $\mathbb{C}$, and $\mathcal{K} = \mathbb{C}((t))$, $\mathcal{O} = \mathbb{C}[[t]]$. They proved that these MV cycles give a (geometric) realization of the basis elements of the (finite-dimansional) irreducible highest weight modules of $G(\mathbb{C})$. Soon afterward, in order to study MV cycles explicitly by combinatorial methods, J. E. Anderson introduced MV polytopes, as the images under the moment map of MV cycles in the affine Grassmannian. Furthermore, J. E. Anderson and I. Mirković proposed a conjecture (the AM conjecture) describing a crystal structure on MV polytopes, which gives a method for generating MV polytopes inductively without making use of the "tropical" Plücker relations. This conjecture was verified in type A case by J. Kamnitzer, who also gave a counterexample for type C case.

In this talk, we give and prove a kind of generalization of the original AM conjecture to the cases of B, C types. To be more precise, we first realize MV polytopes of type B or C as MV polytopes of type A fixed by the natural action of a Dynkin diagram automorphism. The main point is that, in our realization, Kashiwara operators on MV polytopes of type B or C are certain composites of Kashiwara operators on MV polytopes of type A. Therefore, from the original AM conjecture (verified by J. Kamnitzer) applied to MV polytopes of type A fixed by a diagram automorphism, we obtain a polytopal description of the crystal structure on MV polytopes of type B or C, which is analogous to the original one, but indeed holds.

This is a joint work with Professor D. Sagaki of the University of Tsukuba.

Abstract

Group algebras for infinite dimensional Lie groups

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Since an infinite-dimensional Lie group G is not locally compact, it does not possess an invariant measure, so that there is no canonical group C^*-algebra whose representations correspond to the unitary representations of G. However, there is the notion of a host algebra of a topological group G (due to H. Grundling), which is a natural (weaker) replacement of the concept of the group C^*-algebra in the sense that the representations of a host algebra $\mathcal{A}$ are in one-to-one correspondence with certain subclass of continuous unitary representations of G. On the level of states, it means that the states of $\mathcal{A}$ can be viewed as a subset of the states (= normalized continuous positive definite functions) on G.

In this talk we present an approach to host algebras for infinite dimensional Lie groups which is based on complex involutive semigroups. Any locally bounded absolute value α on such a semigroup S leads in a natural way to a C^*-algebra $C^*(S, \alpha)$, and we describe a setting which permits us to conclude that such a C^*-algebra is a host algebra for a Lie group G.

We further explain how to attach to any such host algebra an invariant weak-$*$-closed convex set in the dual of the Lie algebra of G, enjoying certain nice convex geometric properties. If G is the additive group of a locally convex space, one may thus describe all host algebras arising this way in terms of weak-$*$-locally compact subsets of the dual space. This approach further leads to a host algebra whose representations correspond to the regular representations of the canonical commutation relations in infinitely many generators.

Capelli identities for Hermitian symmetric spaces

Kyo Nishiyama

Kyoto University

This is a joint work with Soo Teck Lee (NUS, Singapore) and Akihito Wachi (Hokkaido Institute of Technology).

We consider dual pair (G, G') inside a real symplectic group, where G is an automotphism group of Hermitian symmetric domain X. In this case, we present a formula for harmonic polynomials, which lives in the tangent space of X. We also discuss three different types of dual pairs, and related joint harmonics. These harmonics are killed by certain differential operators. We present a determinantal formula of these differential operators and give analogues of Capelli identity.

Tube domains and basic relative invariants

Takaaki Nomura

Kyushu University

This is a joint work with Hideyuki Ishi. Let w be a complex symmetric matrix of order r, and $\Delta_1(w), \ldots, \Delta_r(w)$ the principal minors of w. If w belongs to the Siegel right half space, then it is known that $\mathrm{Re}\big(\Delta_k(w)/\Delta_{k-1}(w)\big) > 0$ for $k = 1, \ldots, r$. In this talk we study this property in three directions. First we show that this holds for general symmetric right half spaces. Second we present a non-symmetric right half space with this property of arbitary rank ≥ 3. We note that case-by-case verifications up to dimension 10 tell us that there is only one such irreducible non-symmetric tube domain among 133 irreducibles in which 11 are symmetric. The proof of the property reduces to two lemmas. One is entirely generalized to non-symmetric cases as we present in the last part of tha talk. This is the third direction. As a byproduct of our study, we show that the basic relative invariants associated to a homogeneous regular open convex cone Ω studied earlier by Hideyuki Ishi are characterized as the irreducible factors of the determinant of right multiplication operators in the complexification of the clan associated to Ω.

Asymptotic spectral analysis and applications to complex networks

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Since the epoch-making papers by Watts–Strogatz (1998) and by Barabási–Albert (1999) the *network science* has become one of the most fashionable interdisciplinary research areas in current years. We are interested in spectral analysis of various network models to investigate further statistical properties, in particular, along with the quantum probabilistic techniques [1].

In asymptotic spectral analysis of large graphs (or growing graphs) some ideas of quantum probability are useful, for example (i) the method of quantum decomposition; (ii) various concepts of independence and related central limit theorems; (iii) partition statistics and moment-cumulant formulas. I will review these ideas quickly with examples [2] and show an application to a quantum walk [3]. Moreover, I discuss the Erdős–Rényi random graph and its generalization towards more realistic network models [4].

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Limit sets of Kleinian groups and Harmonic analysis

Martin Olbrich

Let X be the quotient of a real semisimple Lie group by a discrete subgroup. Harmonic analysis on function spaces on X is a fascinating subject, which has strong implications for our understanding of various aspects of locally symmetric spaces: spectral theory, topology, arithmetic, dynamics. Classically, one assumes that X has finite volume. We explain what can be done in certain infinite volume cases and highlight the role played by an interesting fractal set: the limit set of the discrete group.

An infinite-dimensional representation of a Lie supergroup and applications to autocorrelations (A. Püttmann, Bochum)

We derive explicit formula for the average $I(t) := \int_K Z(t,k)dk$, where K is one of the classical compact Lie groups $\mathrm{O}_N, \mathrm{SO}_N, \mathrm{Usp}_N \subset \mathrm{U}_N$ equipped with Haar measure dk normalized by $\int_K dk = 1$ and

$$Z(t,k) := \prod_{j=1}^{n} \frac{\mathrm{Det}(e^{\frac{i}{2}\psi_j}\,\mathrm{Id}_N - e^{-\frac{i}{2}\psi_j}k)}{\mathrm{Det}(e^{\frac{1}{2}\phi_j}\,\mathrm{Id}_N - e^{-\frac{1}{2}\phi_j}k)} = e^{\lambda_N} \prod_{j=1}^{n} \frac{\mathrm{Det}(\mathrm{Id}_N - e^{-i\psi_j}k)}{\mathrm{Det}(\mathrm{Id}_N - e^{-\phi_j}k)}$$

depends on the complex parameter $t = (\psi_1, \ldots, \psi_n, \phi_1, \ldots, \phi_n)$ which satifies $\mathfrak{Re}\phi_j > 0$ for all j and $\lambda_N = \frac{N}{2}\sum_{j=1}^{n}(i\psi_j - \phi_j)$.

The integral average $I(t)$ can be regarded as the numerical part of a character of an irreducible representation of a Lie supergroup $(\mathfrak{g}, G)$ restricted to a suitable subset of a maximal torus of G. The Lie superalgebra $\mathfrak{g}$ is the Howe dual partner of the compact group K in the orthosymplectic Lie superalgebra $\mathfrak{osp}(W)$. It is naturally represented on a certain infinite dimensional spinor-oscillator module A_V and the irreducible representation is that on the completion $\mathcal{A}_V^K$ of of the space of K–invariant elements of A_V.

This representation is irreducible and highest weight λ_N, and its character is uniquely determined by analyticity, Weyl group invariance, certain weight constraints and a system of differential equations coming from the Laplace-Casimir invariants of $\mathfrak{g}$.

The exact formula for $I(t)$, which looks very much like a classical Weyl formula, is derived in terms of the roots of the Lie superalgebra $\mathfrak{g}$ and the Weyl group W. Let us state this formula for $K = \mathrm{O}_N, \mathrm{Usp}_N$ without going into the details of the λ–positive even and odd roots $\Delta^+_{\lambda,0}$ and $\Delta^+_{\lambda,1}$ and the Weyl group W. If W_λ is the isotropy subgroup of W fixing the highest weight $\lambda = \lambda_N$, then

$$I(t) = \sum_{w \in W/W_\lambda} e^{w(\lambda_N)(t)} \frac{\prod_{\beta \in \Delta^+_{\lambda,1}}(1 - e^{w(\beta)(t)})}{\prod_{\alpha \in \Delta^+_{\lambda,0}}(1 - e^{w(\alpha)(t)})}.$$

To prove this formula, that holds even for small N, we check that the properties that uniquely characterize $I(t)$ are satisfied by the righthand side.

This completes earlier work of Conrey, Framer, and Zirnbauer for the case of U_N and others that obtained the above formula by different methods for N in the stable range.

Equivariant spectral asymptotics and compact group actions
(Pablo Ramacher, Göttingen University)

The asymptotic distribution of eigenvalues of elliptic operators has been the subject of mathematical research for many years. The first results were obtained by Weyl employing variational techniques. Later, Hörmander extended these results to elliptic pseudodifferential operators on a closed manifold M using the theory of Fourier integral operators. If Q_0 is an elliptic pseudodifferential operator of order m on $\mathrm{L}^2(M)$, which is positive and symmetric, its unique self-adjoint extension Q will have pure point spectrum. Denoting by E_λ the eigenspace of Q belonging to the eigenvalue λ, it can then be shown that the spectral counting function

$$N(\lambda) = \sum_{t \leq \lambda} \dim E_t,$$

which measures the asymptotic distribution of eigenvalues, satisfies Weyl's law

$$N(\lambda) = \frac{\operatorname{vol} B^*M}{(2\pi)^n} \lambda^{n/m} + O(\lambda^{(n-1)/m}), \qquad \lambda \to \infty,$$

where $\operatorname{vol} B^*M$ denotes the volume of a certain subset of the cotangent bundle $T^*(M)$ of M. Let now G be a compact Lie group acting on M by isometries, and assume that Q commutes with the regular representation of G in $\mathrm{L}^2(M)$. It is then natural to ask for an asymptotic formula for

$$N_\kappa(\lambda) = d_\kappa \sum_{t \leq \lambda} \mu_\kappa(t),$$

where $\mu_\kappa(t)$ denotes the multiplicity of the irreducible representation of G belonging to the character $\kappa \in \hat{G}$ of dimension d_κ in the eigenspace E_t. While first order asymptotics for $N_\kappa(\lambda)$ can be obtained in the general case of effective group actions by using heat kernel methods, the derivation of remainder estimates within the framework of Fourier integral operators meets with serious difficulties when singular orbits are present. The reason for this is that the critical set of the considered phase function is no longer a smooth manifold, so that, a priori, the principle of the stationary phase can not be applied. In this talk, we show how to circumvent this obstacle by partially resolving the singularities of the critical set to obtain remainder estimates for $N_\kappa(\lambda)$ in the case of singular group actions.

Paul Ressel

Exchangeable probability measures and positive definite functions

Exchangeability of a "random object" is a strong symmetry condition, leading in general to a convex set of distributions not too far from a "simplex" - a set easily described by its extreme points, in this case distributions with very special properties as for example iid coin tossing sequences in de Finetti's original result. Although in most cases of interest the symmetry is defined via a non-commutative group acting on the underlying space, it very often can be described by a suitable factorization involving an abelian semigroup. The factorizing function typically turns out to be positive definite, and results from Harmonic Analysis on semigroups become applicable. In this way many known theorems on exchangeability can be given another proof, more analytic/algebraic in a sense, but also new results become available. It should be remarked that the results in question are genuinely infinite dimensional, their finite dimensional analogues are not true.

Convolution structures associated with multivariate hypergeometric functions

M. Rösler

Technische Universität Clausthal

Spherical functions on Riemannian symmetric spaces are known to occur as particular cases of hypergeometric functions associated with root systems within the theories of Dunkl, Heckman and Opdam. In this talk, we start from discrete series of geometric cases associated with Grassmann manifolds and their tangent spaces. We interpolate the product formulas of the associated spherical functions which yields commutative hypergroup structures with multivariate hypergeometric functions as characters, beyond the geometric cases. This generalizes well-known one-dimensional convolution structures of Bessel and Jacobi type to higher rank.

On the limit distribution of coupled continuous time random walks

H.P. Scheffler

Fachbereich Mathematik, Universität Siegen

The continuous time random walk (CTRW) model incorporates waiting times J_i between jumps Y_i of a particle. Classical assumptions are that $J_1, J_2, \ldots$ are iid belonging to some domain of attraction of a stable subordinator $D(t)$; $Y_1, Y_2, \ldots$ are iid belonging to the domain of attraction of an (operator) stable Lévy motion $A(t)$ and that (J_i) and (Y_i) are independent.

We present a two-fold generalization of this model by considering general triangular arrays $\Delta = \{(J_i^{(c)}, Y_i^{(c)}) : i \geq 1, c \geq 1\}$ of waiting times and jumps with iid rows and allowing arbitrary dependence between the waiting time $J_i^{(c)}$ before the jump $Y_i^{(c)}$. We assume that the row sums of Δ converge in distribution to some space-time Lévy process $\{(A(t), D(t))\}$. In this general setting the limiting distribution of the generalized CTRW modelled by Δ is of the form $M(t) = A(E(t))$ where $E(t)$ is the hitting time process of the subordinator $D(t)$, as in the classical case. However, since $A(t)$ and $D(t)$ are dependent, $A(t)$ and $E(t)$ are dependent. It turns out the the distribution of $M(t)$ can be represented in terms of $(A(t), D(t))$ even in this general coupled case. Moreover the Fourier-Laplace transform of the distribution of $M(t)$ is the solution to the so-called master equation in statistical physics. Finally the distribution of $M(t)$ is also the mild-solution of a coupled in space and time pseudo-differential equation generalizing fractional PDEs.

UNITARY REPRESENTATIONS AND QUASI-INVARIANT MEASURES ON INFINITE DIMENSIONAL GROUPS

Hiroaki Shimomura

Department of Mathematics, Faculty of Education, Kochi University

This talk concerns two subjects on representations of topological groups G.

First, we assume that G has a σ-finite measure μ on the Borel field $\mathfrak{B}(G)$ whose right admissible shifts form a dense subgroup G_0. Then, we claim that

(1) every continuous unitary representation of G has an irreducible decomposition (cf. [4]) and that

(2) every positive-definite continuous function on G correspondes to a unique (up to equivalence) continuous unitary representation with a cyclic vector through a method similar to that used for the G-N-S construction (cf. [5]).

The resuts remain true, if we go to the inductive limits of such groups.

Finally, we proceed to an important example of infinite-dimensional groups, the group of diffeomorphisms $\mathrm{Diff}_0^*(M)$ on smooth manifolds M, and see that these results are valid under a fairy mild condition.

In the second part of this talk, we assume that G is locally compact with the neutral element e. Let ϕ be a continuous positive-definite function such that $\phi(e) = 1$. The cyclic unitary representation corresponding to ϕ has an irreducible decomposition through Mautner's method, and as a result, we have an extreme decomposition of ϕ to extreme positive-definite continuous functions ϕ_λ with a probability measure space (Λ, μ). Conversely, suppose that we are given a disintegration of ϕ

$$\phi(g) = \int_\Lambda \phi_\lambda(g) d\mu(\lambda) \quad \text{for all} \quad g \in G,$$

with extreme continuous positive-definite functions ϕ_λ and a probability measure space (Λ, μ). Then we ask whether this decomposition is obtained through that of the Mautner method or not, and find a necessary and sufficient condition (NS) for the positive answer.

Integral expressions of indecomposable characters given by Obata (cf. [1]) which was concerend with the work of indecomposable character of Thoma (cf.[6]) has to do with this question. We will show that (NS) does not hold in general, since some of the integral expressions in [1] are the counter-examples.

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LIMIT THEOREMS FOR RANDOM MATRIX ENSEMBLES ASSOCIATED TO SYMMETRIC SPACES

MICHAEL STOLZ, UNIVERSITY OF BOCHUM

This talk surveys recent work which develops aspects of classical random matrix theory in the broader framework of matrix ensembles associated to classical symmetric spaces. It is a classical result of Wigner that for an hermitian matrix with independent entries on and above the diagonal, the mean empirical eigenvalue distribution converges weakly to the semicircle law as matrix size tends to infinity. In joint work [6] with Katrin Hofmann-Credner (Bochum), this has been generalized to random matrices taken from all infinitesimal versions of classical symmetric spaces. Like Wigner's, this result is universal in that it only depends on certain assumptions about the moments of the matrix entries, but not on the specifics of their distributions. In the special case of Gaussian matrix entries, it can be sharpened to a Large Deviations Principle for the empirical eigenvalue distribution ([5], joint work with Peter Eichelsbacher, Bochum), thus generalizing a well-known result of Ben Arous and Guionnet [2].

Joint work [4] with Benoît Collins (Lyon/Ottawa) is devoted to random vectors of the form $(\mathrm{Tr}(A^{(1)}V), \ldots, \mathrm{Tr}(A^{(r)}V))$, where V is a uniformly distributed element of a matrix version of a classical compact symmetric space, and the $A^{(\nu)}$ are deterministic parameter matrices. It is proven that for increasing matrix sizes these random vectors converge to a joint Gaussian limit. This generalizes work of Diaconis et al. (e.g. [1]) on the compact classical groups. The proof uses integration formulae of Collins and Śniady [3], which are firmly rooted in classical invariant theory. This motivates a final remark on another aspect of the interplay between matrix integrals and invariant theory, based on joint work [7] with Tatsuya Tate (Nagoya).

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Michael Voit:
Limit theorems for radial random walks of high dimensions

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Consider a problem of the following kind: Let ν be a fixed probability measure on $[0,\infty[$ satisfying suitable moment conditions; for each dimension $p \in \mathbf{N}$ consider the radial, time homogeneous random walk $(S_k^p)_{k\geq 0}$ on $\mathbf{R}^p$ starting at $0 \in \mathbf{R}^p$, such that at each step of time the jumps are independent with uniformly distributed directions and jump sizes with distribution ν. The task is now to find limit theorems for the $[0,\infty[$-valued random variables $\|S_k^{p_k}\|_2$ for $k \to \infty$ for suitable sequences $(p_k)_{k\in N} \subset \mathbf{N}$ of dimensions with $p_k \to \infty$.

In this talk we present laws of large numbers and a large deviation principle in this setting. The proofs are based on considering the distributions of the $[0,\infty[$-valued random variables $\|S_k^p\|$ as Bessel-type convolution powers of ν which can be analyzed by using Hankel transforms. Using an uniform asymptotic approximation of Bessel functions of high indices in terms of a suitable exponential, one obtains limit relations for Laplace transforms for the distributions of the $\|S_k^p\|_2$ which then leads to limit theorems.

We shall also present an extension of these results to a higher rank case where $\mathbb{R}^p$ is replaced by the space of $p\times q$ matrices over $\mathbb{F} = \mathbb{R}, \mathbb{C}$ or the quaternions, and $[0,\infty[$ by the cone of positive semidefinite matrices. In this setting, similar results hold, where now Bessel functions J_μ of matrix argument and asymptotic results for the J_μ as $\mu \to \infty$ appear. We finally mention that these results admit also interpretations for Dunkl-type Bessel functions of type B_q.

The talk is based on joint work with M. Rösler.

Reference: http://arxiv.org/abs/math/0703520.

Isotropy representations and Howe duality correspondence

Hiroshi Yamashita (Hokkaido University)

This is a continuation of my talks "*Two dual pair methods in the study of generalized Whittaker models for irreducible highest weight modules*" (1999, Kyoto) and "*Isotropy representations for unitary highest weight modules and harmonic analysis on certain compact symmetric spaces*" (2003, Tübingen) delivered at the previous German-Japanese symposia on infinite dimensional harmonic analysis.

The notion of *isotropy representation*, attributed to David Vogan, gives a refinement of the multiplicity in the associated cycle attached to Harish-Chandra modules for real reductive groups G. It plays an essential role to understand infinite dimensional irreducible representations of G in connection with nilpotent orbits in the Lie algebras.

In this talk, I will survey some recent progress on the study of isotropy representations for Harish-Chandra modules with irreducible associated variety. More precisely, we first discuss the isotropy representations for the discrete series by means of the principal symbol of differential operators of gradient-type on Riemannian symmetric spaces. Together with a recent result by Barchini and Zierau, a precise relationship will be clarified between the above principal symbol and the fiber of the moment map defined on certain conormal bundle on generalized flag variety. Secondly, we look at the Howe duality correspondence for the reductive dual pairs where one of the members is compact. This duality theorem, or a classical result of Kashiwara and Vergne can be reproduced to large extent, by using the isotropy representation for the tensor products of the Weil representation.

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