Holographic dual of generalized symmetry, mixed anomaly, and interfaces in $\mathcal{N}=4$ SYM

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Plan of this talk

Part I: $\mathcal{N} = 4$ $su(N)$ SYM theories from (B_2, C_2) boundary conditions and branes in AdS_5

(i) Gauging center symmetry, (ii) Line operators, (iii) 1-form symmetry, (iv) $SL(2,Z)$ duality orbits

Part II: Mixed 't Hooft anomaly from type IIB SUGRA

(i) CP violating anomaly action by gauging 1-form symmetry

Part III: Axionic Janus as interfaces between different θ angles

Motivation

- Recently, nonlocal operators and higher form symmetries (under which they are charged) gained renewed interests in QFT.

- In particular, they provide new insights into the phase structure of QFT via mixed ’t Hooft anomaly involving higher form symmetries.

- Less known, but they can also be used to reveal the intricate structure of the duality web.

Motivation — cont’d

- Meanwhile, string theory hosts, naturally, nonlocal operators (objects) and associated higher form symmetries.

- Thus, via holography (AdS/CFT), one may be able to gain a more intuitive and somewhat simpler understanding of the recent developments in higher form symmetries.

  Hofman-Iqbal

  Many others in the last slide

- Hopefully, this line of thought provides new perspectives on the subject.

Part I: $\mathcal{N}=4$ $su(N)$ SYM theories from (B_2, C_2) boundary conditions and branes in AdS_5

1. Gauging center symmetry

- The $SU(N)$ group has the Z_N center symmetry. By gauging a subgroup of Z_N, we can construct new theories:

$$\mathcal{N}=4\ \ SU(N)/Z_k\ \ \text{SYM theories} \qquad \text{with}\ \ N=kk'$$

- This is not the end of the story and the life is more intricate:

  The $SU(N)/Z_k$ theory are further classified into sub-theories distinguished by the line operator spectrum

  Aharony-Seiberg-Tachikawa

Part I — cont'd

2. Line operator spectrum

Aharony-Seiberg-Tachikawa

- The line operators are Wilson, 't Hooft, and dyonic lines.

$$L_{k,\ell} := \{(z_e, z_m) = \underbrace{\boxed{e(k,0)}}_{k \text{ Wilson lines}} + \underbrace{\boxed{m(\ell, k')}}_{k' \text{ 't Hooft lines } + \ \ell \text{ Witten effect}} \mod N\}$$

- The $su(N)$ theories (center symmetry gauging + line operator spectrum)

$$\boxed{\mathcal{N} = 4 \ \ [SU(N)/Z_k]_\ell \ \ \text{SYM theories}}$$

where $\ell = 0,1,\cdots,k-1$ with $N = kk'$

Part I — cont'd

3. 1-form symmetry

- The most basic $SU(N)$ theory has (electric) Z_N 1-form symmetry.

= gauging magnetic Z_N symmetry

$$W(L) \xrightarrow{Z_N} \underbrace{e^{\frac{2\pi i}{N}}}_{Z_N \text{ charge}} W(L)$$

- The $SU(N)/Z_k$ theories has (electric) $Z_{k'}$ 1-form symmetry:

= gauging magnetic $Z_{k'}$ symmetry

$$\underbrace{W(L)^k}_{Z_k \text{ invariant}} \xrightarrow{Z_{k'}} e^{\frac{2\pi ik}{N}} W(L)^k = \underbrace{e^{\frac{2\pi i}{k'}}}_{Z_{k'} \text{ charge}} W(L)^k$$

- In fact, the $[SU(N)/Z_k]_0$ theory has $Z^e_{k'} \times Z^m_k$ 1-form symmetry:

$$\underbrace{T(L')^{k'}}_{Z_{k'} \text{ invariant}} \xrightarrow{Z_k} e^{\frac{2\pi ik'}{N}} T(L')^{k'} = \underbrace{e^{\frac{2\pi i}{k}}}_{Z_k \text{ charge}} T(L')^{k'}$$

Part I — cont'd

3. 1-form symmetry

- The 1-form symmetry of the $[SU(N)/Z_k]_\ell$ theory

$$\big(\underbrace{Z_{k'}}_{\text{electric}} \times \underbrace{Z_{N/gcd(k',\ell)}}_{\text{dyonic}}\big)/Z_{k'/gcd(k',\ell)}$$

for the line operator spectrum

$$L_{k,\ell} := \{(z_e, z_m) = \underbrace{\boxed{e(k,0)}}_{k\ \text{Wilson lines}} + \underbrace{\boxed{m(\ell, k')}}_{k'\ \text{'t Hooft lines}\ +\ \ell\ \text{Witten effect}} \bmod N\}$$

represented by

$$\left\{ \underbrace{W(L)^k}_{Z_{k'}\ \text{charged}}\ ,\ \underbrace{W(L)^\ell T(L)^{k'}}_{Z_{N/gcd(k',\ell)}\ \text{charged}} \right\} \quad \textbf{with} \quad \underbrace{\left(W(L)^k\right)^\ell = \left(W(L)^\ell T(L)^{k'}\right)^k}_{Z_{k'/gcd(k',\ell)}\ \text{charged}}$$

Part I — cont'd

- In the diagonal basis, the 1-form symmetry of the $[SU(N)/Z_k]_\ell$ theory

$$\boxed{Z_{N/\gcd(k,k',\ell)} \times Z_{\gcd(k,k',\ell)}}$$

Gaiotto-Kapustin-Seiberg-Willett

under which the following line operators are charged:

$$W(L)^{pk+\ell}T(L)^{k'} \xrightarrow{Z_{N/\gcd(k,k',\ell)}} e^{\frac{2\pi i}{N/\gcd(k,k',\ell)}}\left(W(L)^{pk+\ell}T(L)^{k'}\right)$$

$$W(L)^{\frac{\delta N}{\gcd(k,k',\ell)}}T(L)^{\frac{\gamma N}{\gcd(k,k',\ell)}} \xrightarrow{Z_{\gcd(k,k',\ell)}} e^{\frac{2\pi i}{\gcd(k,k',\ell)}}\left(W(L)^{\frac{\delta N}{\gcd(k,k',\ell)}}T(L)^{\frac{\gamma N}{\gcd(k,k',\ell)}}\right)$$

Bergman-SH

where there always exists $^\exists p \in Z$ such that $\gcd(pk+\ell, k') = \gcd(k,k',\ell)$

$$\delta\frac{k'}{\gcd(k,k',\ell)} - \gamma\frac{pk+\ell}{\gcd(k,k',\ell)} = 1$$

Part I — cont'd

✻ Gravity dual description

- The line operator spectrum of all $[SU(N)/Z_k]_\ell$ theories and its center symmetry are all encoded in the topological theory   Witten & Aharony-Witten

$$S_{top} = \int_{AdS_5 \times S^5} B_2 \wedge dC_2 \wedge dC_4 = \frac{N}{2\pi} \int_{AdS_5} B_2 \wedge dC_2$$

where (B_2, C_2) are a canonical conjugate pair like (x, p) upon quantisation

$$[b, c] = \frac{2\pi i}{N} \qquad \textbf{with} \qquad b = \int_S B_2 \ , \quad c = \int_S C_2$$

- The admissible boundary conditions

equivalent to Dirac quantisation condition

$$Z_e\, b + Z_m\, c = 0 \ , \quad Z'_e\, b + Z'_m\, c = 0 \qquad \textbf{with} \qquad \underbrace{Z_e Z'_m - Z'_e Z_m = N}_{\text{two BCs commute mod } N}$$

Part I — cont'd

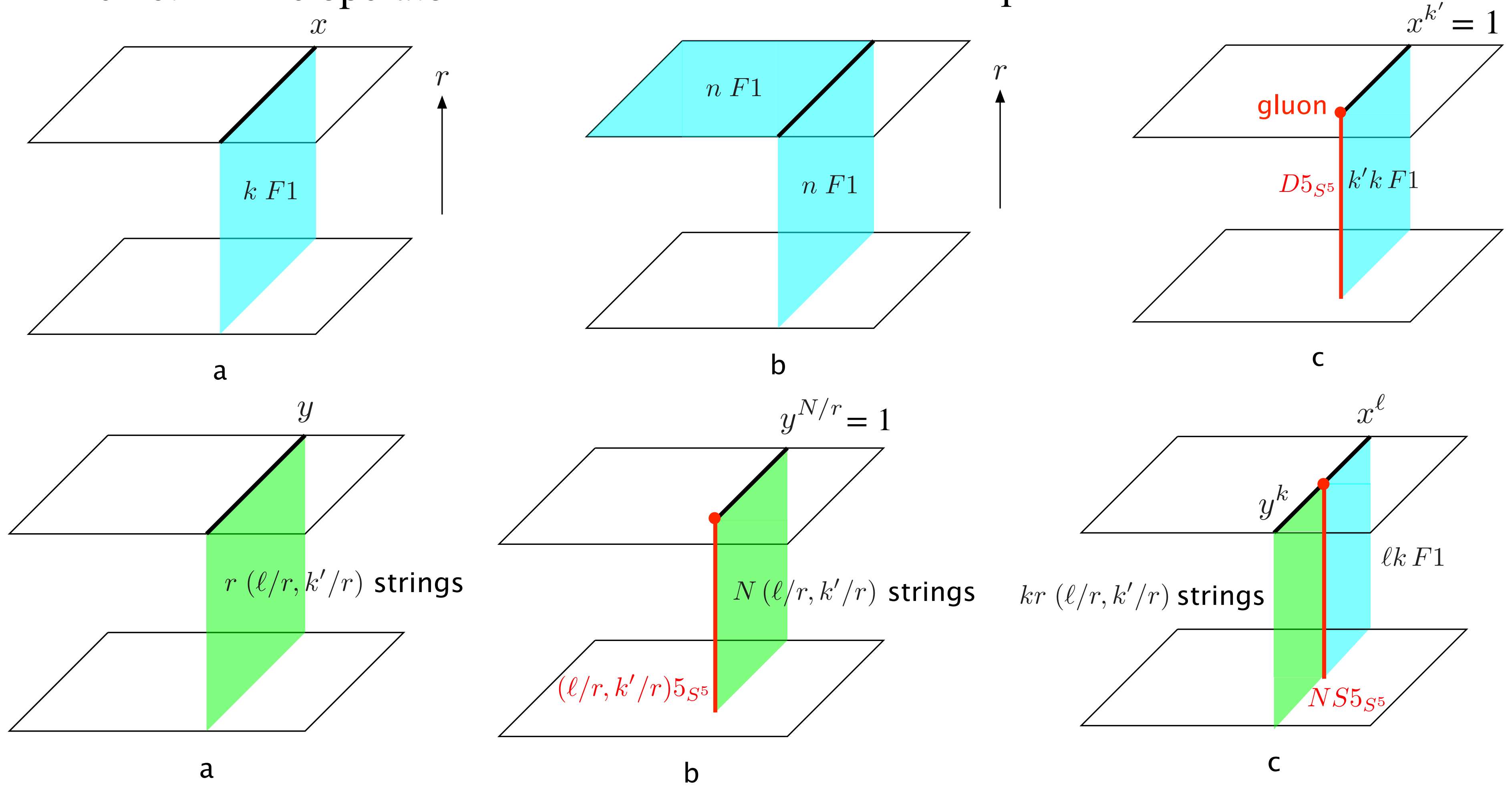
By an $SL(2,Z)$ rotation, the BCs can be brought into the canonical form

Bergman-SH

$$\begin{pmatrix} Z_e & Z_m \\ Z'_e & Z'_m \end{pmatrix} \longrightarrow \begin{pmatrix} k & 0 \\ \ell & k' \end{pmatrix} \quad : \quad \underbrace{kb = 0}_{k\ \mathrm{F1}} \ , \quad \underbrace{k'c + \ell b = 0}_{(\ell, k')\ \mathrm{string}}$$

where $\ell = 0, 1, \cdots, k-1$

"Dirichlet" = line operator "Neumann" = surface operator



$$x \leftrightarrow kb = 0$$
$$y \leftrightarrow k'c + \ell b = 0$$
$$r = gcd(k', \ell)$$
$$n \neq mk \ , N$$

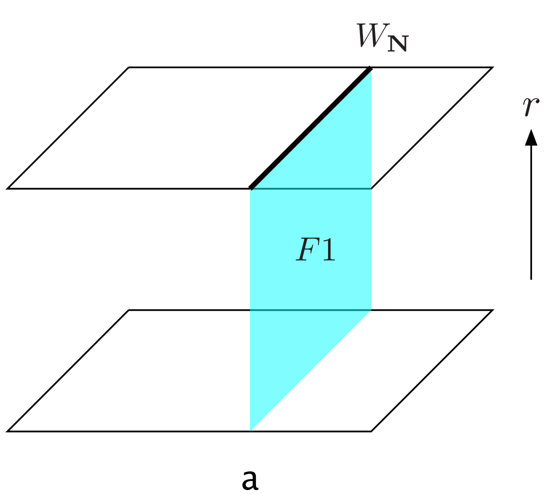
Part I — cont'd

NB: The BCs on (line or point) D5 and NS5-branes wrapping S^5

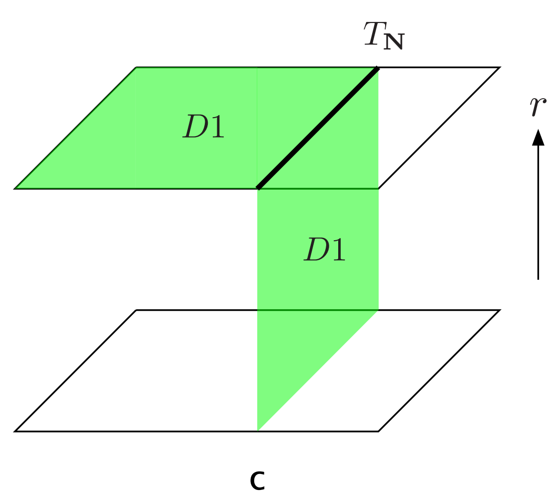
As an illustration, consider the $SU(N) = [SU(N)]_0$ theory: cousins of baryon vertex

"Dirichlet" wrapped $D5$ point ("gluon") : $A_1 = \int_{S^5} C_6 = 0$ (Hodge dual of Neumann $C_2 \neq 0$)
on which N F1s ending

"Neumann" wrapped $NS5$ line : $A_1' = \int_{S^5} B_6 \neq 0$ (Hodge dual of Dirichlet $B_2 = 0$)
on which N D1s ending

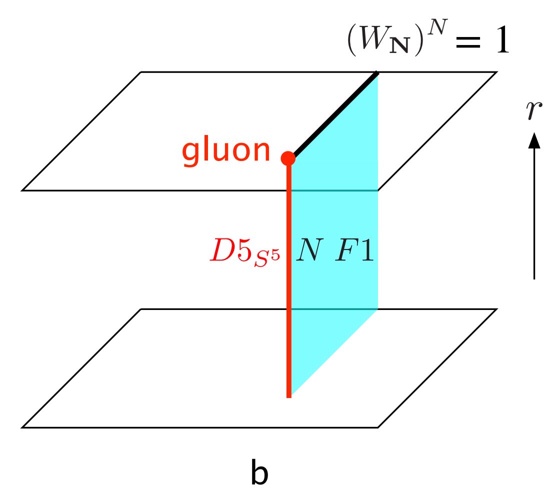


$b = 0$



$Nc = 0 \leftrightarrow nc \neq 0$

$n = 0,1,\cdots,N-1$



$Nb = 0$

$D1$ surface = $AdS_3 \times S^1$ $D3$ with F Gukov-Witten, Drukker-Gomis-Matsuura

Part I — cont'd

4. $SL(2,Z)$ duality orbits from gravity dual

See Aharony-Seiberg-Tachikawa for the original field theory discussions

- The different $[SU(N)/Z_k]_\ell$ theories are connected by $SL(2,Z)$.

- However, not all the $[SU(N)/Z_k]_\ell$ theories belong to a single $SL(2,Z)$ orbit. There are "islands" of $SL(2,Z)$ orbits distinguished by the 1-form symmetry or the $SL(2,Z)$ invariant $\gcd(k,k',\ell)$:

$$Z_{N/\gcd(k,k',\ell)} \times Z_{\gcd(k,k',\ell)}$$

- This can be most manifestly understood from the $SL(2,Z)$ transformations of the boundary conditions (in the diagonal basis of the 1-form symmetry):

$$\boxed{\begin{array}{l} Z_{N/\gcd(k,k',\ell)}: \\ Z_{\gcd(k,k',\ell)}: \end{array} \quad \begin{pmatrix} pk+\ell & k' \\ \frac{\delta N}{\gcd(k,k',\ell)} & \frac{\gamma N}{\gcd(k,k',\ell)} \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} \equiv M_D \begin{pmatrix} b \\ c \end{pmatrix} = 0}$$

$$\begin{array}{lll} T: & B \to B\,, & C \to C+B \\ S: & B \to -C\,, & C \to B \end{array}$$

Part I — cont'd

4. $SL(2,Z)$ duality orbits from gravity dual

- The duality orbit can be understood from the following relation:

$$M_D \underbrace{\begin{pmatrix} \frac{pk+\ell}{gcd(k,k',\ell)} & \frac{k'}{gcd(k,k',\ell)} \\ \delta & \gamma \end{pmatrix}}_{SL(2,Z)\text{ duality rotation}} = \underbrace{\begin{pmatrix} gcd(k,k',\ell) & 0 \\ 0 & N/gcd(k,k',\ell) \end{pmatrix}}_{[SU(N)/Z_{gcd(k,k',\ell)}]_0\text{ theory}}$$

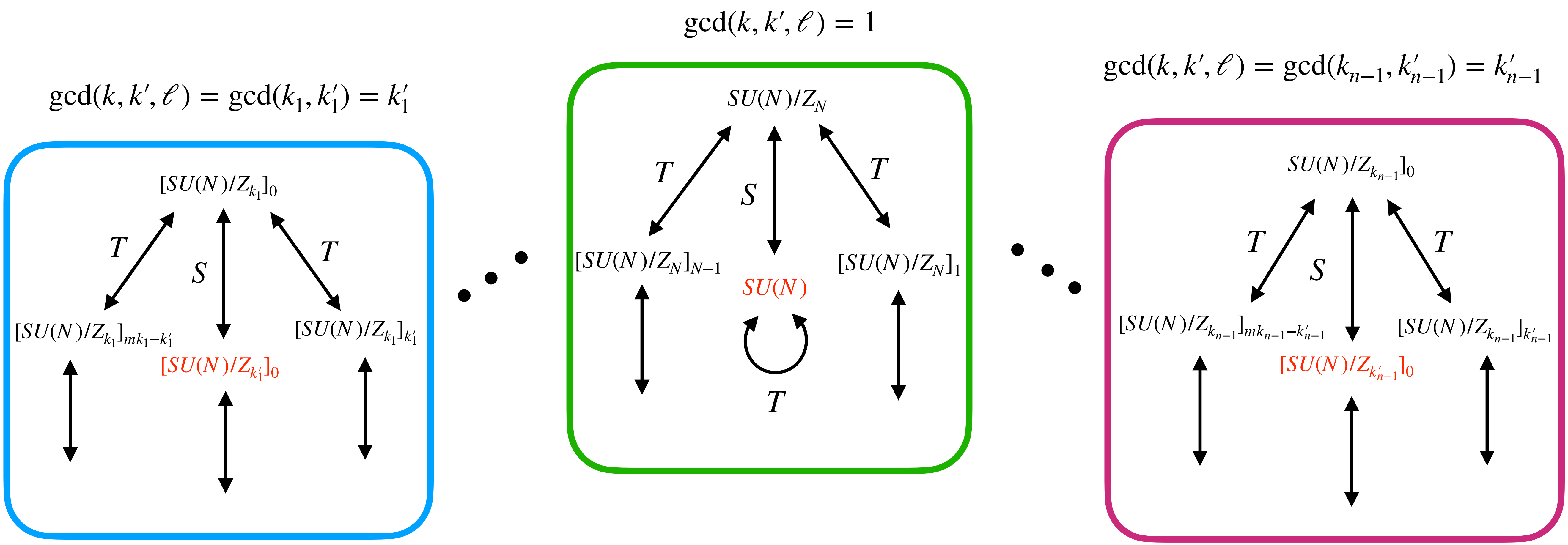
Starting from the $[SU(N)/Z_{\mathbf{k}}]_0$ theory with $\mathbf{k} = gcd(\mathbf{k},\mathbf{k}') = gcd(k,k',\ell)$, all $su(N)$ theories with the same $gcd(k,k',\ell)$ can be generated by $SL(2,Z)$ duality transformations.

Part I — cont'd

4. $SL(2,Z)$ duality orbits

Bergman-SH

$$\boxed{\#\text{ of } SL(2,Z) \text{ orbits} \quad = \quad \#\text{ of } k \text{ satisfying } k = \gcd(k,k')}$$



$SL(2,Z)$ duality web of $\mathcal{N}=4$ $su(N)$ SYM theories

Part II: Mixed 't Hooft anomaly from type IIB SUGRA

1. Field theory

Gaiotto-Kapustin-Kormagodski-Seiberg

- There is a mixed 't Hooft anomaly by gauging the (electric) $Z_{k'}$ subgroup of the 1-form symmetry in the presence of θ, which breaks CP at $\theta = \pi$.
  For the $SU(N)$ theory ($k' = N$)

Gaiotto-Kapustin-Seiberg-Willett

$$Z[\theta + 2\pi] = Z[\theta] \exp\left[2\pi i \,\frac{N-1}{N} \int_X \frac{\mathcal{P}(\mathsf{B})}{2}\right]$$

where $\mathsf{B} \in H^2(X, Z_N)$ the background 2-form gauge field, $\mathcal{P}(\,\cdot\,)$ the Pontryagin square operation; $\mathcal{P}(\mathsf{B})/N \simeq$ fractional instanton number by Z_N gauging.

- The anomaly action (to be reproduced by gravity dual)

$$S_{5d} = 2\pi i \,\frac{N-1}{N} \int \frac{d\theta}{2\pi} \frac{\mathcal{P}(\mathsf{B})}{2} \quad \xrightarrow{SU(N)/Z_k} \quad 2\pi i \,\frac{N(N-1)}{k'^2} \int \frac{d\theta}{2\pi} \frac{\mathcal{P}(\mathsf{B})}{2}$$

where $\mathsf{B} \in H^2(X, Z_{k'})$ for the latter

Part II — cont'd

✻ Gravity dual description

Bergman-SH

- The CS action in type IIB SUGRA reproduces the anomaly action:

$$S_{top} = \int_{AdS_5 \times S^5} B_2 \wedge dC_2 \wedge dC_4 \xrightarrow{\tilde{F}_3 = dC_2 - C_0 dB_2 = 0} -2\pi N \int_{AdS_5} dC_0 \wedge B_2 \wedge B_2$$

where $\mathsf{B} = k' B_2 \in H^2(X, Z_{k'})$ and $C_0 = \theta/2\pi$

- To be more precise, the $\mathcal{O}(N^2)$ part is missing. However, on spin manifolds X, $\int_X \mathscr{P}(\mathsf{B})/2 \in \mathbb{Z}$ and the anomaly indeed agrees.

Part III: Axionic Janus as interfaces between different θ angles

1. Field theory

- In the case of pure $SU(N)$ YM theory, the mixed 't Hooft anomaly implies that CP at $\theta = \pi$ is spontaneously broken (out of three logical possibilities, nontrivial gapless theory, gapped TFT, or SSB).

NB: can't be a gapped trivial theory due to a nontrivial anomaly

- For pure $SU(N)$ YM theory, the IR theory is gapped and it is known that CP is SSB for $N \to \infty$. Hence a domain wall exists between the $\theta = \pm\pi$ vacua.

- However, since $\mathcal{N} = 4$ SYM is a CFT, there cannot be SSB but it can still "saturate" the anomaly because it's a nontrivial gapless theory.

Part III: Axionic Janus as interfaces between different θ angles

1. Field theory

Gaiotto-Komargodski-Seiberg

- Even though there is no domain wall, a non-dynamical interface, which is nonetheless similar to the domain wall, can exist between $\theta = \pm\pi$ simply by varying θ over the space.

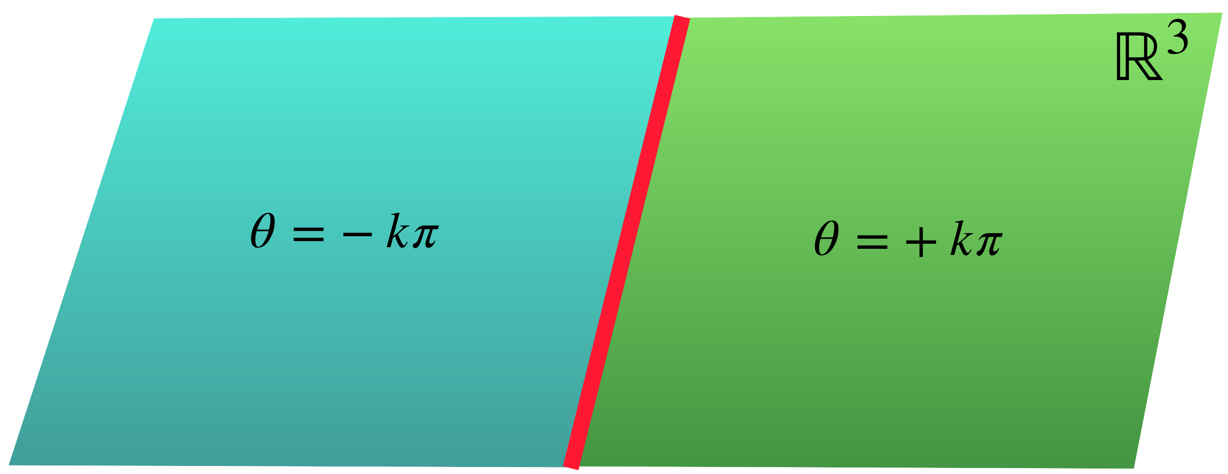
- Since $\mathcal{N} = 4$ SYM is a CFT, without introducing a scale by hand, the interface must be sharp, $\nabla\theta \to \infty$.

- The sharp jump of θ adds to the action the $SU(N)_k$ CS term at the interface. So the interface theory is plausibly the $SU(N)_k$ CS theory, which is level-rank dual to the $U(k)_{-N}$ CS theory.

Part III — cont'd

✻ Gravity dual description

- Since the D7 charge $Q_{D7} = \int dC_0$, the jump of θ can be described by D7-branes in $AdS_5 \times S^5$.



k D7 branes wrapping S^5 = 2 dimensional wall in $\mathbb{R}^3$

	0	1	2	3	4	5	6	7	8	9
D3	×	×	×	×						
D7	×	×	×			×	×	×	×	×

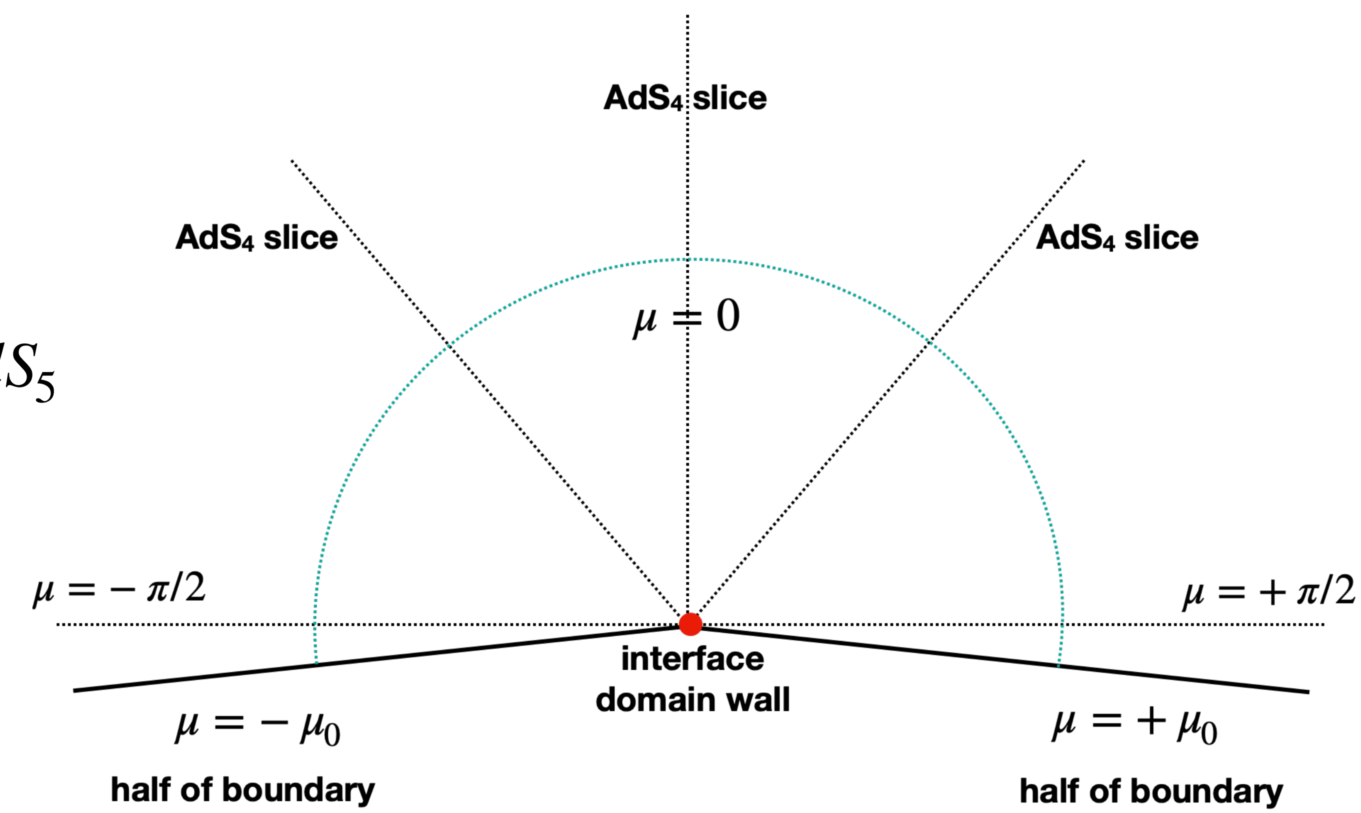
$DN = 6$ non-SUSY

Part III — cont'd

✻ Gravity dual description

- The interface has two faces and there exists a geometry that exactly has such a feature — Janus geometry (the Roman two-faced God) Bak-Gutperle-Hirano

Janus = deformed AdS_5
with two faces

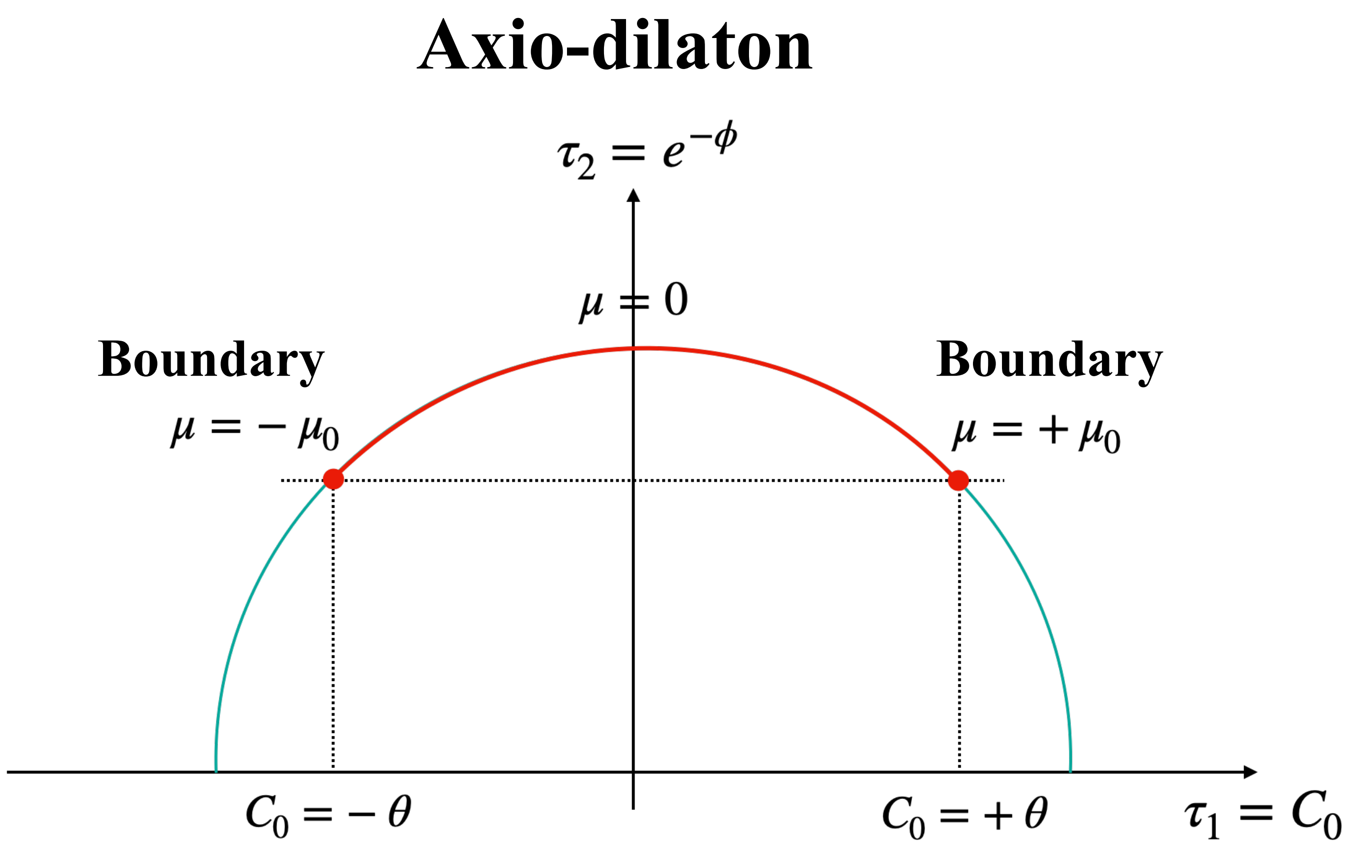


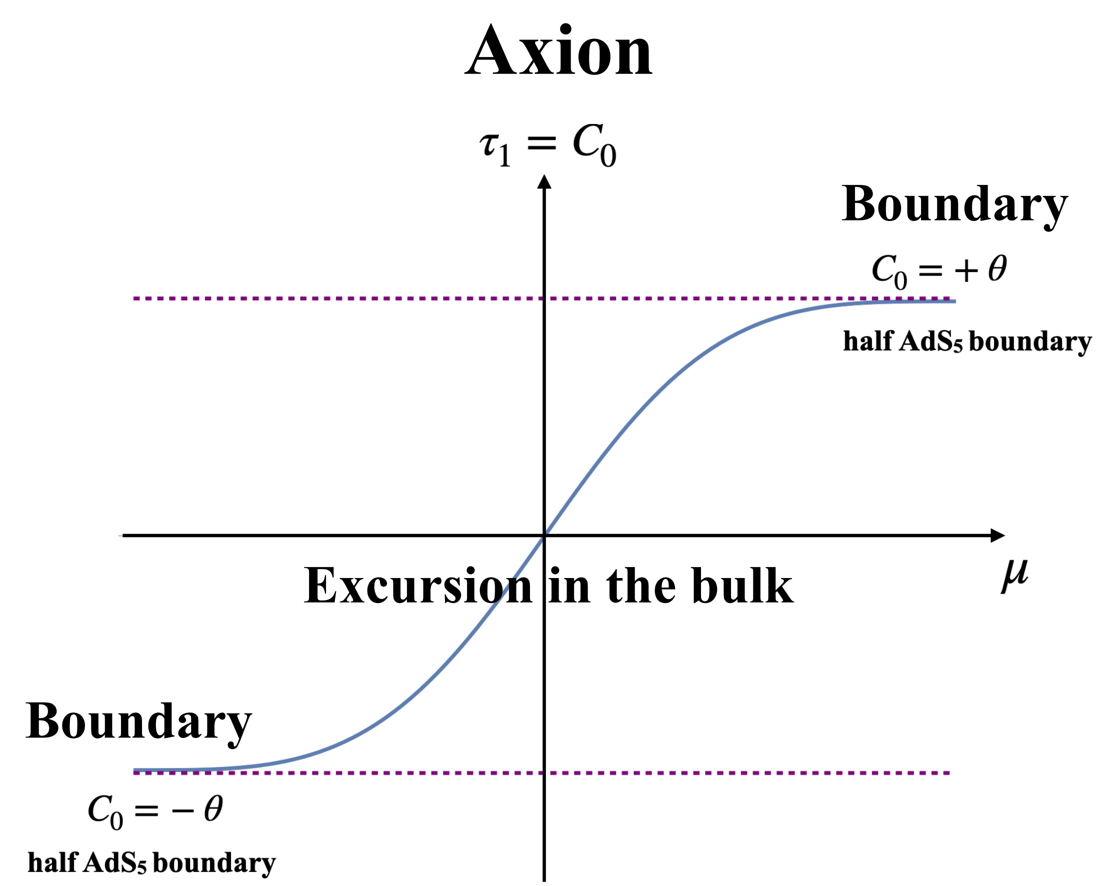
- The original Janus is purely dilatonic, but it has been generalized to include the axion. The one of our interest is the Janus for which the dilaton (dual to the gauge coupling) is constant on the boundary. D'Hoker-Estes-Gutperle

Part III — cont'd

✻ Gravity dual description

- The relevant part of type IIB SUGRA is that of gravity $g_{\mu\nu}$ and axio-dilaton $\tau = C_0 + ie^{-\phi}$





The dilaton = YM coupling does not vary in the boundary, whereas the axion = θ angle jumps across the interface

Another illustration of a sadden jump of the axion across the interface in the boundary

Part III — cont'd

✻ Gravity dual description

- The interface theory is described by the D7 $U(k)_{-N}$ CS action

$$S_{D7} = \int_{R^{1,2}\times S^5} C_4 \wedge \mathrm{Tr}_{U(k)}(F \wedge F) = -N\int_{R^{1,2}} \mathrm{Tr}_{U(k)}(A \wedge F)$$

- This agrees with the field theory expectation: The sharp jump of θ adds to the action the $SU(N)_k$ CS term at the interface which is level-rank dual to the $U(k)_{-N}$ CS theory.

Discussions

- $so(N), spin(N), sp(N)$ theories worked out Bergman-SH
- ABJM theories Bergman-Tachikawa-Zafrir
- Klebanov-Strassler theory Apruzzi-van Beest-Gould-Schaefer-Nameki
- Witten-Sakai-Sugimoto model Argurio-Bertolini-Bigazzi-Cotrone-Niro

- The $SL(2,Z)$ is not a symmetry of type IIB string theory on $AdS_5 \times S^5$ with the boundary conditions properly taken into account. It maps one theory to another. Nevertheless, given the recent discussions on the ensemble interpretation of holography, it might be interesting to consider an ensemble of $\mathcal{N} = 4$ $su(N)$ SYM theories with a fixed value of $gcd(k, k', \ell)$ that are connected by $SL(2,Z)$ transformations.

Thank you!

Part III — cont'd

✻ Gravity dual description

Ansatz

$$ds^2 = h(\mu)\left(d\mu^2 + ds^2_{AdS_4}\right) + d\Omega_5^2$$

$$F_5 = 2h(\mu)^{5/2} d\mu \wedge \omega_{AdS_4} + 2\omega_{S^5}$$

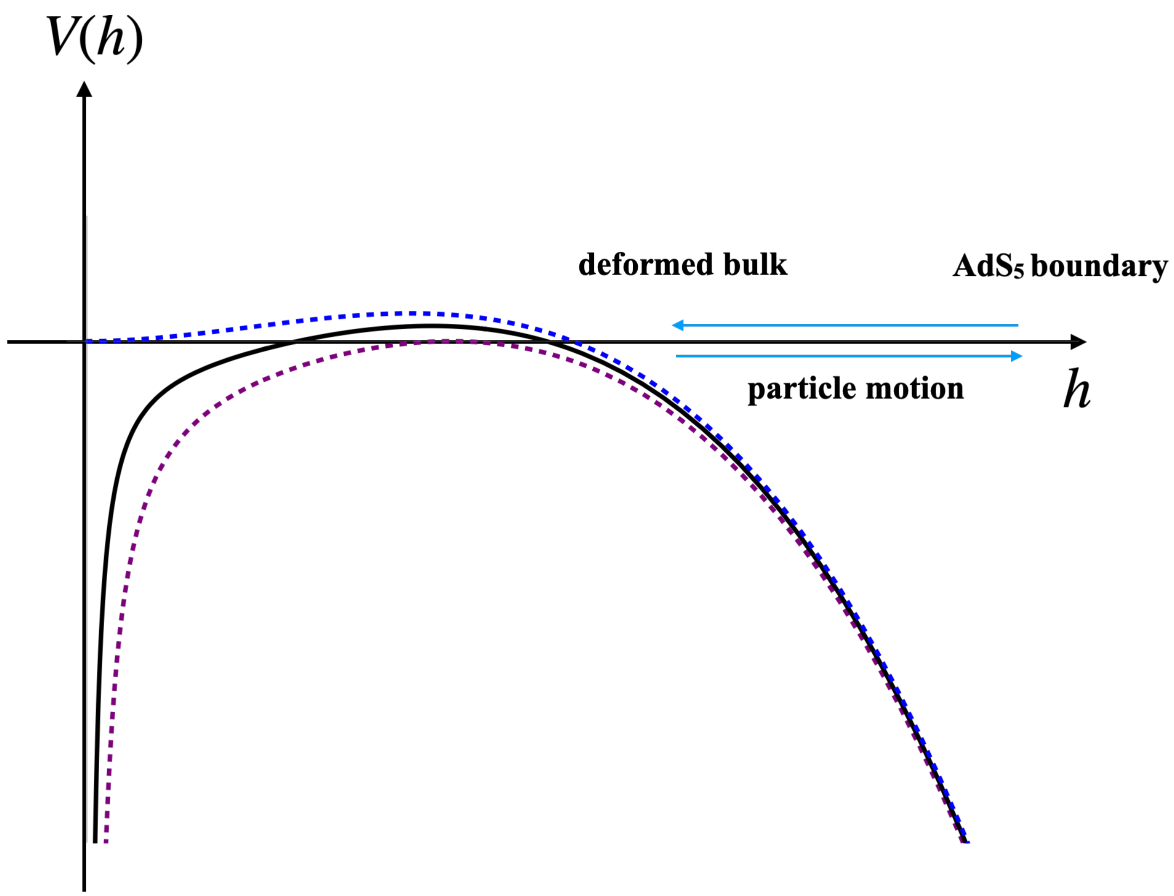
$$\frac{\tau''}{\tau'} + \frac{3h'}{2h} + i\frac{\tau'}{\mathrm{Im}(\tau)} = 0$$

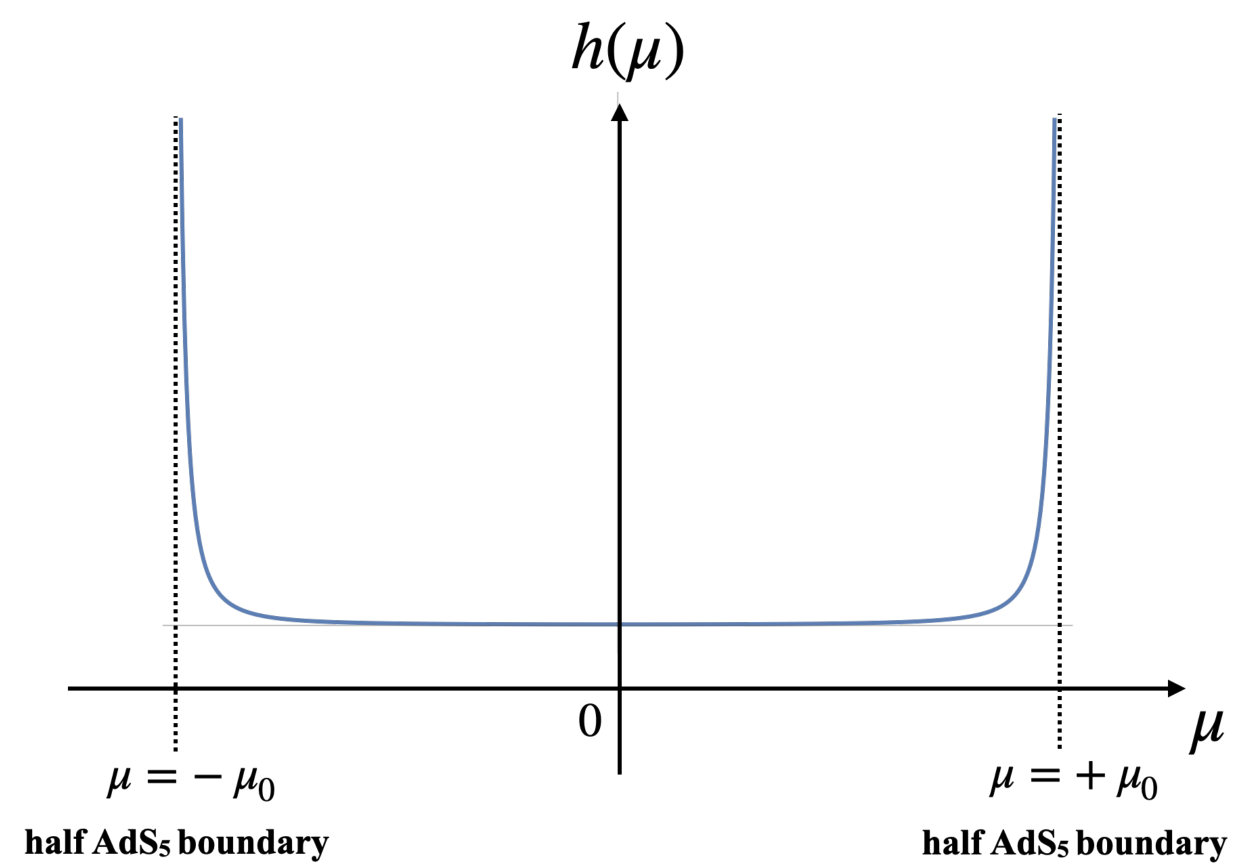
Solution

$$h'^2 - 4h^3 + 4h^2 = \frac{c_0^2}{6h}$$

$$|\tau'|^2/(\mathrm{Im}(\tau))^2 = c_0^2/h^3$$

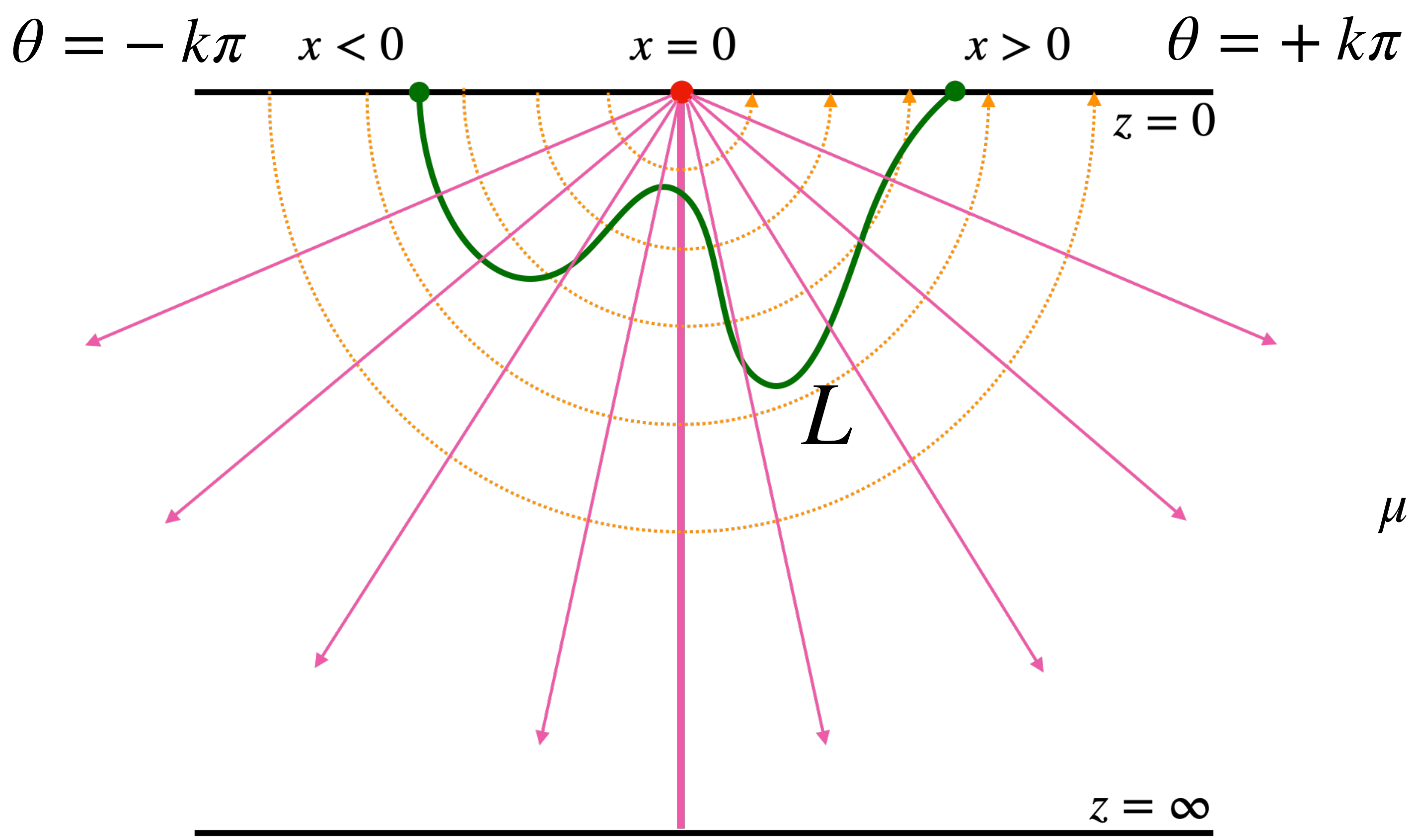
$$|\tau(\mu)|^2 = r^2 \qquad (r \in \mathbb{R})$$





Part III — cont'd

✻ Gravity dual description



$$\mu \xrightarrow{z\to 0} 2\mu_0\left(\Theta(x) - \frac{1}{2}\right)$$

- The D7 flux: $F = dC_0 \xrightarrow{z\to 0} F = \dfrac{\theta}{\mu_0}d\mu = 2\theta\delta(x)dx - \dfrac{2\theta}{\pi x}dz$

- The jump of θ: $Q_7 = \displaystyle\int_L dC_0 = C_0(\mu_0) - C_0(-\mu_0) = 2\theta = k \quad \Longrightarrow \quad \theta_{YM} = 4\pi\theta = 2k\pi$