Argument shift in $U\mathfrak{gl}_n$

(M, π) $\quad f, g \in C^\infty(M)$

$\{f, g\} = \pi(df, dg)$

$[\pi, \pi] = 0$

$Z(C^\infty(M), \{,\}) = \{ f \in C^\infty(M) \mid \{f, g\} = 0 \}$

$M = \mathfrak{g}^*, \quad \pi = x_k C^k_{ij} \partial_i \otimes \partial_j$

$Z(C^\infty(\mathfrak{g})) = \{ f \in C^\infty(\mathfrak{g}^*) \mid f(g^{-1} x g) = f(x) \}$

$x \in \mathfrak{g}^*, \ g \in G$

$$Z(C^\infty(\mathfrak{g}^*)) = C^\infty(\mathfrak{g}^*)^G$$

Assume ξ v field

$$\mathcal{L}_\xi(\mathcal{L}_\xi \pi) = 0 \qquad (\star)$$

Theorem: $\forall f, g \in Z(C^\infty(M))$

$\forall p, q \geq 0$

$$\{\xi^p(f), \xi^q(g)\} = 0$$

Example: $M = \mathfrak{g}^*$, $\xi = \xi^i \partial_i$

M.F. ξ-generic, $\Rightarrow$ maximal

Vinberg:

$$\begin{tikzcd}
& C(\mathfrak{g}^*) \cong S(\mathfrak{g}) \\
\mathfrak{g} \arrow[ur, hook] \arrow[dr, hook] & \\
& U\mathfrak{g}
\end{tikzcd}$$

PBW thm: $U\mathfrak{g} \simeq S\mathfrak{g}$

G acts on $U\mathfrak{g}$ & $S\mathfrak{g}$

$$Z(U\mathfrak{g}) \simeq Z_\pi(S\mathfrak{g})$$

$$U\mathfrak{g} \supset \hat{A}_\xi \qquad \underbrace{\text{PBW}} \qquad A_\xi$$

1994 Nazarov – Olshanski

Quantum M-F algebra

2004 Rybnikov

2001 Tarasov $\mathfrak{g} = \mathfrak{gl}_d$

generators of certain type of the $Z_\pi(S\,\mathfrak{gl}_d)$,

$\sigma(z^p(e_i)) \subset U\mathfrak{gl}_d$

2010 – 2014 A. Molev Yakimova

ζ — missing

$\mathfrak{g} = \mathfrak{gl}_d$

$$\zeta : U\mathfrak{gl}_d \to U\mathfrak{gl}_d$$

$$[\zeta^p(f), \zeta^q(g)]^\dagger = 0$$

$$\forall f, g \in Z(U\mathfrak{gl}_d)$$

Quasiderivations of $U\mathfrak{gl}_d$

2010 G.P.S.

(1) Coordinates $e_{ij} \in \mathfrak{gl}_d$

$$\hat{\partial}_{pq} e_{ij} = \delta_{pj} \delta_{qi}$$

$$\hat{\partial}_{pq}(fg) = \hat{\partial}_{pq} f \cdot g + f \hat{\partial}_{pq} g + \sum_{k=1}^{d} \hat{\partial}_{kq} f \, \hat{\partial}_{pk} g$$

(2) $\hat{D} : U\mathfrak{gl}_d \to \mathrm{Mat}_d(U\mathfrak{gl}_d)$

$\hat{D} \sim (\hat{\partial}_{pq})$

$$\begin{tikzcd} S\mathfrak{gl}_d \arrow[r, "\sigma = PBW"] \arrow[drr, "\exp(D)"'] & U\mathfrak{gl}_d \arrow[r, "1+\hat{D}"] & \mathrm{Mat}_d(U\mathfrak{gl}_d) \arrow[d, "PBW^{-1}"] \\ & & \mathrm{Mat}_d(S\mathfrak{gl}_d) \end{tikzcd}$$

$\xi = \mathrm{Tr}(\xi D)$

$\xi : C(\mathfrak{gl}_d) \to C(\mathfrak{gl}_d)^*$

$D = (\partial_{pq})$

$\xi(\sigma(c_i)) =$
$=$ lin comb. of $\sigma(\xi(c_i))$

Theorem: $c_i \in Z_\pi(S\mathfrak{gl}_d)$

$\sigma(c_i) \in Z(U\mathfrak{gl}_d)$

$\hat{\xi} = \mathrm{Tr}(\xi \hat{D}) : U\mathfrak{gl}_d \to U\mathfrak{gl}_d$

$$[\hat{\xi}^p(\sigma(c_i)), \hat{\xi}^q(\sigma(c_j))] = 0$$