# Burns holography

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Work with K. Costello & N. M. Paquette
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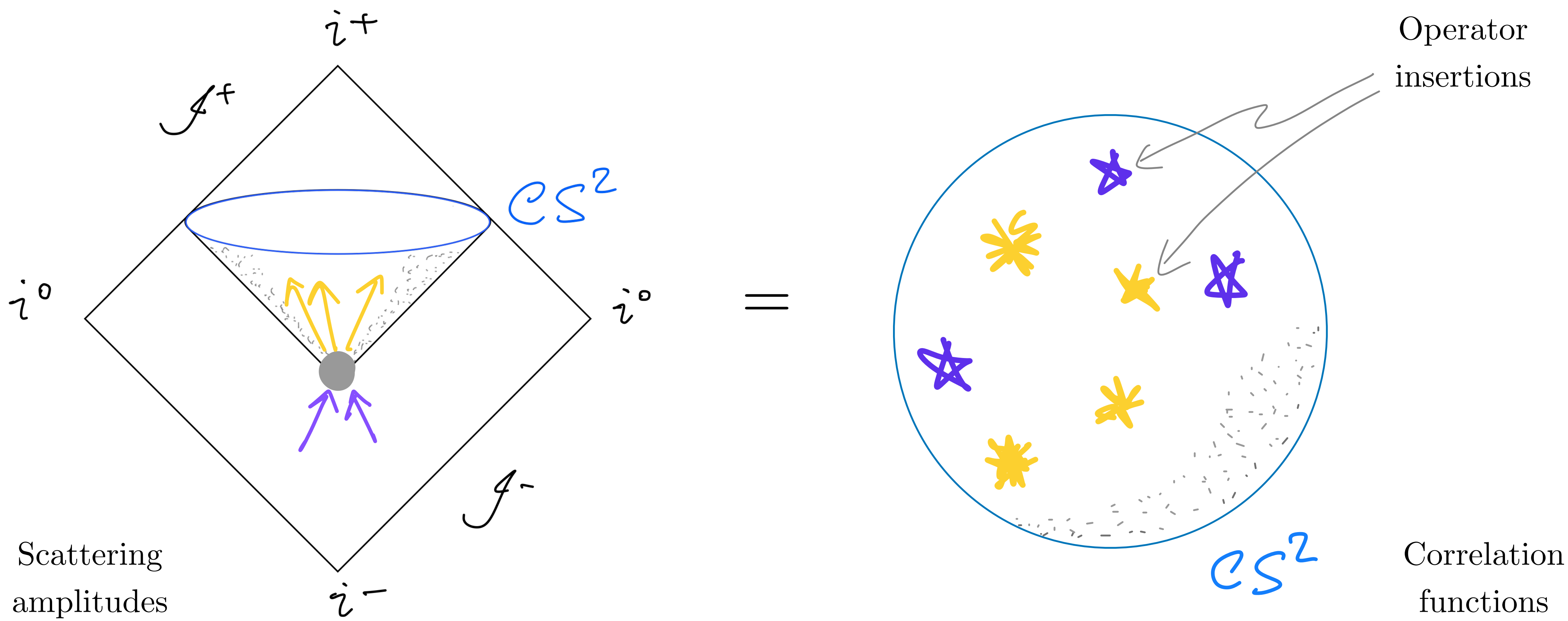
# Celestial holography

[Strominger '17]
[Pasterski, Pate, Raclariu '21]

Quantum gravity in asymptotically flat spacetimes $=$ Celestial conformal field theories



- Today: build an actual example!

| | | |
|---|---|---|
| Mabuchi gravity<br>+<br>4d WZW model<br>on Burns space | = | Chiral algebra on<br>D1 branes in the<br>twistor space of<br>flat space |

# Summary of the talk

- An overview of the duality
- A “derivation” from D branes
- The dual chiral algebra

# Overview

## Burns space

- Burns metric is a Euclidean signature, asymptotically flat, Kähler metric that is scalar-flat (but not Ricci-flat)

Kähler potential

$$K = \|u\|^2 + N \log \|u\|^2 \qquad u_i \in \mathbb{C}^2 - 0$$

[Burns '86]
[LeBrun '88]

Burns metric

$$g = \|\mathrm{d}u\|^2 + \frac{N}{\|u\|^4} |u_1 \mathrm{d}u_2 - u_2 \mathrm{d}u_1|^2$$

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- Extends smoothly to $\widetilde{\mathbb{C}}^2 = \mathrm{blowup}_0(\mathbb{C}^2)$

- Isometry group $\mathrm{U}(1) \times \mathrm{SU}(2) \longrightarrow$ complexifies to $\mathbb{C}^* \times \mathrm{SL}_2(\mathbb{C})$

## The bulk theory

[Mabuchi '86] [Donaldson '85] [Nair '91] [Yang '77]
[Losev, Moore, Nekrasov, Shatashvili '95]

- $\mathcal{K} = K + \rho$
- $\mathrm{g} : \widetilde{\mathbb{C}}^2 \to \mathrm{SO}(8)$

$$S_{4\mathrm{d}} = \int_{\widetilde{\mathbb{C}}^2} \mathrm{Ric}(\mathcal{K}) \wedge \partial\mathcal{K} \wedge \bar{\partial}\mathcal{K} + \int_{\widetilde{\mathbb{C}}^2} \partial\bar{\partial}\mathcal{K} \wedge \mathrm{tr}\big(\mathrm{g}^{-1}\partial\mathrm{g} \wedge \mathrm{g}^{-1}\bar{\partial}\mathrm{g}\big) - \frac{1}{3}\int_{\widetilde{\mathbb{C}}^2\times[0,1]} \partial\bar{\partial}\mathcal{K} \wedge \mathrm{tr}\big(\tilde{\mathrm{g}}^{-1}\mathrm{d}\tilde{\mathrm{g}}\big)^3$$

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- The Wess-Zumino term imposes quantization of $N$.
- Classically describes scalar-flat Kähler metrics coupled to self-dual Yang-Mills with gauge group SO(8).

## The boundary theory

- Sp($N$) gauge theory with SO(8) flavor symmetry.
- Consists of weight $\left(\frac{1}{2}, 0\right)$ symplectic bosons

$$X_{iab}(z) \in \mathbb{C}^2 \otimes \wedge^2 \mathbb{C}^{2N} \qquad X_{iab}(z)\, X_{jcd}(w) \sim \frac{1}{z-w}\, \varepsilon_{ij} \varepsilon_{a[c|} \varepsilon_{b|d]}$$

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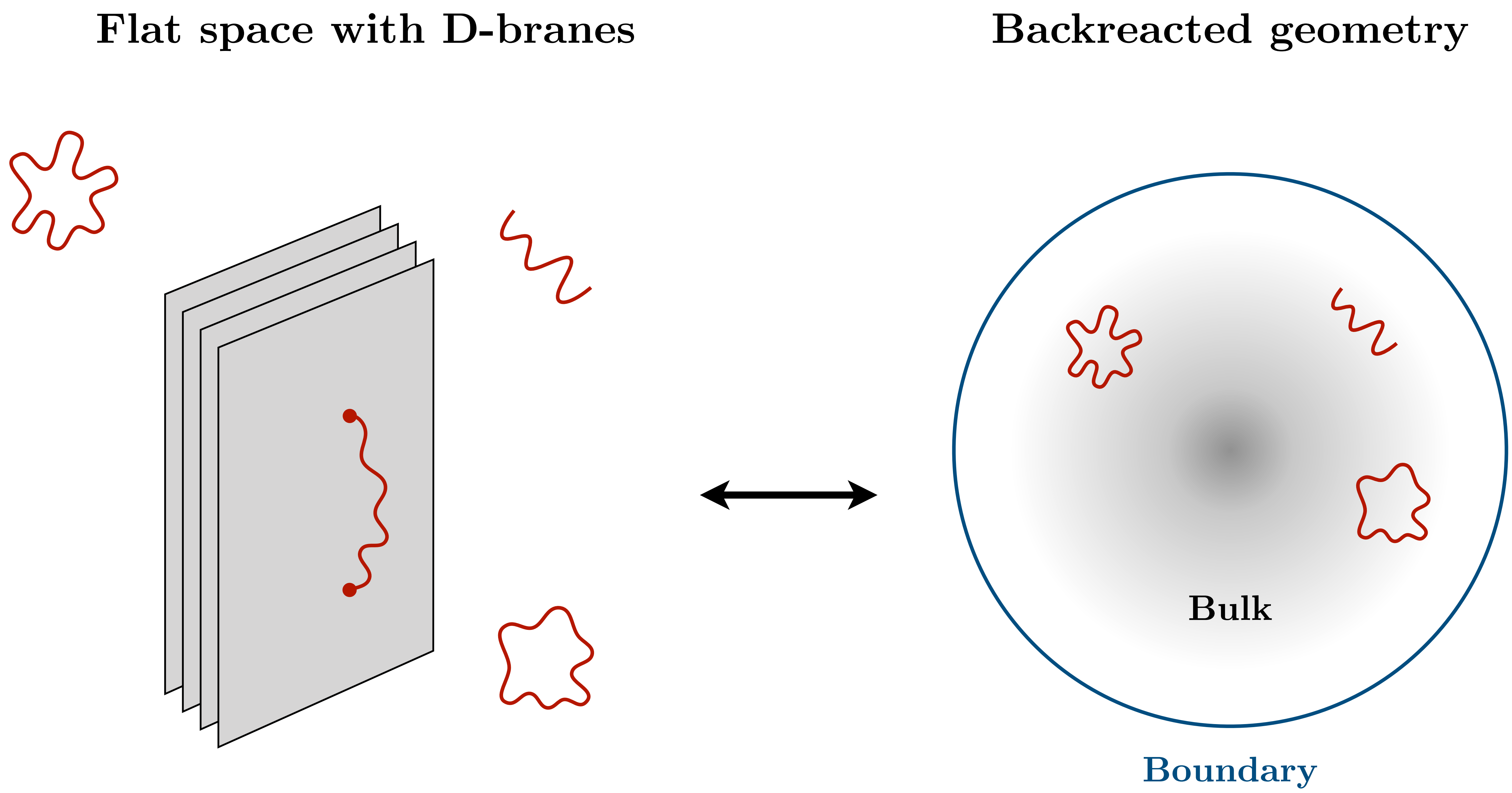
*Defects at north and south poles*

$$X \sim 1/z \text{ at } z = 0, \infty$$

$$I \sim 1/\sqrt{z} \text{ at } z = 0, \infty$$

"Ramond puncture"

# Braneology




Maldacena's argument for AdS/CFT

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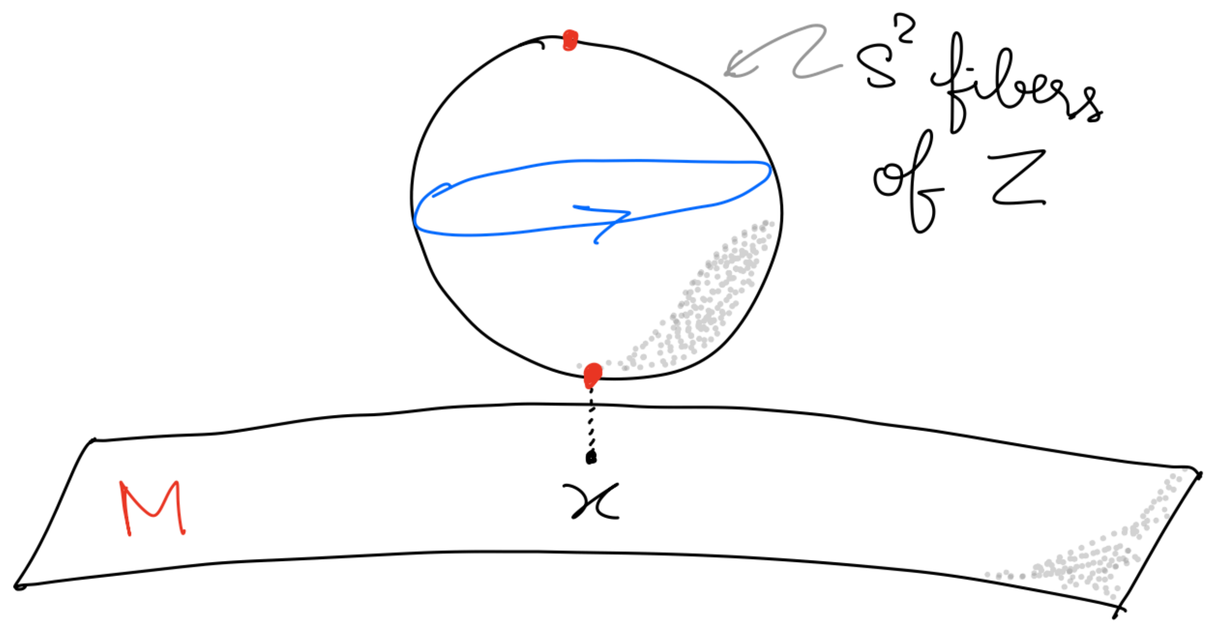
- If $g$ is also Kähler, then $Z$ encodes its Kähler form in a meromorphic 3-form $\Omega$ with two double poles on each fiber.

$$\partial\bar{\partial}K = \frac{1}{2\pi \mathrm{i}} \oint \Omega$$

[Pontecorvo '92] [LeBrun '92]



- We study type I topological B-model on $Z$.

[Costello, Li '19]

[Costello '21]

$$S_{6\text{d}} = \int_Z \partial^{-1}(\mu \lrcorner \Omega)\, \bar{\partial}(\mu \lrcorner \Omega) + \frac{1}{3}\int_Z (\mu^3 \lrcorner \Omega) \wedge \Omega + \int_Z \operatorname{tr}\left(\mathcal{A} \wedge \bar{\partial}\mathcal{A} + \frac{2}{3}\mathcal{A}^3\right) \wedge \Omega + \begin{matrix}\text{Green-Schwarz}\\ \text{coupling}\end{matrix}$$

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- Boundary conditions:
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- KK reduces to Mabuchi + 4d WZW model on $M$.

[Costello '21] [Bittleston, Skinner '20]

[Costello, Paquette, AS '23]

- Twistor space of $\mathbb{C}^2$ with its Euclidean metric:

$$\mathbb{PT} = \mathscr{O}(1) \oplus \mathscr{O}(1) \to \mathbb{CP}^1$$

$$v_1 \qquad v_2 \qquad z$$

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- Meromorphic 3-form: $\quad \Omega = \dfrac{\mathrm{d}z \wedge \mathrm{d}v_1 \wedge \mathrm{d}v_2}{z^2}$

$$\frac{1}{2\pi \mathrm{i}} \oint_{z=0} \Omega = \frac{1}{2\pi \mathrm{i}} \oint_{z=0} \frac{\mathrm{d}z \wedge (\mathrm{d}u_1 - z\,\mathrm{d}\bar{u}_2) \wedge (\mathrm{d}u_2 + z\,\mathrm{d}\bar{u}_1)}{z^2}$$

$$= \mathrm{d}u_1 \wedge \mathrm{d}\bar{u}_1 + \mathrm{d}u_2 \wedge \mathrm{d}\bar{u}_2$$

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- New holomorphic coordinates: $v_i \ , \ w_i = \dfrac{v_i}{z} - \dfrac{N \hat{v}_i}{\|v\|^2}$

$\mathbb{SL}_2(\mathbb{C})$ $\quad w_1 v_2 - w_2 v_1 = N$

Geometric transition!

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$$v_i = u_i + \vartheta \hat{u}_i \sigma \,, \quad w_i = \hat{u}_i + \vartheta u_i \sigma^{-1} \,, \quad \vartheta = \sqrt{1 + \frac{N}{\|u\|^2}}$$

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- $\mathbb{SL}_2(\mathbb{C})$ has holomorphic volume form

$$\Omega = \mathrm{Res}_{\mathbb{SL}_2(\mathbb{C})} \frac{\mathrm{d}v_1 \wedge \mathrm{d}v_2 \wedge \mathrm{d}w_1 \wedge \mathrm{d}w_2}{w_1 v_2 - w_2 v_1 - N}$$

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- Can obtain Burns metric's Kähler form:

$$\frac{1}{2\pi \mathrm{i}} \oint_{\sigma=0} \Omega = \partial\bar{\partial} K \,, \qquad K = \|u\|^2 + N \log \|u\|^2$$

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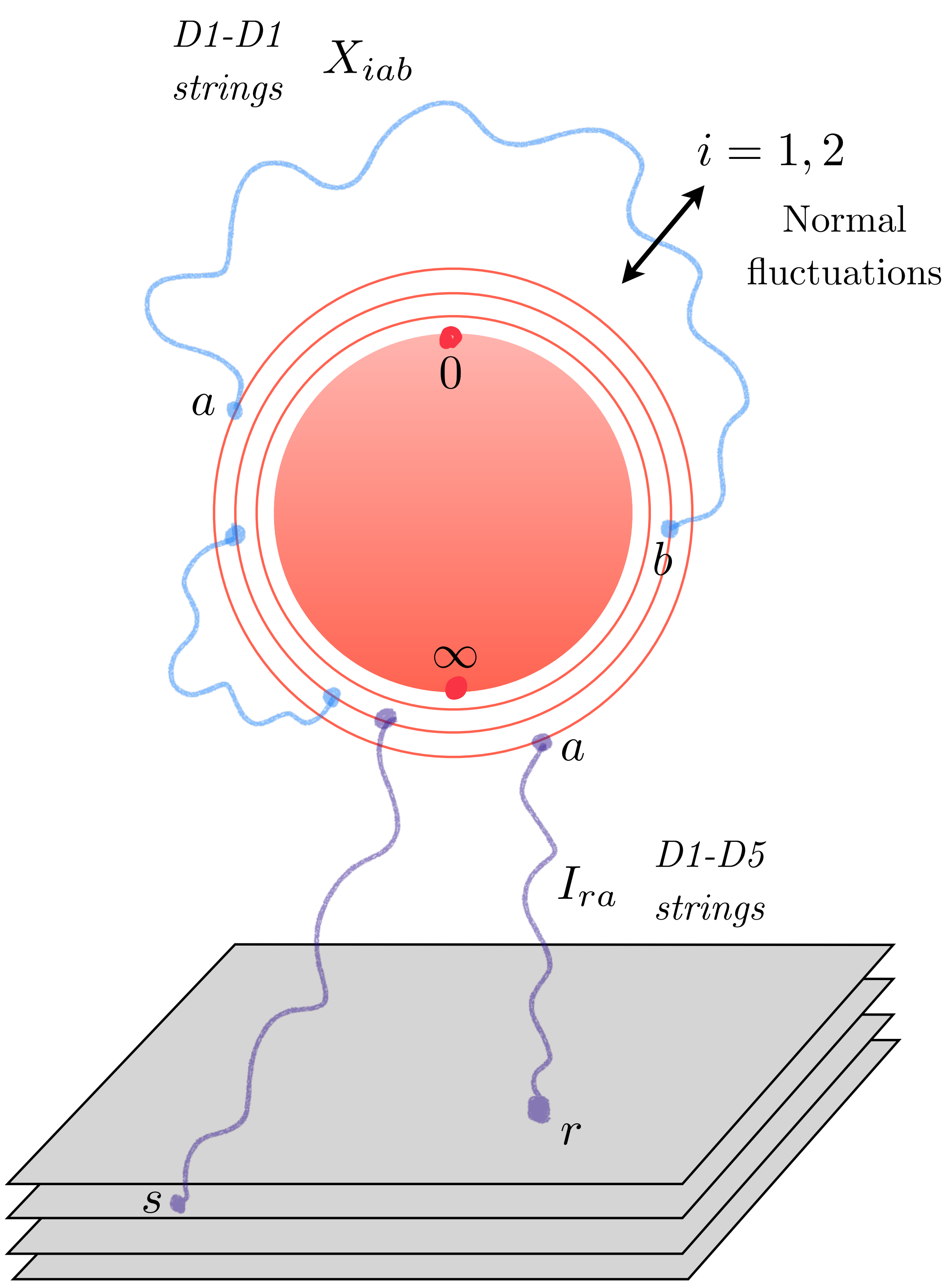
- Thus, KK reduction gives Mabuchi + 4d WZW on Burns space.

# Chiral algebra

Dual chiral algebra is obtained as the worldvolume theory of D1 branes.

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$+$ Sp$(N)$ ghosts

$+$ defects at $0, \infty$

BRST invariant single-trace operators at large $N$

| Conformal weight in $z$ | | |
|---|---|---|
| $\frac{1}{2}(k+l+2)$ | $J_{rs}[k,l](z) = I_r X_1^{(k} X_2^{l)} I_s$ | dual to soft modes of 4d WZW |
| $\frac{1}{2}(k+l)$ | $E[k,l](z) = \mathrm{Tr}\, X_1^{(k} X_2^{l)}$ | dual to soft modes of Mabuchi gravity |
| $\frac{1}{2}(k+l+2)$ | $F[k,l](z) = \mathrm{Tr}\, X_1^{(k} X_2^{l)} (X_1 \partial X_2 - X_2 \partial X_1) + \text{ghosts}$ | |

These are completely explicit Burns space realizations of Strominger's infinite dimensional holographic symmetry algebras.

[Strominger '21]

[Guevara, Himwich, Pate Strominger '21]

## Defect boundary conditions

Bulk-brane couplings before backreaction

$$\int_{v=0} \mathrm{d}z\, z^{k+l} J_{rs}[k,l]\, \frac{\partial^{k+l}\mathcal{A}^{rs}}{\partial v_1^k \partial v_2^l}$$

[Costello, Paquette '22]
[Paquette, Williams '21]
[Costello, Paquette '20]

Integrand remains finite at $z = \infty$ because

$$(z, v_i) \mapsto \left(\frac{1}{z}, \frac{v_i}{z}\right) \implies J[k,l](z) \mapsto z^{k+l+2} J[k,l](z)$$

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Has 1st order zeroes at the poles

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Most general allowed poles:

$$J_{rs}[k,l] = I_r X_1^{(k} X_2^{l)} I_s \sim \frac{1}{z^{k+l+1}} \text{ at } z = 0, \infty$$

We can engineer the order $k+l+1$ pole while preserving $\mathrm{Sp}(N)$ symmetry by giving $I$ and $X$ poles of order $\frac{1}{2}$ and 1 respectively.

## Testing the duality: an example

- Gauge invariant operators:

Soft currents

[Strominger '21]

[Guevara, Himwich, Pate Strominger '21]

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- Planar correlator without defects reproduces 2-pt amplitude of Hawking, Page & Pope '80.

$$\left\langle J_{pq}(\omega_1, z_1, \bar{z}_1)\, J_{rs}(\omega_2, z_2, \bar{z}_2)\right\rangle = -\frac{N}{z_{12}^2}\, J_0\left(\sqrt{4N\omega_1\omega_2 z_1 z_2 \frac{\bar{z}_{12}}{z_{12}}}\right) \mathrm{tr}(\mathfrak{t}_{pq}\mathfrak{t}_{rs})$$

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[Gaiotto, Budzik '22]
[Hartnoll, Policastro '04]
[Donaldson, Friedman '89]

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- Are there multi-centered generalizations of our duality?

  [Gaiotto, Budzik '22]
  [Hartnoll, Policastro '04]
  [Donaldson, Friedman '89]

- Can we discover dualities for self-dual Einstein gravity?

  [Skinner '13]
  [Bittleston, Heuveline, Skinner '23]