Deformed Prepotential, Quantum Integrable System and Liouville Field Theory

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Introduction

N=2 supersymmetric gauge theory is very interesting framework where various interesting results have been found: e.g.

- exact effective action related with the Seiberg-Witten curve
- instanton counting (Nekrasov partition function)
- relation with integrable systems

M5-branes wrapped on Riemann surface

[Witten '97, Gaiotto '09]

N=2, SU(2) quiver SCFTs can be induced on worldvolume of 2 M5-branes wrapped on Riemann surfaces $\Sigma_{g,n}$:

genus $\longrightarrow$ "genus" of quiver diagram

punctures $\longrightarrow$ flavor symmetries

$q_i = e^{\pi i \tau_i}$

complex structures $\longrightarrow$ gauge coupling constants

Low energy effective theory

- *Seiberg-Witten curve* is double cover of the Riemann surface $\Sigma_{g,n}$ on which M5-branes wrap:

$$x^2 = \phi(t), \quad t; \text{coordinate on Riemann surface}$$

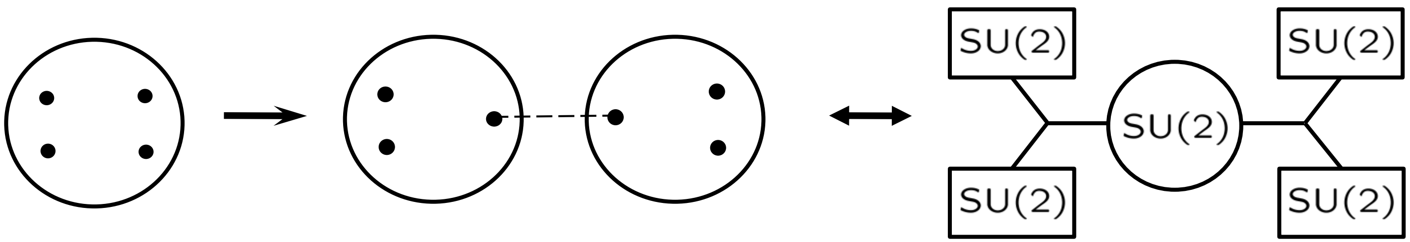
- *meromorphic one form* is given by $\lambda_{\mathrm{SW}} = x dt$

Using these, the low energy prepotential is

$$a_i = \oint_{A_i} \lambda_{\mathrm{SW}}, \quad \frac{\partial \mathcal{F}}{\partial a_i} = \oint_{B_i} \lambda_{\mathrm{SW}}$$

SU(2) gauge theory with 4 flavors

A simple example: 2 M5-branes on $\Sigma_{0,4}$, *a sphere with 4 punctures induce SU(2) gauge theory with 4 flavors*:



In this case, the curve obtained from M-theory is

$$x^2 = \left(\frac{m_0}{t} + \frac{m_1}{t-1} + \frac{m_2}{t-q}\right)^2 + \frac{1}{t(t-1)(t-q)}\left((m_4^2 - (\sum m_i)^2)t + qU\right)$$

Classical integrable systems

The relation between the Seiberg-Witten theory and the classical integrable system has been studied

[Gorsky et al., Donagi-Witten]
[Martinec-Warner, Itoyama-Morozov, ….]

Seiberg-Witten curve $\Longleftrightarrow$ spectral curve

meromorphic one form $\Longleftrightarrow$ symplectic form

Pure Yang-Mills theory: periodic Toda system
N=2* gauge theory: elliptic Calogero-Moser system

Quantization of integrable systems

[Nekrasov-Shatashvili]

N=2 low energy effective theory can be described in omega-background via Nekrasov partition function:

$$Z_{\text{Nek}}(a, m_i; \epsilon_1, \epsilon_2) = \exp\left(-\frac{\mathcal{F}}{\epsilon_1\epsilon_2} + \dots\right)$$

In the case where ϵ_1 finite and $\epsilon_2 \to 0$,

$$Z_{\text{Nek}}(a, m_i; \epsilon_1, \epsilon_2) = \exp\left(-\frac{1}{\epsilon_1\epsilon_2}(\mathcal{F}(\epsilon_1) + \mathcal{O}(\epsilon_2))\right)$$

related with the integrable system

(ϵ_1 plays the role of the Planck constant.)

Schrodinger equation and deformed prepotential

[Mironov-Morozov, arXiv:0910.5670]

It was suggested that the deformed prepotential can be obtained from the periods

$$\begin{aligned} 2\pi i\hat{a} &= \oint_A P(z;\epsilon_1)dz, \\ \frac{1}{2}\frac{\partial\hat{\mathcal{F}}(\epsilon_1)}{\partial\hat{a}} &= \oint_B P(z;\epsilon_1)dz, \end{aligned}$$

where P is calculated from the WKB ansatz

$$\mathcal{H}\Psi(z) = E\Psi(z), \quad \Psi(z) = \exp\left(-\frac{1}{\epsilon_1}\int^z P(z';\epsilon_1)dz'\right).$$

Relation with the Liouville theory

[Alday-Gaiotto-Tachikawa]

M5-brane theory on $\mathbb{R}^4 \times \Sigma_{g,n}$

compactify on $\Sigma_{g,n}$ [Gaiotto]

4d N=2 gauge theory $\longleftrightarrow$ **2d theory on $\Sigma_{g,n}$?**

2d Liouville field theory on $\Sigma_{g,n}$

(Nekrasov partition function) = (n-point conformal block)

Surface and loop operators from the Liouville

[Alday-Gaiotto-Gukov-Tachikawa-Verlinde]

The partition function in the presence of a surface operator was conjectured to be identified with

$$\Psi(a,z) = \left\langle \Phi_{2,1}(z) \prod_{i=1}^{n} V_{\alpha_i}(z_i) \right\rangle .$$

where $\Phi_{2,1}(z)$ is the degenerate field.

Loop operators correspond to monodromy operation:

$$\begin{aligned} \Psi(a, z+A) &= \exp\left(-\frac{2\pi i a}{\epsilon_1}\right)\Psi(a,z), \\ \Psi(a, z+B) &= \Psi(a+\frac{\epsilon_2}{2}, z) \end{aligned}$$

Relation with quantum integrable system:
Schrodinger equation

The first point is that the differential equation which is satisfied by the conformal block with degenerate field due to the null condition:

$$(b^2 L_{-2} + (L_{-1})^2)\Phi_{2,1}(z) = 0$$

in the limit ϵ_1 finite and $\epsilon_2 \to 0$, can be identified with the Schrodinger equation of one-dimensional system

$$\mathcal{H}\Psi(z) = E\Psi(z)$$

Relation with quantum integrable systems: deformed prepotential

The second point is that the proposal due to Mironov-Morozov is *equivalent* to the expected monodromy of the conformal block with the degenerate field, in the limit ϵ_1 finite and $\epsilon_2 \to 0$

$$\begin{aligned}\Psi(z+A) &= \exp\left(-\frac{2\pi i a}{\epsilon_1}\right)\Psi(z),\\ \Psi(z+B) &= \exp\left(-\frac{1}{2\epsilon_1}\frac{\partial\mathcal{F}(\epsilon_1)}{\partial a}\right)\Psi(z),\end{aligned}$$

<u>Contents</u>

1. *Deformed prepotential from integrable system*

2. *AGT relation and its extension*

3. *Schrodinger equation from Liouville theory*

4. *Deformed prepotential from Liouville theory*

5. *Conclusion*

1. Deformed prepotential from integrable system

Proposal by Mironov-Morozov

Let us consider the following Schrodinger system:

$$\mathcal{H}\Psi(z) = E\Psi(z),$$

where the Hamiltonian is $\mathcal{H} = -\epsilon_1^2\partial_z^2 + V(z;\epsilon_1)$.

The wave-function is given by

$$\Psi(z) = \exp\left(-\frac{1}{\epsilon_1}\int^z P(z';\epsilon_1)dz'\right).$$

This is the exact WKB ansatz.

Proposal by Mironov-Morozov

Then, *the deformed prepotential can be given by the similar relation to Seiberg-Witten theory:*

$$
\begin{aligned}
2\pi i \hat{a} &= \oint_A P(z;\epsilon_1)dz, \\
\frac{1}{2}\frac{\partial \hat{\mathcal{F}}(\epsilon_1)}{\partial \hat{a}} &= \oint_B P(z;\epsilon_1)dz,
\end{aligned}
$$

This is very nontrivial proposal and there is no proof of this.

As we will see, we can check this at lower order in ϵ_1 .

Prescription to obtain prepotential (1)

First of all, in terms of P, the Schrodinger equation is

$$-P^2 + \epsilon_1 P' + V(z;\epsilon_1) = E,$$

$$\left(\begin{aligned} P(\epsilon_1; z) &= \sum_{k=0}^{\infty} \epsilon_1^k P_k(z), \\ V(z;\epsilon_1) &= \sum_{k=0}^{\infty} \epsilon_1^k V_k(z). \end{aligned} \right)$$

Then, we obtain

$$-P_0^2 + V_0 = E, \qquad -2P_0P_1 + P_0' + V_1 = 0,$$

which lead to

$$P_0 = \sqrt{V_0 - E}, \quad P_1 = \frac{V_1}{2P_0} + \frac{1}{2}(\log P_0)', \quad \ldots.$$

Prescription to obtain prepotential (2)

The contour integral can be written

$$\oint P = \hat{\mathcal{O}} \oint P_0 = (1 + \epsilon_1 \hat{\mathcal{O}}_1 + \ldots) \oint P_0$$

Then,

1. calculate A- and B-cycle integrals at $\epsilon_1 = 0$:

$$2\pi i a(E) = \oint_A P_0 dz, \quad \frac{1}{2}\frac{\partial \mathcal{F}}{\partial a}(E) = \oint_B P_0 dz,$$

2. act the operator $\hat{\mathcal{O}}$ on the ones obtained above

$$\hat{a}(E;\epsilon_1) = \hat{\mathcal{O}}[a(E)], \quad \frac{\partial \hat{\mathcal{F}}}{\partial \hat{a}}(E;\epsilon_1) = \hat{\mathcal{O}}\left[\frac{\partial \mathcal{F}}{\partial a}(E)\right],$$

3. invert as $E = E(\hat{a})$ and substitute it into $\frac{\partial \hat{\mathcal{F}}}{\partial \hat{a}}(E)$

N=2* theory and elliptic Calogero-Moser

We consider the Hamiltonian:

$$\mathcal{H} = -\epsilon_1^2 \partial_z^2 + m(m - \epsilon_1)\mathcal{P}(z),$$

which is the same as the one of the elliptic Calogero-Moser system.

The one form can be written

Same form as the Seiberg-Witten curve of N=2* theory

$$\begin{aligned} P_0 &= \sqrt{m^2 \mathcal{P}(z) - E}, \\ \oint P_1 dz &= -\frac{m}{2} \oint \frac{\mathcal{P}(z)}{P_0} dz = -\frac{1}{2} \frac{\partial}{\partial m} \oint P_0 dz. \end{aligned}$$

Let $a(E)$ and $\frac{\partial \mathcal{F}}{\partial a}(E)$ be the ones obtained at $\epsilon_1 = 0$

Then, at the next order,

$$\widehat{a}(E) = a(E)|_{m^k \to m^k - k\epsilon_1 m^{k-1}/2}, \quad \frac{\partial \widehat{\mathcal{F}}}{\partial \widehat{a}}(E) = \frac{\partial \mathcal{F}}{\partial a}(E)|_{m^k \to m^k - k\epsilon_1 m^{k-1}/2}.$$

By inverting the first equation and substituting it into the second, we obtain

$$\widehat{\mathcal{F}}(\widehat{a}, \epsilon_1) = \mathcal{F}(a)|_{a \to \widehat{a},\ m^k \to m^k - k\epsilon_1 m^{k-1}/2} + \mathcal{O}(\epsilon_1^2)$$

This is the correct behavior of the Nekrasov function:

$$\begin{aligned}\widehat{\mathcal{F}}_{\text{inst}}(\widehat{a}) \;&=\; \frac{m^4}{2\widehat{a}^2} q + \frac{m^4(96\widehat{a}^4 - 48\widehat{a}^2 m^2 + 5m^4)}{64\widehat{a}^6} q^2 + \dots \\ &\quad - \left[\frac{m^3}{\widehat{a}^2} q + \frac{m^3(48\widehat{a}^4 - 36\widehat{a}^2 m^2 + 5m^4)}{16\widehat{a}^6} q^2 + \dots\right] \epsilon_1 + \mathcal{O}(\epsilon_1^2),\end{aligned}$$

N=2 gauge theory with four flavors

The potential is

$$V = \frac{\tilde{m}_1^2 - \frac{\epsilon_1^2}{4}}{z^2} + \frac{m_0(m_0-\epsilon_1)}{(z-1)^2} + \frac{m_1(m_1-\epsilon_1)}{(z-q)^2} - \frac{m_0(m_0-\epsilon_1) + m_1(m_1-\epsilon_1) + \tilde{m}_1^2 - \tilde{m}_0^2}{z(z-1)}.$$

Then, the one form becomes

$$P_0 = \sqrt{V_0 - \frac{(1-q)E}{z(z-1)(z-q)}}, \qquad P_1 = -\frac{1}{2}\left(\frac{\partial}{\partial m_0} + \frac{\partial}{\partial m_1}\right) P_0 + d\left(\frac{1}{2}\log P_0\right), \ldots$$

Similarly, we can obtain deformed prepotential.

2. AGT relation and its extension

Four-point conformal block

Let us consider the four-point correlation function which has the following form

$$\langle V_{\alpha_1}(\infty)V_{\alpha_2}(1)V_{\alpha_3}(q)V_{\alpha_4}(0)\rangle^{\text{full}} = \int \frac{d\alpha^{int}}{2\pi} C(\alpha_1^*, \alpha_2, \alpha^{int}) C(\alpha^{int*}, \alpha_3, \alpha_4) \\ \times \left| \langle V_{\alpha_1}(\infty)V_{\alpha_2}(1)V_{\alpha_3}(q)V_{\alpha_4}(0)\rangle \right|^2$$

The conformal block which we will consider behaves as

$$\langle V_{\alpha_1}(\infty)V_{\alpha_2}(1)V_{\alpha_3}(q)V_{\alpha_4}(0)\rangle = q^{\Delta_\alpha^{int} - \Delta_{\alpha_3} - \Delta_{\alpha_4}} \mathcal{B}(q)$$

where *B* is expanded as $\mathcal{B}(q) = 1 + \mathcal{O}(q)$.

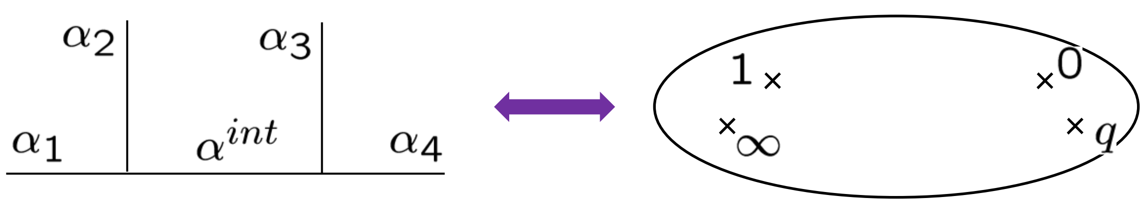
AGT relation (SU(2) theory with 4 flavors)

[Alday-Gaiotto-Tachikawa]

A simple example of the AGT relation is as follows:

the conformal block which we have seen above can be identified with the Nekrasov partition function of SU(2) theory with 4 flavors

$$\mathcal{B}(\alpha^{int}, \alpha_i, q; b) = Z_{\text{inst}}^{N_f=4}(a, m_i, q; \epsilon_1, \epsilon_2)$$



Identification of the parameters

- External momenta : mass parameters

$$\Delta_{\alpha_1} = \frac{Q^2}{4} - \frac{\tilde{m}_0^2}{\hbar^2}, \quad \Delta_{\alpha_2} = \frac{m_0}{\hbar}(Q - \frac{m_0}{\hbar}), \quad \Delta_{\alpha_3} = \frac{m_1}{\hbar}(Q - \frac{m_1}{\hbar}), \quad \Delta_{\alpha_4} = \frac{Q^2}{4} - \frac{\tilde{m}_1^2}{\hbar^2}.$$

- Internal momenta : vector multiplet scalar vev

$$\Delta_p^{int} = \frac{Q^2}{4} - \frac{a_p^2}{\hbar^2},$$

- (bare coupling) = (complex structure) = q

- deformation parameters ; Liouville parameter

$$\epsilon_1 = \frac{\hbar}{b}, \quad \epsilon_2 = \hbar b.$$

AGT relation (N=2* gauge theory)

N=2*, SU(2) gauge theory is the theory with an adjoint hypermultiplet with mass m.

The partition function of this theory was identified with the torus one point conformal block:

$$Z_{\text{inst}}^{N=2^*}(a, m, q; \epsilon_1, \epsilon_2) = \mathcal{B}(\alpha^{int}, \alpha, q; b)$$

where the identification is

$$\Delta_{\alpha^{int}} = \frac{Q^2}{4} - \frac{a^2}{\hbar^2}, \quad \Delta_\alpha = \frac{m}{\hbar}(Q - \frac{m}{\hbar}).$$

Seiberg-Witten curve

The curve can be obtained by the insertion of the energy-momentum tensor:

$$x^2 = \phi_2(z) \equiv \frac{\langle T(z) \prod_{i=1}^{n} V_{\alpha_i}(z_i) \rangle}{\langle \prod_{i=1}^{n} V_{\alpha_i}(z_i) \rangle},$$

and taking a limit where $\alpha^{int}, \alpha^i \gg b, 1/b$.

This limit corresponds to the gauge theory limit $\epsilon_{1,2} \to 0$.

Seiberg-Witten curve (4 flavor theory)

In the case of a sphere with 4 punctures, the insertion leads to

$$\phi_2(z) = \sum_{i=1}^{4} \left(\frac{\Delta_i}{(z-z_i)^2} + \frac{1}{z-z_i}\frac{\partial}{\partial z_i} \right) \langle \prod_{i=1}^{4} V_{\alpha_i}(z_i) \rangle / \langle \prod_{i=1}^{4} V_{\alpha_i}(z_i) \rangle .$$

Then, after some algebra, we obtain

$\phi_2(z) = \widehat{\phi}_2(z) \left\langle \prod_{i=1}^{4} V_{\alpha_i}(z_i) \right\rangle / \left\langle \prod_{i=1}^{4} V_{\alpha_i}(z_i) \right\rangle$ with

$$\begin{aligned} \widehat{\phi}_2 \;&=\; \frac{\Delta_4}{z^2} + \frac{\Delta_3}{(z-q)^2} + \frac{\Delta_2}{(z-1)^2} \\ &\quad - \frac{1}{z(z-1)(z-q)} \left((1-q)\frac{\partial}{\partial \ln q} + (z-q)\left(\sum_{i=2}^{4} \Delta_i - \Delta_1\right) \right) . \end{aligned}$$

Extension of the AGT relation

The partition function in the presence of the surface operator was conjectured to coincide with the conformal block with the degenerate field insertion:

$$\Psi(\alpha^{int}, \alpha_i, z) = \left\langle \Phi_{2,1}(z) \prod_{i=1}^{4} V_{\alpha_i}(z_i) \right\rangle .$$

where $\Phi_{2,1}(z)$ is the primary field with momentum $-\frac{b}{2}$
The degenerate field satisfies the null state condition:

$$(b^2 L_{-2} + (L_{-1})^2)\Phi_{2,1}(z) = 0$$

Degenerate conformal block

We note that the degenerate conformal block may behave as

$$\Psi = \exp\left(-\frac{1}{\epsilon_1\epsilon_2}(\mathcal{F}(\epsilon_1) + \epsilon_2\mathcal{W}(z;\epsilon_1) + \mathcal{O}(\epsilon_2^2))\right).$$

The second term indicates the expected behavior of the surface operator.

In what follows, we will consider the case with ϵ_1 finite and $\epsilon_2 = 0$ which would corresponds to the quantum integrable system.

3. *Schrodinger equation from Liouville theory*

The case of a sphere with four punctures

The null state condition $(b^2 L_{-2} + (L_{-1})^2)\Phi_{2,1}(z) = 0$ in the case of a sphere with four punctures leads to the differential equation:

$$0 = \left[b^{-2}\partial_z^2 + \hat{\phi}_2 - \frac{1}{z(z-1)(z-q)} \left((2z-1)\frac{\partial}{\partial z} + \Delta \right) \right] \Psi(z),$$

where

$$\begin{aligned} \hat{\phi}_2 \;\; = \;\; & \frac{\Delta_4}{z^2} + \frac{\Delta_3}{(z-q)^2} + \frac{\Delta_2}{(z-1)^2} \\ & - \frac{1}{z(z-1)(z-q)} \left((1-q)\frac{\partial}{\partial \ln q} + (z-q)(\sum_{i=2}^{4} \Delta_i - \Delta_1) \right). \end{aligned}$$

Let us go back to the gauge theory parameters:

$$\Delta_\alpha \to \Delta_\alpha \hbar^2, \quad \text{and} \quad \frac{\hbar}{b} = \epsilon_1, b\hbar = \epsilon_2$$

In this scaling, e.g. $\Delta_{\alpha_1}\hbar^2 = \frac{\epsilon_1^2}{4} - \tilde{m}_0^2 + \mathcal{O}(\epsilon_2).$

Then, by taking the limit where ϵ_1 finite and $\epsilon_2 \to 0$,

$$\left(-\epsilon_1^2\partial_z^2 + V(z;\epsilon_1)\right)\Psi(z) = \frac{(1-q)}{z(z-1)(z-q)}\frac{\partial\mathcal{F}(\epsilon_1)}{\partial \ln q}\Psi(z),$$

where V is the same as the potential above.

Note that the limit is crucial to obtain the form of the Schrodinger equation.

The case of a torus with one puncture

We will skip the detailed derivation of the differential equation. The result is

$$\left(-b^{-2}\partial_z^2 - (\Delta_{\alpha'} - \frac{1}{2} - \frac{b^2}{4})\mathcal{P}(z) - \eta_1(1 + \frac{3b^2}{2}) + \frac{1}{2\pi\Im\tau}\right)\Psi(z|\tau) = \frac{2i}{\pi}\frac{\partial}{\partial\tau}\Psi(z|\tau).$$

Again, we go back to the gauge theory parameters and then take the limit ϵ_1 finite and $\epsilon_2 \to 0$

$$\left(-\epsilon_1^2\partial_z^2 + m(m-\epsilon_1)\mathcal{P}(z)\right)\Psi(z|\tau) = 4\frac{\partial\mathcal{F}(\epsilon_1)}{\partial \ln q}\Psi(z|\tau),$$

This is the Hamiltonian of the elliptic Calogero-Moser system.

4. *Deformed prepotential from Liouville theory*

Differential equation

Let us solve the differential equation obtained above:

$$\left(-\epsilon_1^2\partial_z^2 + V(\epsilon_1)\right)\Psi(z) = g(z)u(\epsilon_1)\Psi(z),$$

where g(z) is an irrelevant factor which depends on z.

Note that the difference between this equation and the Schrodinger equation is

$$\begin{aligned} E \to u(\epsilon_1) &\equiv \frac{\partial \mathcal{F}(\epsilon_1)}{\partial \ln q} \\ &= \sum_{k=0}^{\infty} \epsilon_1^k u_k \end{aligned}$$

WKB ansatz

Let us make an ansatz: $\Psi(z) \sim \exp\left(-\frac{1}{\epsilon_1}\int^z x(z';\epsilon_1)dz'\right)$

Then, the differential equation leads to

$$-x^2 + \epsilon_1 x' + V(z;\epsilon_1) = g(z)u(\epsilon_1).$$

which implies

$$x_0 = \sqrt{V_0 - g(z)u_0}, \quad x_1 = \frac{1}{2x_0}(x_0' + V_1 - g(z)u_1),$$

where

$$x = \sum_{k=0}^{\infty} \epsilon_1^k x_k, \quad V = \sum_{k=0}^{\infty} \epsilon_1^k V_k.$$

Contour integrals

At lower order, the contour integral of one form is

$$\begin{aligned} \oint x dz &= \oint x_0 dz + \epsilon_1 \oint \frac{x_0' + V_1}{2x_0} dz + \epsilon_1 u_1 \frac{\partial}{\partial u_0} \oint x_0 dz \\ &= \hat{\mathcal{O}} \left[\oint x_0 dz \right] + \epsilon_1 u_1 \frac{\partial}{\partial u_0} \oint x_0 dz + \mathcal{O}(\epsilon_1^2), \end{aligned}$$

If we assume the proposal:

$$\begin{aligned} 2\pi i \hat{a}(E) &= \hat{\mathcal{O}} \left[\oint_A P_0(z) dz \right], \\ \frac{1}{2} \frac{\partial \hat{\mathcal{F}}(\epsilon_1)}{\partial \hat{a}}(E) &= \hat{\mathcal{O}} \left[\oint_B P_0(z) dz \right], \end{aligned}$$

we obtain

$$\frac{1}{2\pi i} \oint_A x dz = \hat{a}(E \to u_0) + \epsilon_1 u_1 \frac{\partial}{\partial u_0} \hat{a}(E \to u_0) + \dots,$$

Thus,

$$\frac{1}{2\pi i}\oint_A x dz = \hat{a}(E \to u(\epsilon_1)), \quad \frac{1}{2}\oint_B x dz = \frac{\partial \hat{\mathcal{F}}}{\partial \hat{a}}(E \to u(\epsilon_1)),$$

Here we come to the important relation.

The final remark is the following. We already know the form of u: indeed we can show that

$$u(\epsilon_1; a) = E(\hat{a}; \epsilon_1)|_{\hat{a}\to a}.$$

Therefore, we obtain

$$\frac{1}{2\pi i}\oint_A x dz = a, \quad \frac{1}{2}\oint_B x dz = \frac{\partial \mathcal{F}(\epsilon_1)}{\partial a},$$

Expected monodromy

From the Liouville side, monodromy of $\Psi(z)$ has been considered: [Alday et al., Drukker-Gomis-Okuda-Teschner]

$$\begin{aligned}\Psi(a, z+A) &= \exp\left(\frac{2\pi i a}{\epsilon_1}\right)\Psi(a,z)\\ \Psi(a, z+B) &= \Psi(a+\frac{\epsilon_2}{2}, z)\end{aligned}$$

In the limit where ϵ_1 finite and $\epsilon_2 \to 0$, these become $\left(\Psi = \exp\left(-\frac{\mathcal{F}(\epsilon_1)}{\epsilon_1\epsilon_2} - \frac{1}{\epsilon_1}\mathcal{W}(z;\epsilon_1) + \mathcal{O}(\epsilon_2^2)\right)\right)$

$$\begin{aligned}\mathcal{W}(z+A;\epsilon_1) &= \mathcal{W}(z;\epsilon_1) + 2\pi i a,\\ \mathcal{W}(z+B;\epsilon_1) &= \mathcal{W}(z;\epsilon_1) + \frac{1}{2}\frac{\partial \mathcal{F}(\epsilon_1)}{\partial a}.\end{aligned}$$

These are exactly what we have derived!

5. Conclusion

We have considered the proposal that the deformed prepotential can be obtained from the Schrodinger equation.

We have derived the Schrodinger equation from the conformal block with the degenerate field by making use of the AGT relation

We have seen that the proposal above is equivalent to the expected monodromy of the conformal block.

Future directions

- Higher order check and other SU(2) theories
- SU(N)/Toda generalization: Surface operators and degenerate field insertion
- Direct check that the Nekrasov partition function with surface operator satisfies the differential equation
- Deformation to N=1 ?

Thank you very much for your attention!