Breaking Instantons to Monopoles

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International Seminars on Topological Solitons
Nagoya University
September 25, 2023

KL and P.Yi, 9706023

Zhihao Duan, KL, June Nahmgoong, Xin Wang 2103.06003

Hee-Cheol Kim, KL, Kaiwen Sun, Xin Wang to appear

D0 on D4 branes

KL and P.Yi, 9706023

Instantons on R⁴

Anti self dual $F + {*F} = 0$

ADHM construction

$$[B_1, B_1^\dagger] + [B_2, B_2^\dagger] + qq^\dagger - \tilde{q}^\dagger \tilde{q} = \zeta$$

$$[B_1, B_2] + q\tilde{q} = 0$$

Instanton moduli space = hyper Kahler space

For $SU(2)$ gauge group with single instanton, dim = 8 = 4+4

center of mass motion + relative motion $\mathbb{C}^2/\mathbb{Z}_2$

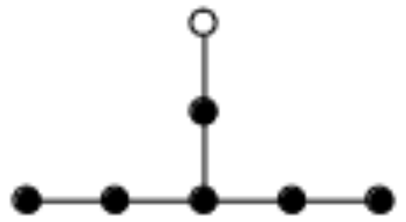
Instantons of group G

Rank r_G, dimension d_G

dual Coxeter number $h_G^\vee$

k instanton zero mode $4kh_G^\vee = 4 + 4(kh_G^\vee - 1)$

Dynkin diagram G

$G^{(1)}$	$h_G^\vee$	dynkin
A_r	$r+1$	
B_r	$2r-1$	
C_r	$r+1$	
D_r	$2r-2$	
E_6	12	
E_7	18	
E_8	30	
G_2	4	
F_4	9	

D0-D4 branes

K D0 branes in N D4 branes

5d $\mathcal{N} = 2$ SYM with SU(N) gauge group

$$(A_\mu, \phi, \lambda) + (\Phi_a, \Psi)$$

Instanton current $J = \frac{1}{32\pi^2} * \mathrm{Tr}(F \wedge F)$

Instanton of mass $8\pi^2/g_5^2 = 1/R_5$

Kaluza-Klein modes on a circle of radius R_5

6d (2,0) SCFTs for N M5 branes on a circle $x_5 \sim x_5 + 2\pi R_5$

6d (2,0) SCFTs

6d (2,0) SCFTs on N M5 branes

Tensor theory: (B, ϕ, λ) + (Φ_a, Ψ) such that H=dB=*H

M2 branes between 2 M5 branes =self-dual strings

6d (2,0) SCFTs= A,D,E types (type II on $\mathbb{C}^2/\Gamma_{A,D,E}$

Degrees of freedom: $h_G^\vee d_G$

D0-D4 branes on a circle

$$x_4 \sim x_4 + 2\pi R_4$$

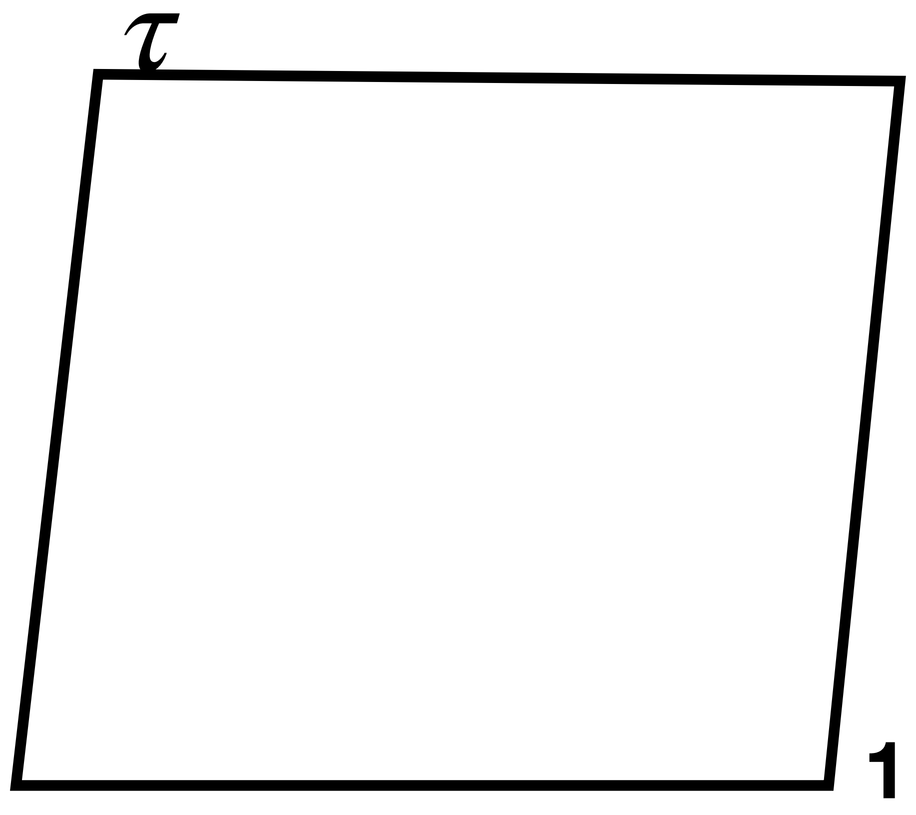
4d $\mathcal{N} = 4$ SYM with SU(N) gauge group + KK modes

Coupling constant: $\tau = \theta/2\pi + 4\pi i/g_4^2$

$$1/g_4^2 = 2\pi R_4/g_5^2,\ \alpha_4 = g_4^2/4\pi = g_5^2/8\pi^2 R_4 = R_5/R_4$$

S-duality: $\tau \leftrightarrow -1/\tau$

$$R_4 \leftrightarrow R_5$$



D0-D4 branes on S^1

Gauge holonomy $\langle A_4 \rangle = (v_1, v_2, \cdots, v_N)$ $v_1 \geq v_2 \geq \cdots v_N \geq v_{N+1} = v_1 + 1/R_4$

T-duality: D1-D3 branes with D3 branes position v_i on the dual circle

For each simple root $\boldsymbol{\alpha}_i = e_i - e_{i+1},\ (i = 1,\ldots,N-1),$ and $\boldsymbol{\alpha}_0 = e_N - e_1$

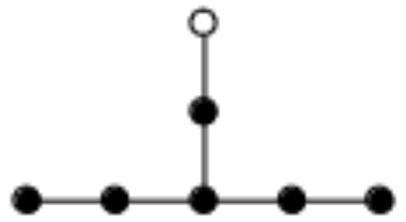
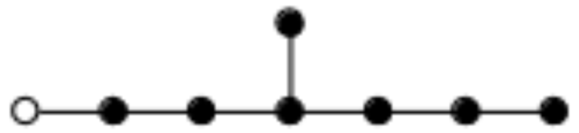
a fundamental monopole of charge $\boldsymbol{\alpha}_i^\vee = 2\alpha_i/\alpha_i^2 = \alpha_i$ and mass

$$\frac{4\pi}{g_4^2}\boldsymbol{\alpha}_i \cdot \mathbf{v} = \frac{4\pi}{g_4^2}(v_i - v_{i+1})$$

The comarks are $a_i = 1$ and so the total magnetic charge =0 and total mass is

$$\frac{4\pi}{g_4^2} \cdot \frac{1}{R_4} = \frac{1}{R_5}$$ is the instanton mass

Dynkin diagram G

$G^{(1)}$	$h_G^\vee$	dynkin
A_r	$r+1$	
B_r	$2r-1$	
C_r	$r+1$	
D_r	$2r-2$	
E_6	12	
E_7	18	
E_8	30	
G_2	4	
F_4	9	

5d theory of $G^{(1)}$ on S^1

Gauge holonomy $\langle A_4 \rangle = \mathbf{v} \cdot \mathbf{H},\ \alpha_i^\vee \cdot \mathbf{v} \geq 0,\ 1/R_4 + \alpha_0^\vee \cdot \mathbf{v} \geq 0$

A fundamental monopole for each roots $\alpha_i,\ i = 0,1,...,r$

Comarks $a_i^\vee$: $\sum_{i=0}^{r} a_i^\vee \boldsymbol{\alpha}_i^\vee = 0$ and $a_0^\vee = 1$, dual Coxeter number $h_G^\vee = \sum_{i=0}^{r} a_i$

k instanton zero mode $4kh_G^\vee = 4 + 4(kh_G^\vee - 1)$

Instanton mass= $\dfrac{4\pi}{g_4^2} \sum_i a_i \alpha_i^\vee \cdot \mathbf{v} = \dfrac{1}{R_5}$

Different Fractionalization

Consider a single fundamental string wrapping on a circle N times

Effective circle circumference is $2\pi RN$

Allowed KK momentum is $\frac{1}{NR}$ with total KK momentum being the integer multiple of 1/R.

Similarly, one can imagine N M5 branes on a circle, regarded as a single M5 brane wrapping the circle N times, would lead to 1/NR KK momentum.

5=3+2, leading to KK momentum 1/3R or 1/2R, for example.

D0+D4+F1(Supertube)

Dyonic Instantons

KK momentum on selfdual strings in (2,0) theory = a wave on a circle

Wave carrying $J_1 : (x_1, x_2), \quad J_2 : (x_3, x_4)$ angular momentum

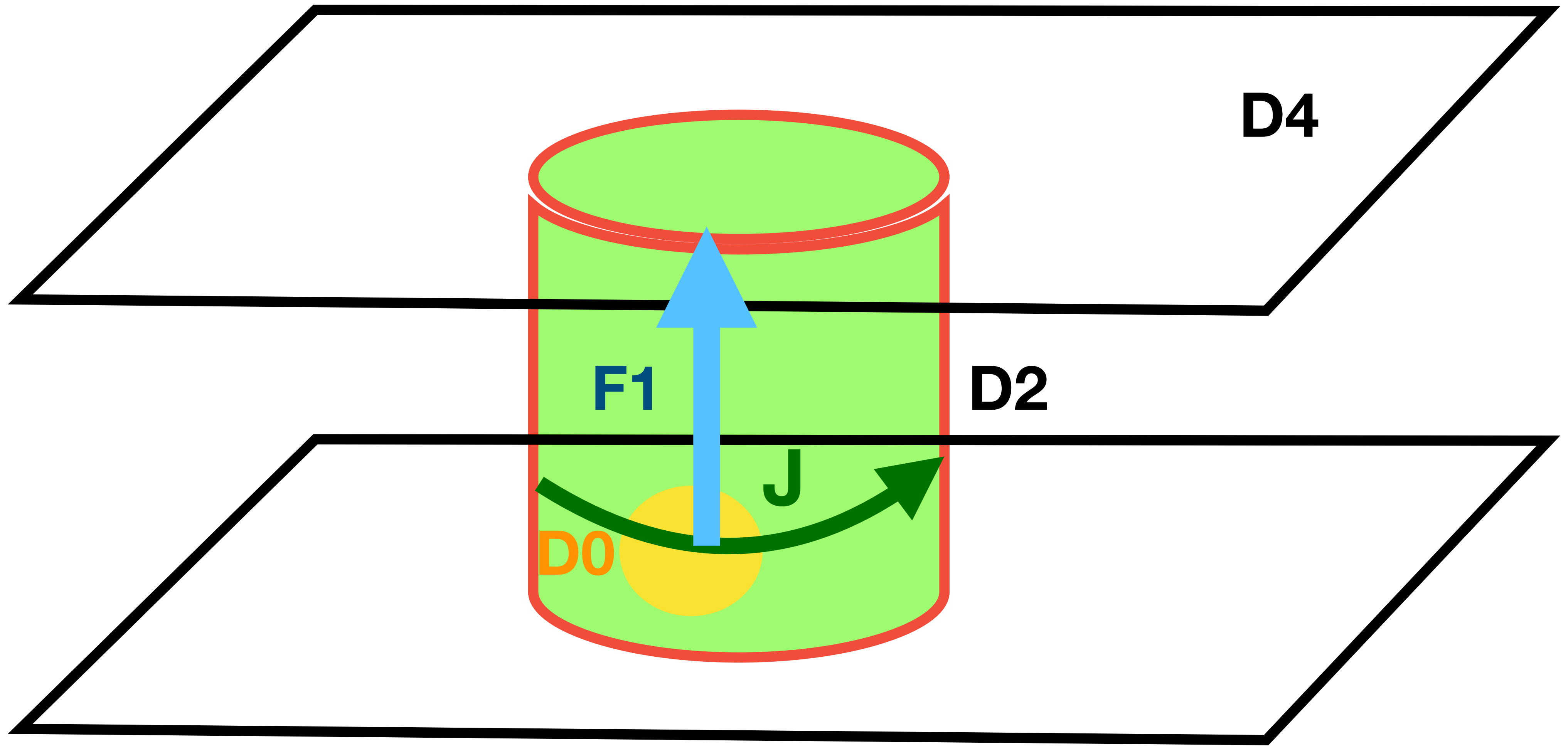
Supertube: D2 brane circle connecting two D4 branes (Meyer's effect)

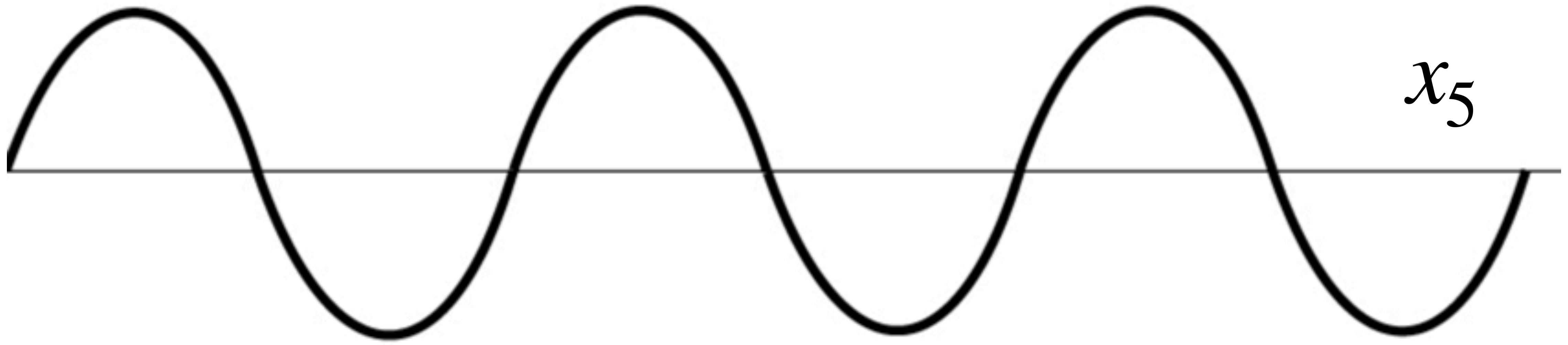
D0 on D2=magnetic field, F1 on D2=electric field

Poyinting vector: angular momenta J_1, J_2

Nekrasov partition function

D0+D4+F1(Supertube)





Solve the Laplacian for adjoint scalar in the instanton background

For SU(2) gauge group, the dyonic instanton is characterized by

$$D^\mu D_\mu \Phi = 0, \ \langle \Phi^a \rangle_\infty = v\delta^{a3}.$$

Two D4 branes meet at the curves defined by $\Phi(x_\mu) = 0$.

Seok Kim, KL 20xx

Twisting (2,0) SCFTs

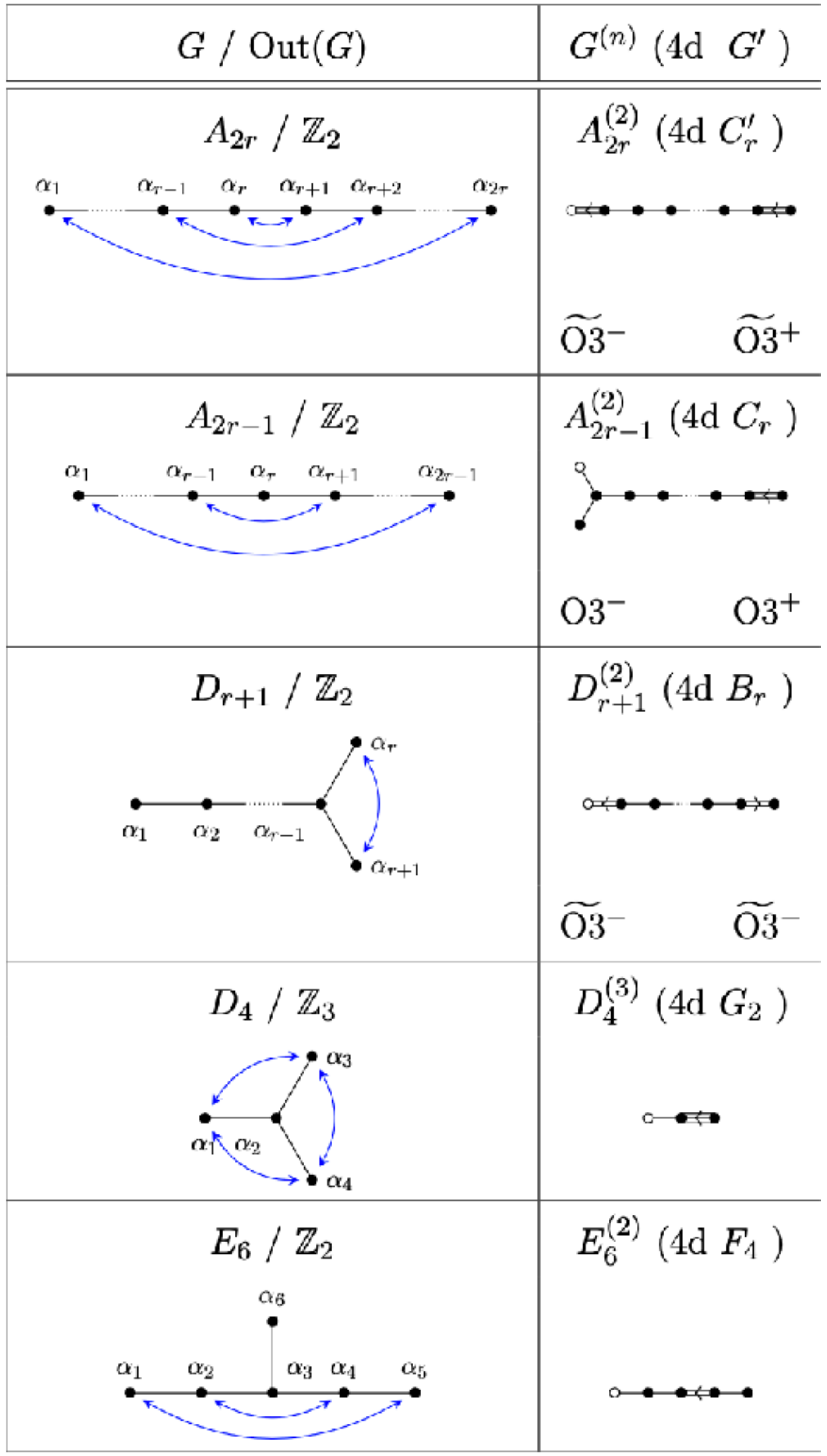
Zhihao Duan, KL, June Nahmgoong, Xin Wang 2103.06003

5d theory of $G^{(n)}$ on S^1 with Twist

Outer-automorphism $\phi(x_4 + 2\pi R_4) = \sigma(\phi(x_4))$

Twisted affine algebra with twisted Dynkin diagram

$$\begin{array}{lcl}
A_{2r}^{(2)}: \ \mathbf{adj} \text{ of } A_{2r} & \longrightarrow & \text{long}_k \oplus \text{short}_{\frac{k}{2}} \oplus \text{special}_{k\pm\frac{1}{4}} \oplus 1_{k+\frac{1}{2}} \ \text{ of } C_r' \\
A_{2r-1}^{(2)}: \ \mathbf{adj} \text{ of } A_{2r-1} & \longrightarrow & \text{long}_k \oplus \text{short}_{\frac{k}{2}} \ \text{ of } C_r \\
D_{r+1}^{(2)}: \ \mathbf{adj} \text{ of } D_{r+1} & \longrightarrow & \text{long}_k \oplus \text{short}_{\frac{k}{2}} \ \text{ of } B_r \\
E_6^{(2)}: \ \mathbf{adj} \text{ of } E_6 & \longrightarrow & \text{long}_k \oplus \text{short}_{\frac{k}{2}} \ \text{ of } F_4 \\
D_4^{(3)}: \ \mathbf{adj} \text{ of } D_4 & \longrightarrow & \text{long}_k \oplus \text{short}_{\frac{k}{3}} \ \text{ of } G_2 \ ,
\end{array}$$



The simple roots are

$$\beta_j,\ j = 0,1,\ldots,r'$$

Comarks $\displaystyle\sum_{j=0}^{r'} b_j^\vee \beta_j^\vee = 0$

β_0 is a short root and $b_0 = 1$

Dual Coxeter number

$$h_G^\vee = \sum_i a_i^\vee = \sum_j b_j^\vee$$

$h_G^\vee$ does not change under twist

5d N=2 SYM of B,C,C’,G,F types

S-dual in 4-dim = the change of the compactification

S-dual of Twisted Dynkin diagram

5d $N = 2$ theories of B, C, C', G, F on a circle

G / Out(G)	$G^{(n)}$ (4d G')	$G^{\vee(1)}$	5d $G^{\vee}$
A_{2r} / $\mathbb{Z}_2$	$A_{2r}^{(2)}$ (4d C'_r) $\widetilde{O3}^-$ $\widetilde{O3}^+$	$(C_r^{(1)})_\pi$ $O3^+$ $\widetilde{O3}^+$	$(C_r)_\pi$ $\widetilde{O4}^+$
A_{2r-1} / $\mathbb{Z}_2$	$A_{2r-1}^{(2)}$ (4d C_r) $O3^-$ $O3^+$	$B_r^{(1)}$ $O3^-$ $\widetilde{O3}^-$	B_r $\widetilde{O4}^-$
D_{r+1} / $\mathbb{Z}_2$	$D_{r+1}^{(2)}$ (4d B_r) $\widetilde{O3}^-$ $\widetilde{O3}^-$	$(C_r^{(1)})_0$ $O3^+$ $O3^+$	$(C_r)_0$ $O4^+$
D_4 / $\mathbb{Z}_3$	$D_4^{(3)}$ (4d G_2)	$G_2^{(1)}$	G_2
E_6 / $\mathbb{Z}_2$	$E_6^{(2)}$ (4d F_4)	$F_4^{(1)}$	F_4

Twisting (2,0) Theories

S-dual in 4-dim = the change of the compactification

S-dual of Twisted Dynkin diagram

5d $N = 2$ theories of B, C, C', G, F on a circle

Preservation of DOF

$$\frac{h_G^\vee \cdot d_G}{n_G} = h_{H^\vee}^\vee d_{H^\vee}$$

SU(6) 6*35: SO(7) 5*21

	d_G	$h_G^\vee$	$\|\vec{\rho}\|^2$
A_r	$(r+1)^2-1$	$r+1$	$\frac{1}{12}r^3+\frac{1}{4}r^2+\frac{1}{6}r$
B_r	$2r^2+r$	$2r-1$	$\frac{1}{3}r^3-\frac{1}{12}r$
C_r	$2r^2+r$	$r+1$	$\frac{1}{6}r^3+\frac{1}{4}r^2+\frac{1}{12}r$
D_r	$2r^2-r$	$2r-2$	$\frac{1}{3}r^3-\frac{1}{2}r^2+\frac{1}{6}r$
G_2	14	4	$\frac{14}{3}$
F_4	52	9	39
E_6	78	12	78
E_7	133	18	$\frac{399}{2}$
E_8	248	30	620

(2,0) and (1,1) LSTs

Hee-Cheol Kim, KL, Kaiwen Sun, Xin Wang to appear soon

T-duality between (2,0) and (1,1) LSTs

A,D,E types

(2,0) A-type LST: N NS5 branes of type IIA= N M5 branes on M-circle

F1= M2 on M-circle= N fundamental self-dual strings $\sum_{i=0}^{N-1} \alpha_i = 0$

(1,1) A-type LST: N NS5 branes in type IIB

6d (1,1) SYM of gauge group SU(N)

Instanton strings = F1

T-duality between (2,0) and (1,1) LSTs

(2,0) LST on a circle of radius R_5

=(1,1) LST on a circle of radius $\tilde{R}_5 = \ell_s^2 / R_5$

instanton string of (1,1) LST on a circle =compost of 5d magnetic monopole strings

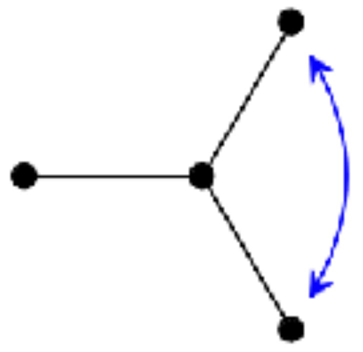
T-duality: KK momentum modes <-> winding modes

(2,0) wrapped self-dual string<-> (1,1) fractional momentum

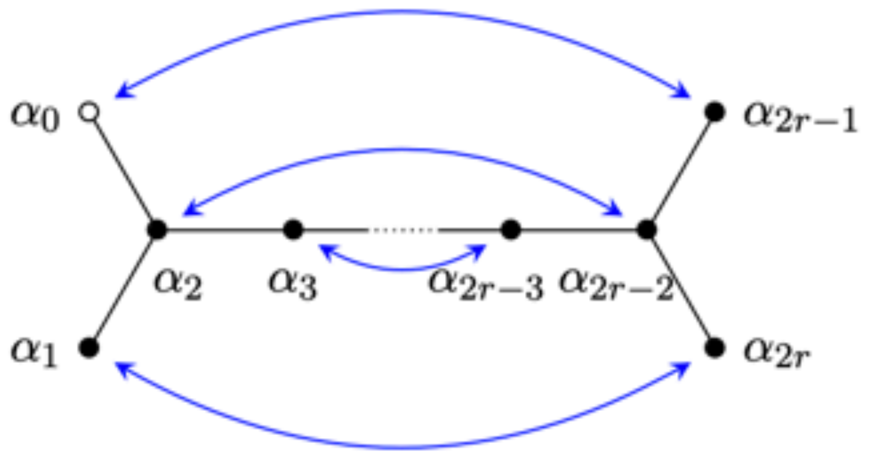
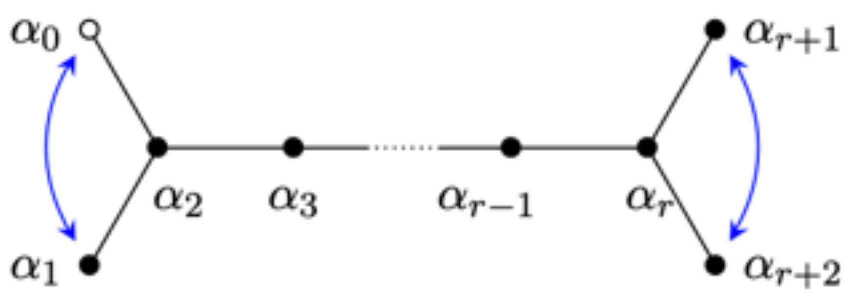
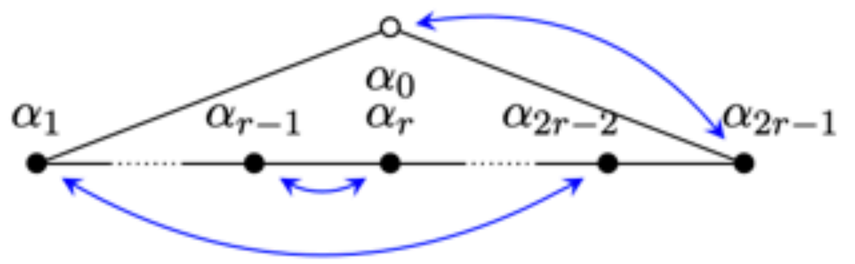
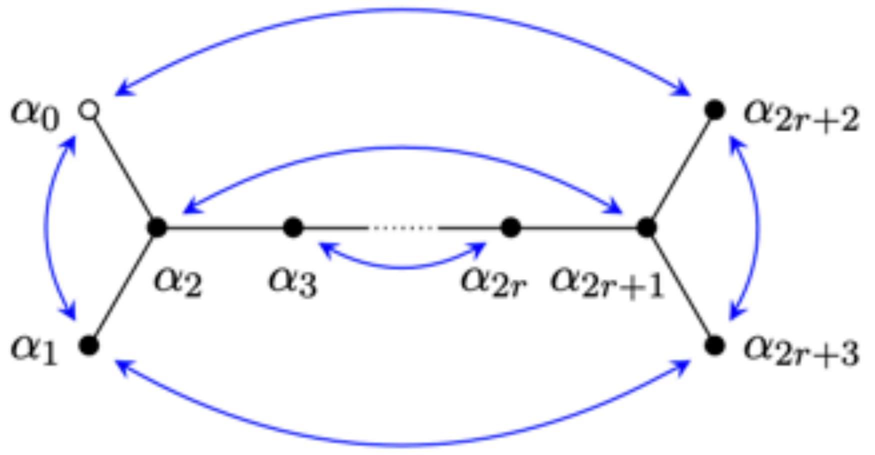
(2,0) integer KK modes <-> (1,1) instanton strings wapping the circle

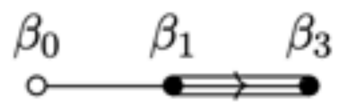
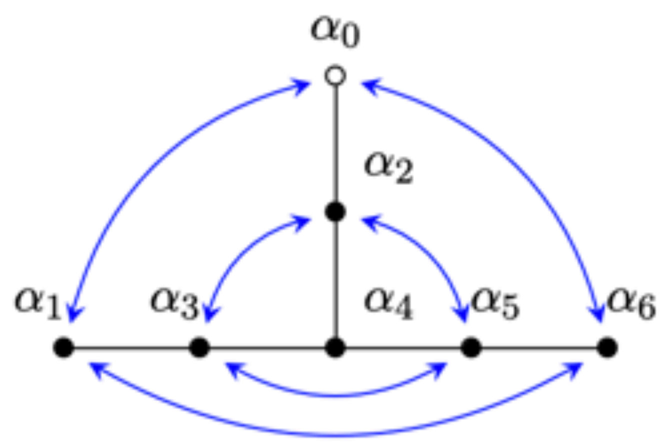
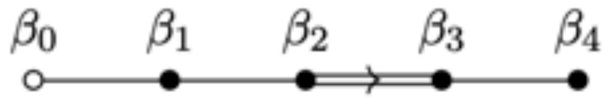
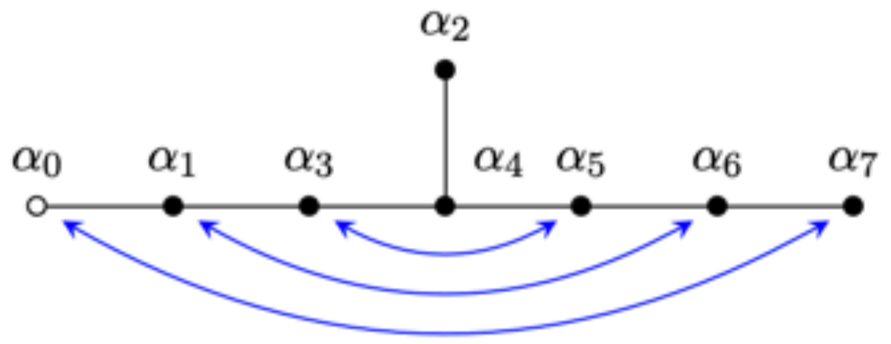
T-duality with twist

Lessions from twisting of (2,0) SCFTs

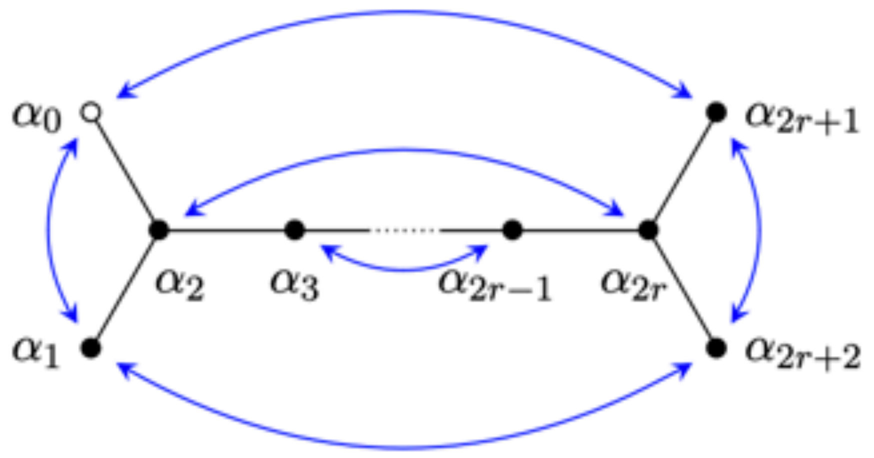
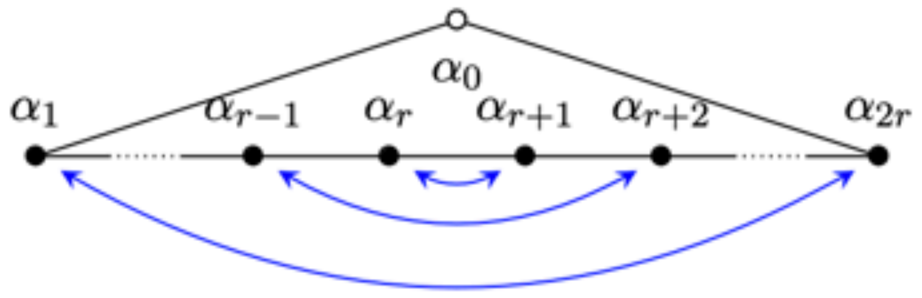
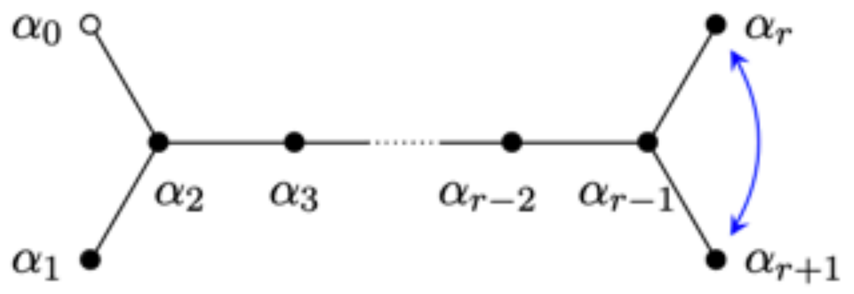
(2,0) SCFT	twisted
	 π $\widetilde{O4}^{+}$
	 $\widetilde{O4}^{-}$
	 0 $O4^{-}$

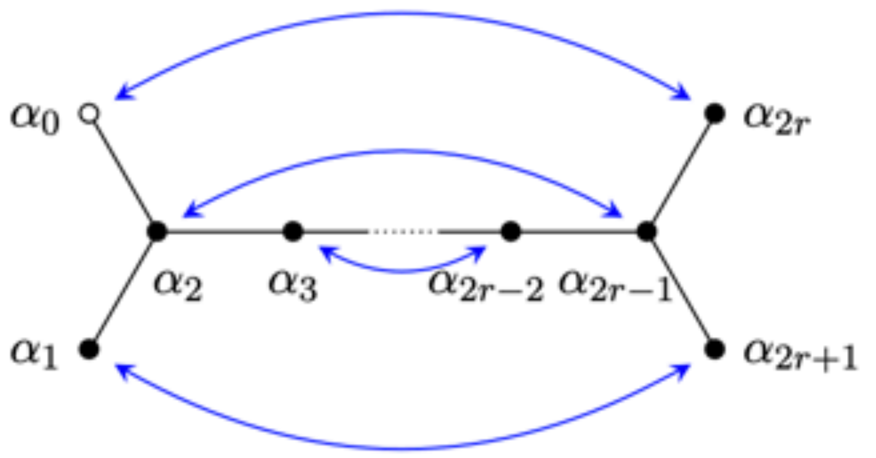
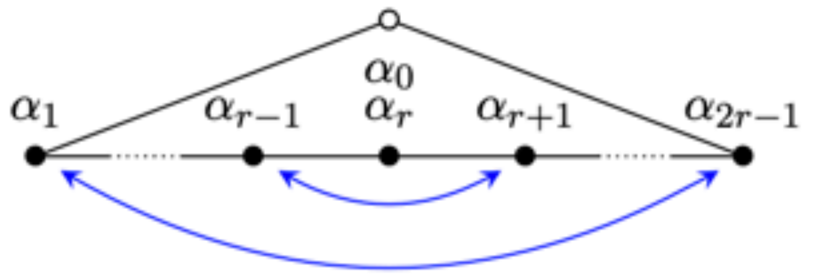
(1,1) LSTs for B,C,C',G,F SYM

(1,1)		(2,0)
$B_r^{(1)}$ (2r-1) 	$\widetilde{O5}^-$ $O4^- + \widetilde{O4}^-$	$D_{2r}/\mathbb{Z}_2$ 2(2r-1) 
$(C_r^{(1)})_0$ (r+1) 	O5$^+$ $O4^+ + O4^+$	$D_{r+2}/\mathbb{Z}_2$ 2(r+1) 
$(C_r^{(1)})_0$ (r+1)  π, π	O5$^+$ $\widetilde{O4}^+ + \widetilde{O4}^+$	$A_{2r+1}\mathbb{Z}_2$ 2(r+1) 
$(C_r^{(1)})_\pi$ (r+1) 	$\widetilde{O5}^+$ $O4^+ + \widetilde{O4}^+$	$D_{2r+3}/\mathbb{Z}_2 \times \mathbb{Z}_2$ $4 \cdot (r+1)$ 

(1,1)		(2,0)
$G_2^{(1)}$ 4 		$E_6/\mathbb{Z}_3$ $12 = 3 \cdot 4$ 
$F_4^{(1)}$ 9 		$E_7/\mathbb{Z}_2$ $2 \cdot 9$ 

Twisting of (1,1) LSTs for A,D,E SYM

(1,1)		**(2,0)**
$A^{(2)}_{2r,0}$ $(2r+1)$ 	$\widetilde{O4}^- + O4^+$	$D_{2r+2}/\mathbb{Z}_4$ $2(2r+1)$ 
$A^{(2)}_{2r,\pi}$ $(2r+1)$ 	$\widetilde{O4}^- + \widetilde{O4}^+$	$A^{(1)}_{2r}/\mathbb{Z}_2$ $(2r+1)$ 
$A^{(2)}_{2r-1,0}$ $2r$ 	$O4^- + O4^+$	$D_{r+1}/\mathbb{Z}_2$ $2r$ 

(1,1)		**(2,0)**
$A^{(2)}_{2r-1,\pi}$ $2r$ 	$O4^- + \widetilde{O4}^+$	$D_{2r+1}/\mathbb{Z}_4$ $2\cdot 2r$ 
$D^{(2)}_{r+1}$ $2r$ 	$\widetilde{O4}^- + \widetilde{O4}^-$	$A_{2r-1}/\mathbb{Z}_2$ $2r$ 

Conclusion

Instanton can be fractionalized to magnetic monopoles

It appears in 4d YM on a circle or 5d YM on a circle

Instanton strings can be broken to monopole strings in 6d (1,1) LST on a circle

Partition functions involving monopoles and instants are considered.

Questions

4d BPS quiver of LSTs on $R^{1+3} \times T^2$

Massless monopoles and monopole bubbling

Monopole walls and Fermions

Monopole string junction=self-dual string junctions

3d magnetic monopole operators as hyper-multiplets