Liquid crystal topological defects – particle physics resemblance

Repair classical EM (Faber):

1) $\int_0^\infty E^2 r^2 dr \propto \int_0^\infty r^{-4} r^2 dr = \infty$

Deform/**regularize charge to finite energy** (**Higgs-like potential** allowing deformation)

2) **Gauss law** charge $q \in \mathbb{R}$, nature $q \in \mathbb{Z}e$

Interpret **curvature as electric field** to count **(quantized) topological charge** with Gauss law.

Skyrmion-like with SO(3) ellipsoid vacuum

3 distinguishable axes like biaxial nematic

Use **real symmetric tensor field** $M = ODO^T$

~**Higgs** e.g. $V = \sum_i (\lambda_i - \Lambda_i)^2$ for D **shape**

Getting **3 leptons**, baryons, nuclei …

with **unified wave-like vacuum dynamics**:

EM >> quantum phase >> GEM

EM	quantum phase	GEM
1st axis rotations	~Klein-Gordon/Dirac? 1st axis twists (~Berry)	0th axis in 4D tiny boosts

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

spin/particle + phase $\quad \partial_\mu$? $\quad$ Lagrangian/Hamiltonian

Jarek Duda **article** **demo** **video** **github**

E,B Ahar. Bohm

$\partial_\mu A_\nu - \partial_\nu A_\mu$

Maxwell: $\Box A_\mu \propto J_\mu$ $\quad A \quad$ gauge ?

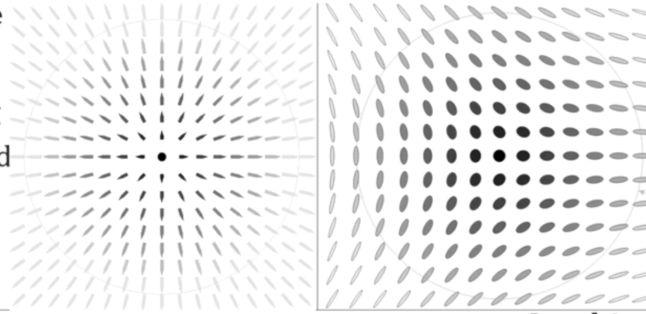
extended quantum phase for topological charge quantiziation

(dual) $A^*_\mu = [M, M_\mu]$ $\cong$affine connection $F^* \cong$ curvature

QM+EM $\quad$ EM

$\psi_{2D} \xrightarrow{M=ODO^T} 3D \quad \psi$

$\Box\psi \propto -\psi \quad | \quad \hat{\mathbf{P}} = -i\hbar\nabla - q\mathbf{A}$



2D charge (+1) as director hedgehog

regularized $\vec{n} \to 0$ by Higgs pot. $V(\vec{n}) = (\|\vec{n}\|^2 - 1)^2$

2 axes in 2D

regularized $V(M) = \sum_i (\lambda_i^M - \Lambda_i)^2$ to finite energy

e.g. Landau-de Gennes' (in 3D: vortex/fluxon)

2D charge/spin -1/2 $= \frac{1}{2\pi} \oint_L (n_2 n_1' - n_1 n_2')\, dL$

$\vec{n} \equiv \vec{n}(x) \qquad \|\vec{n}\| = 1$

electric/topological (3D winding number) **3D charge in 3D**

Gauss law $\qquad$ Jacobian $\mathcal{S} \to S^2$

$$= \oint_{\mathcal{S}} E \cdot dA = \frac{e_0}{4\pi} \oint_{\mathcal{S}(u,v)} du\, dv\, (\partial_u \vec{n} \times \partial_v \vec{n}) \cdot \vec{n}$$

curvature

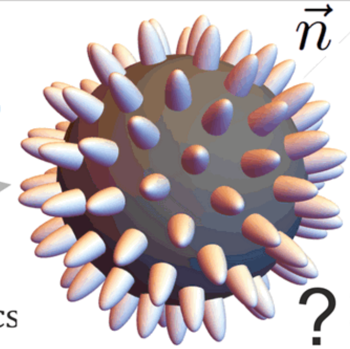
uniaxial nematic

unitary vector in 3D of director field $\vec{n}(x)$

1 distingused axis

charge + Coulomb

Maxwell equations from director dynamics



$\vec{n}$?

vacuum $S^2 \to SO(3)$

biaxial nematic

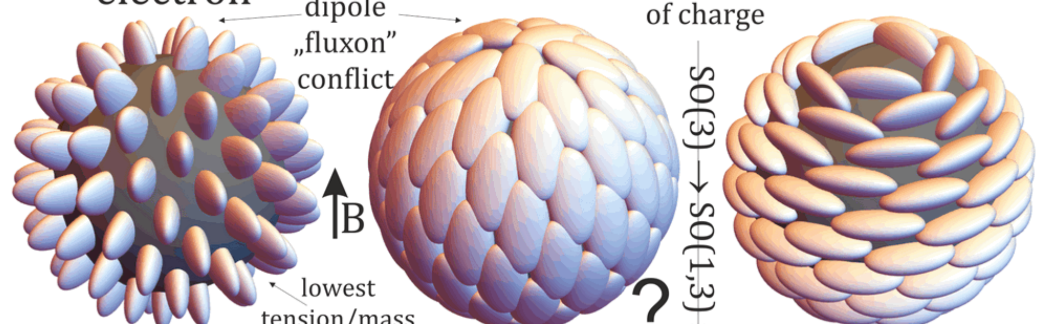
$1 \to$ **3 in 3D** distinguished axes

„uniaxial + quantum phase" (for „pilot wave")

field of real symmetric M

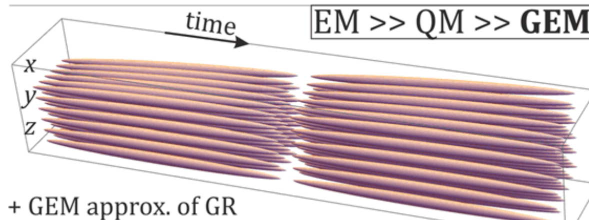
3 charges with magnetic dipoles

electron $\quad$ magnetic dipole „fluxon" conflict $\quad$ muon $\quad$ 3 realizations of charge $\quad$ tau



B $\quad$ lowest tension/mass $\quad$?

$SO(3) \to SO(1,3)$



+ GEM approx. of GR from tiny perturbations (boost) of 0th axis

EM >> QM >> **GEM**

$3 \to$ **4 axes** in **4D spacetime**

0th: local time direction

2nd set of **Maxwell equations** for its tiny perturbations: much weaker (longer axis), no mass/energy quantization

energy density: **EM** | **uniaxial** (Higgs potential) | **biaxial/general**, M - matrix (real, symmetric)

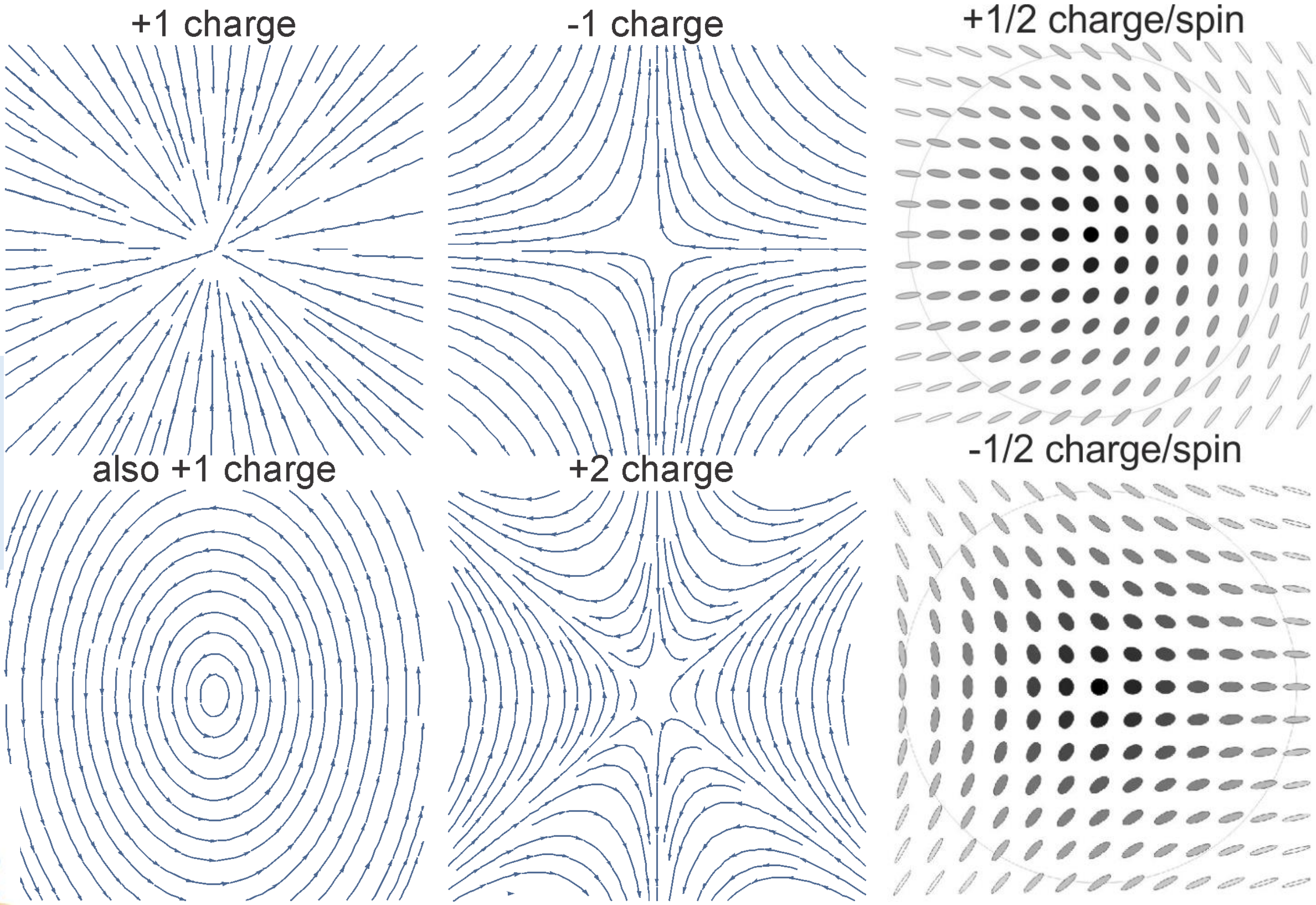
quantized charge $\quad$ curvature $\quad$ (regularization) $\quad$ curvature

$$\mathcal{H} \sim \sum_{\mu,\nu=0}^{3} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \to \sum_{\mu,\nu=0}^{3} \|\partial_\mu \vec{n} \times \partial_\nu \vec{n}\|^2 + (\|\vec{n}\|^2 - 1)^2 \to \sum_{\mu,\nu=0}^{3} \|\partial_\mu A_\nu - \partial_\nu A_\mu\|^2 + V(M)$$

$$A_\mu = M M_\mu - M_\mu M$$

final 4D Lagrangian? $\boldsymbol{\mathcal{L} = -F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} - V_{\mathrm{Higgs}}(M)}$ $\qquad (R \sim)\ F_{\mu\nu\alpha\beta} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$

2D topological charge



fluxon – **quant** of magnetic field

resembles **spin** ($\rightarrow \mu$)   also **½**

$$2\pi k = \Delta\varphi = \frac{q}{\hbar}\oint_{\partial S} A\cdot dl = \frac{q}{\hbar}\oint_{S} B\cdot dS$$

quantum rotation operator

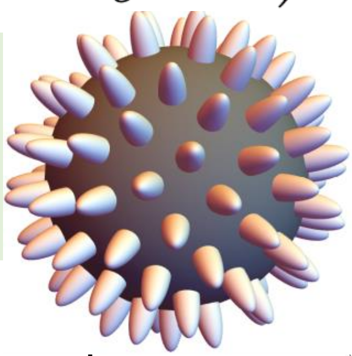
spin s particle by θ angle rotates

quantum phase: $\psi \rightarrow \psi\, e^{-is\theta}$

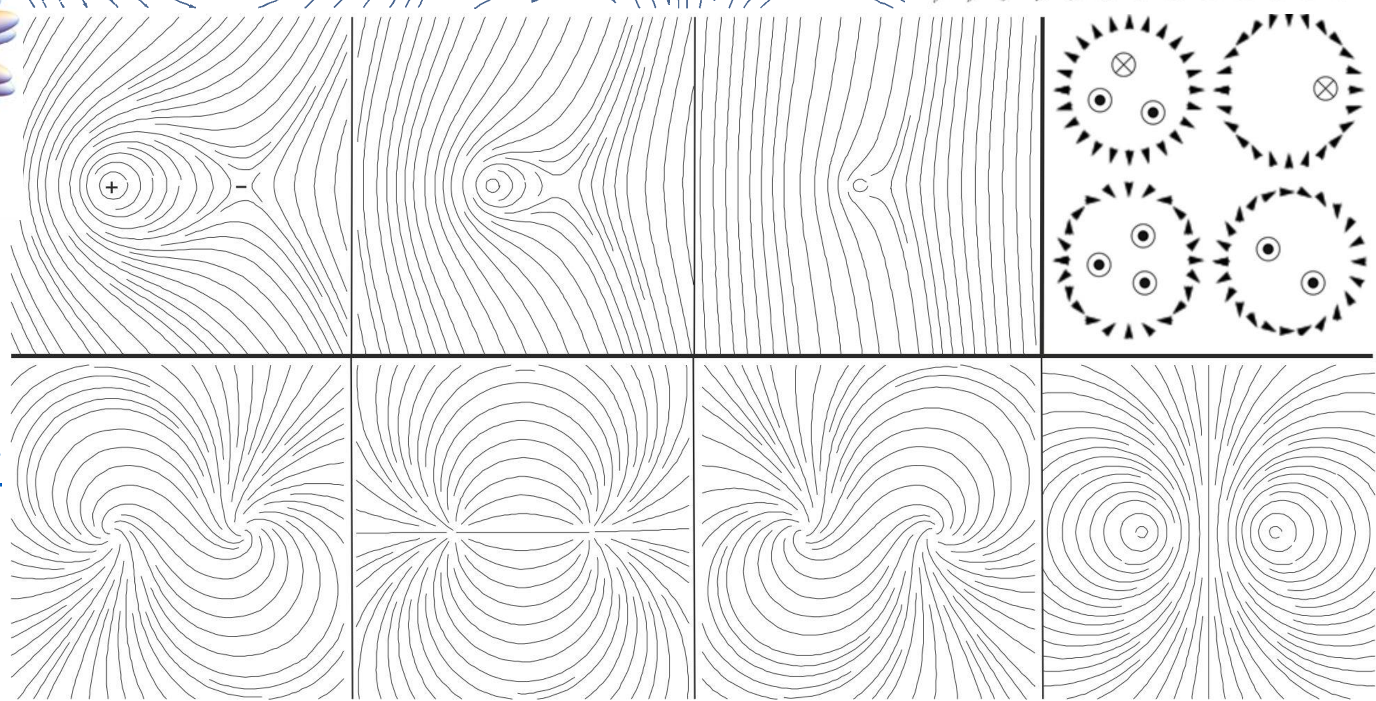
(½ spin) bispinor rotation by ϕ:

$$S[\Lambda_{\mathrm{rot}}] = \begin{pmatrix} e^{+i\phi\cdot\sigma/2} & 0 \\ 0 & e^{+i\phi\cdot\sigma/2} \end{pmatrix}$$

3D topological charge: electric



$V(r)$ distance dependence of field stress/energy gives **long-range attraction/repulsion**:



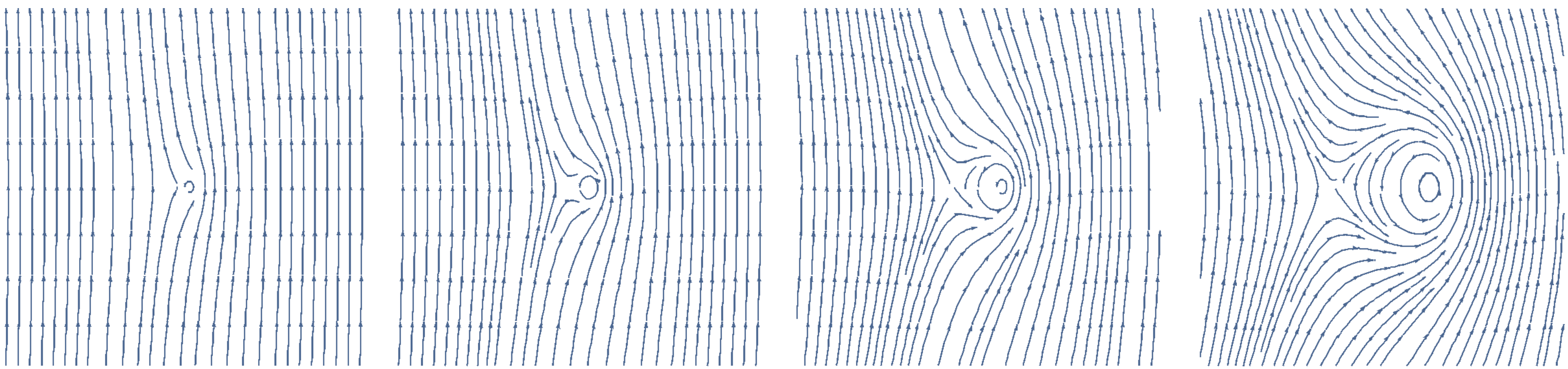
de Broglie clock/zitterbewegung

evolution of **quantum phase** around - forming pilot wave

electromagnetism	topological charge/index/degree
charge conservation	the number of **covering** of $\boldsymbol{f}: \boldsymbol{S}^{\boldsymbol{D-1}} \to \boldsymbol{S}^{\boldsymbol{D-1}}$ **Hopf thm**: f, g are homotopic $\Leftrightarrow deg\ f = deg\ g$ **Poincare-Hopf thm.**: $\sum_i deg_{x_i} = \chi(S)$ **Riemann-Hurwitz thm.**: $\chi(n \text{ covering of } S) = n\, \chi(S)$
Gauss law (in **3D**): **charge** inside $S = \oint_S E \cdot dS$ charge is real number	**argument principle in 2D:** $\oint_C (\ln f)' dz = 2\pi i (N - P)$ general - quantized **Gauss-Bonnet theorem**: $\oint_S R \cdot dS = 2\pi n\, \chi(S)$ (R: curvature/Jacobian)
Coulomb: same charges repel, opposite attract $F \propto 1/r^2$	to **reduce stress of the field** entire field causes attraction/repulsion

$\vec{\Gamma}_i = (\partial_i \vec{u}) \times \vec{u}$ $\quad \vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ $\quad \mathcal{L}_{EM} = -\frac{\alpha\hbar c}{16\pi} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu}$ Faber's quantized EM

Local rotation axis, curvature, EM Lagrangian with $F_{\mu\nu} \sim R^*_{\mu\nu}$ $(E \leftrightarrow^* B)$



QED $\hbar c$ tiny → unify, also gravity

$$\mathcal{L}_{QED} = -\boxed{\hbar c\bar\psi\gamma^\mu\partial_\mu\psi} - mc^2\bar\psi\psi - \boxed{F_{\mu\nu}F^{\mu\nu}/4}$$

$$\mathcal{L} = -\underbrace{\sum_{\alpha\beta\mu\nu} F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta}}_{\text{EM-like}} - \underbrace{V(M)}_{\text{Higgs-like}}$$

SO(1,3) minimum

for $F_{\mu\nu\alpha\beta} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$

charge quantizaton

F: **shape D** weighted curvature R

Landau-de Gennes/EM-like Lorentz-invariant Lagrangian,

vacuum for long-range interactions

V=0: $\boxed{M = O\, D\, O^T}$ $O \in \mathrm{SO}(1,3)$

stress-energy tensor+Higgs

O: 3D rotation + 4D boost

$\boxed{D = \mathrm{diagonal}(g, 1, \delta, 0)}$

gravity	+ EM	+ QM
g>>1 boosts	1	1>>δ>0
~1/G	1	Planck

Aharonov-Bohm E, B ← $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ← A gauge?

Maxwell: $\Box A_\mu \propto J_\mu$

extended quantum phase for topological charge quantiziation

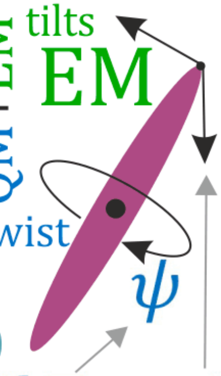
(dual) $A^*_\mu = [M, M_\mu]$

≅affine connection

$F^* \cong$ curvature

$\psi_{2D} \xrightarrow{M = ODO^T} 3D$

$\Box\psi \propto -\psi$ | $\hat{\boldsymbol{P}} = -i\hbar\boldsymbol{\nabla} - qA$




```
M̄μ = {{0, g Γ̃¹μ, g Γ̃²μ, g Γ̃³μ}, {-g Γ̃¹μ, 0, Γ³μ, -Γ²μ}, {-g Γ̃²μ, Γ³μ, 0, δ Γ¹μ}, {-g Γ̃³μ, -Γ²μ, δ Γ¹μ, 0}}; (* g >> 1 >> δ > 0 approximation *)
(F̄μν = FullSimplify[coms[M̄μ, M̄μ /. μ → ν], sub]) // MatrixForm
```

→ clock ? EM/QM - gravity interaction

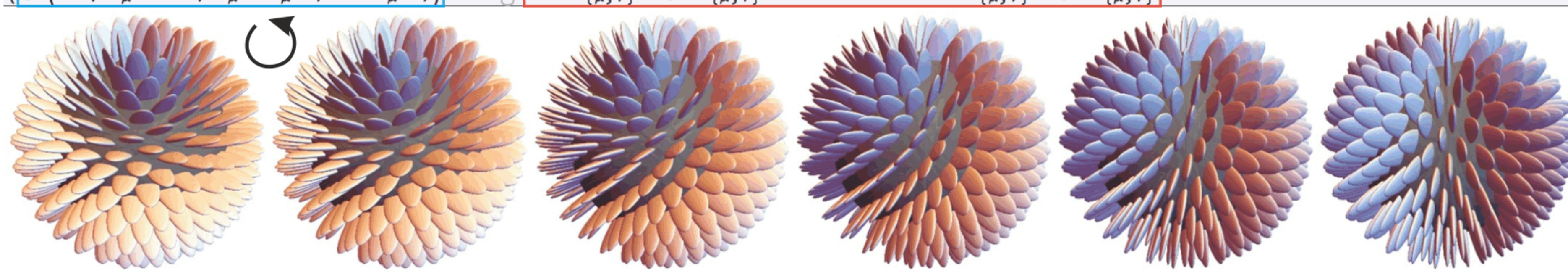
$$\begin{pmatrix} 0 & g\left(\Gamma^3_\nu\tilde\Gamma^2_\mu - \Gamma^2_\nu\tilde\Gamma^3_\mu - \Gamma^3_\mu\tilde\Gamma^2_\nu + \Gamma^2_\mu\tilde\Gamma^3_\nu\right) & g\left(\Gamma^3_\nu\tilde\Gamma^1_\mu + \delta\Gamma^1_\nu\tilde\Gamma^3_\mu - \Gamma^3_\mu\tilde\Gamma^1_\nu - \delta\Gamma^1_\mu\tilde\Gamma^3_\nu\right) & g\left(-\Gamma^2_\nu\tilde\Gamma^1_\mu + \delta\Gamma^1_\nu\tilde\Gamma^2_\mu + \Gamma^2_\mu\tilde\Gamma^1_\nu - \delta\Gamma^1_\mu\tilde\Gamma^2_\nu\right) \\ g\left(\Gamma^3_\nu\tilde\Gamma^2_\mu - \Gamma^2_\nu\tilde\Gamma^3_\mu - \Gamma^3_\mu\tilde\Gamma^2_\nu + \Gamma^2_\mu\tilde\Gamma^3_\nu\right) & 0 & \delta R^3_{\{\mu,\nu\}} + g^2\tilde R^3_{\{\mu,\nu\}} & \delta R^2_{\{\mu,\nu\}} - g^2\tilde R^2_{\{\mu,\nu\}} \\ g\left(\Gamma^3_\nu\tilde\Gamma^1_\mu + \delta\Gamma^1_\nu\tilde\Gamma^3_\mu - \Gamma^3_\mu\tilde\Gamma^1_\nu - \delta\Gamma^1_\mu\tilde\Gamma^3_\nu\right) & -\delta R^3_{\{\mu,\nu\}} - g^2\tilde R^3_{\{\mu,\nu\}} & 0 & R^1_{\{\mu,\nu\}} + g^2\tilde R^1_{\{\mu,\nu\}} \\ g\left(-\Gamma^2_\nu\tilde\Gamma^1_\mu + \delta\Gamma^1_\nu\tilde\Gamma^2_\mu + \Gamma^2_\mu\tilde\Gamma^1_\nu - \delta\Gamma^1_\mu\tilde\Gamma^2_\nu\right) & -\delta R^2_{\{\mu,\nu\}} + g^2\tilde R^2_{\{\mu,\nu\}} & -R^1_{\{\mu,\nu\}} - g^2\tilde R^1_{\{\mu,\nu\}} & 0 \end{pmatrix}$$

+/- squared energy contributions

QM: tilt-twist

EM: tilt-tilt

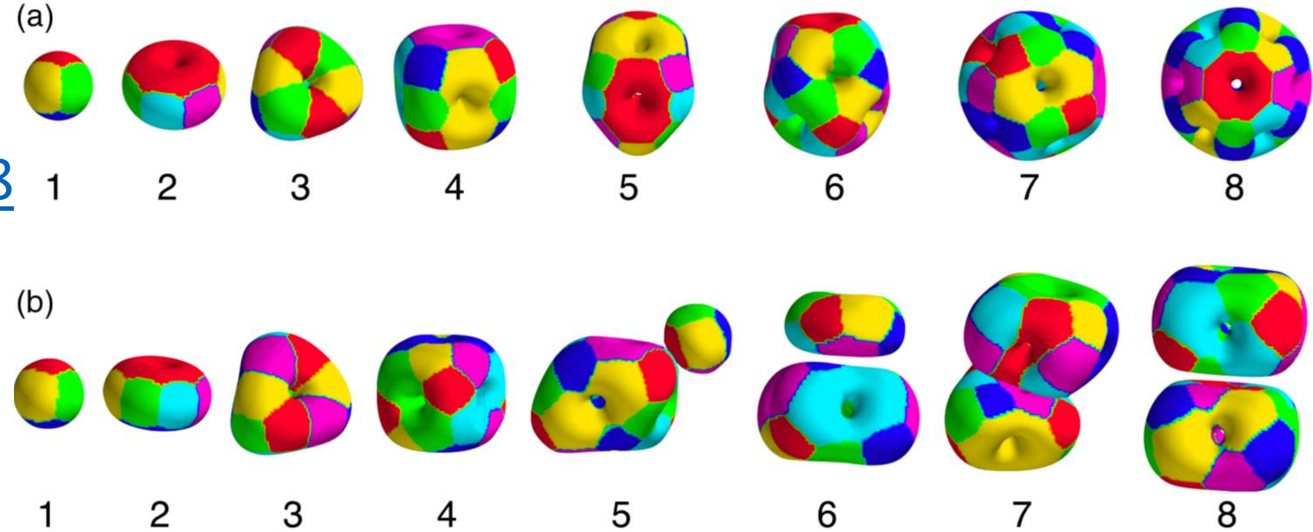
$\tilde R \equiv R^g$ gravity



electron clock(?): phase ψ time evolution

Popular skyrmion models

Solid state, nucleus: PRL 2018



U – tensor field (of matrix)

$\Gamma_i = \partial_i U\, U^{-1}$ local rotation

$E_{kin} = c_1 \sum_i \mathrm{Tr}(\Gamma_i \Gamma_i) + \cdots$

$E_{pot} \propto \mathrm{Tr}(\mathbf{1} - U)$ for unitary – single minimum $U = \mathbf{1}$

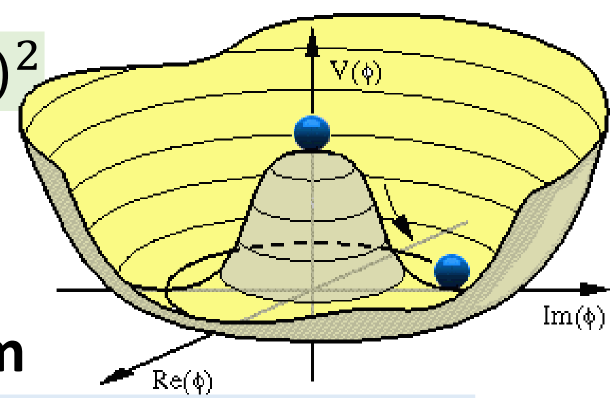
Vacuum (far from particles) filled with $U \approx \mathbf{1}$, only short-range interaction!

No EM, charge (**proton = neutron**), no long-range e.g. Coulomb interaction

(**proton** lighter than **neutron**)

For long-range: use **topologically nontrivial vacuum** (minimum of potential)

E.g. **Higgs potential** “Mexican hat”: $V(\vec{n}) = (||\vec{n}||^2 - 1)^2$



- zero is not minimum (inflation, charge regularization)
- **Dynamics in minimum** corresponds to massless particles (Goldstone bosons), like **electromagnetism**

$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu,\ \mathcal{L}_{EM} = -\frac{\alpha\hbar c}{16\pi} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu}$ EM with quantized charge (Faber)

Liquid crystal long-range interactions due to nontrivial vacuum like for Higgs $V(\vec{n}) = (|\vec{n}|^2 - 1)^2$

$F \sim 1/D$: "**Annihilation** dynamics of topological defects induced by microparticles in nematic liquid crystals" Soft Matter

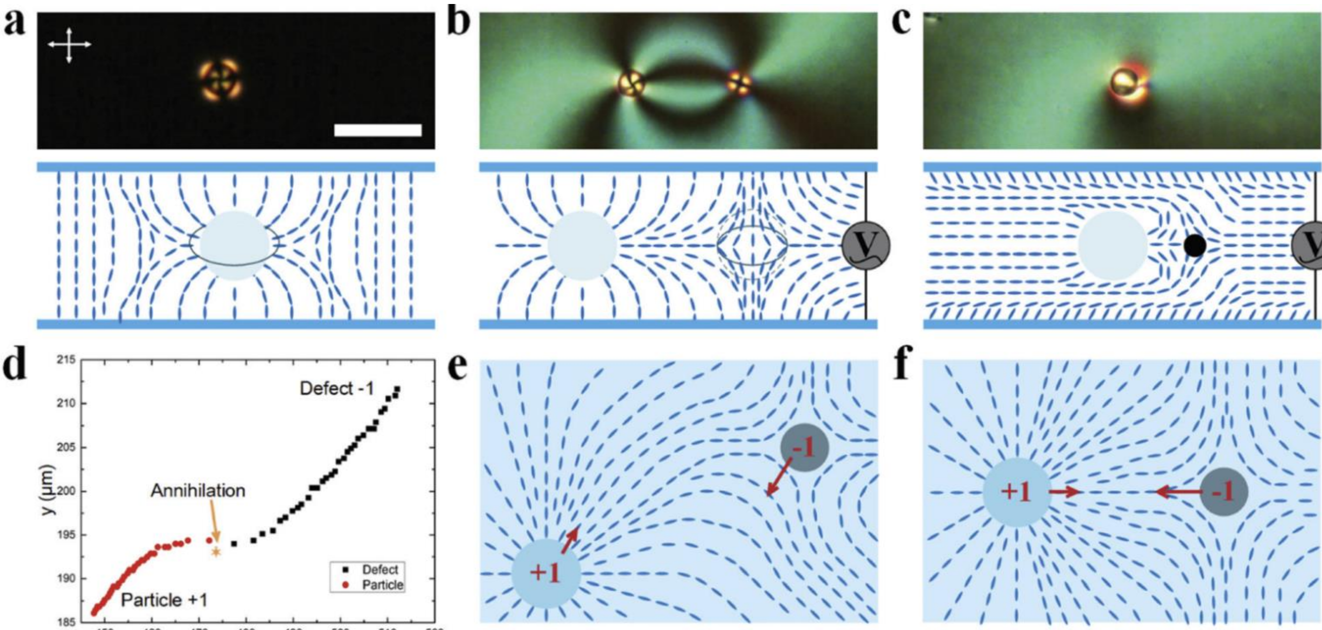
Coulomb: "**Coulomb**-like interaction in nematic emulsions induced by external torques exerted on the colloids" PRE

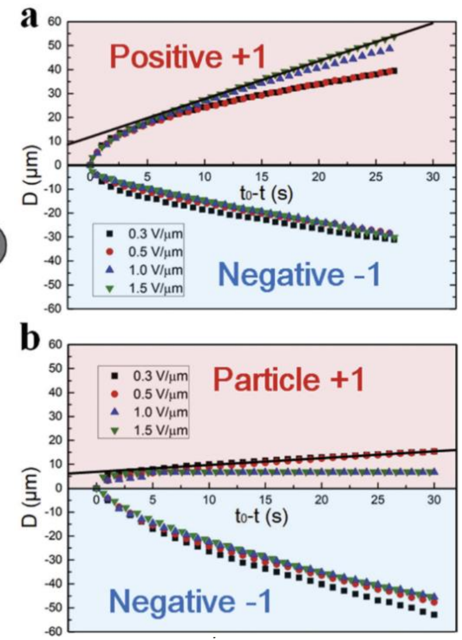
"**Coulomb**-like elastic interaction induced by symmetry breaking in nematic liquid crystal colloids" Scientific Reports

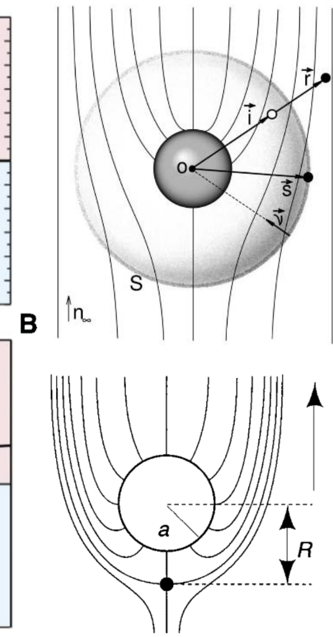
dipole-dipole: "Novel Colloidal Interactions in Anisotropic Fluids" Science

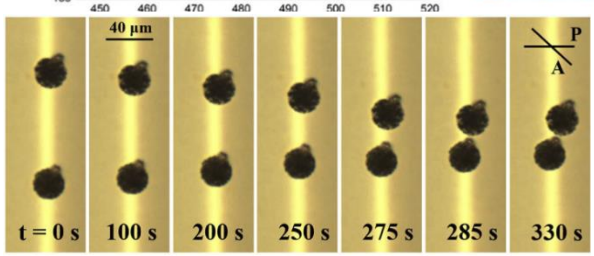
quadrupole-quadrupole: "**Long-range forces** and aggregation of colloid particles in a nematic liquid crystal" PRE

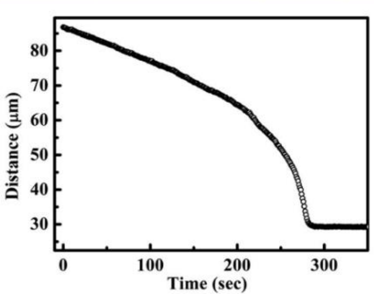
Quantum computers on liquid crystal topological defects? (Science Advances)

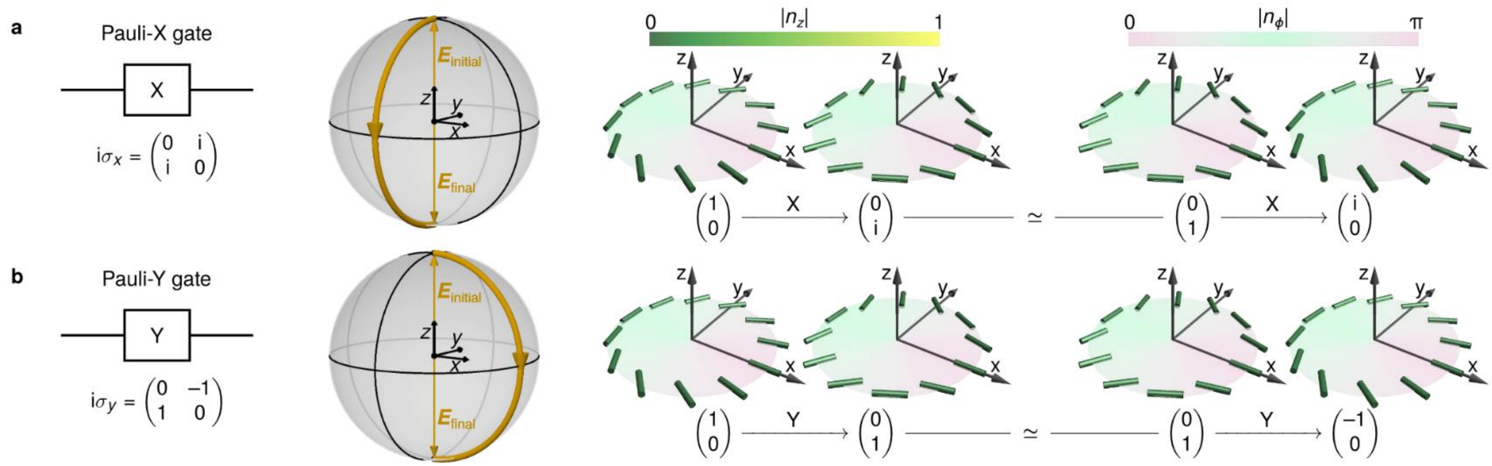




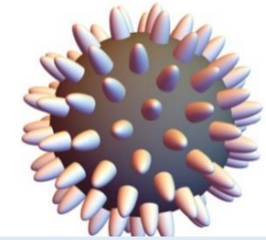


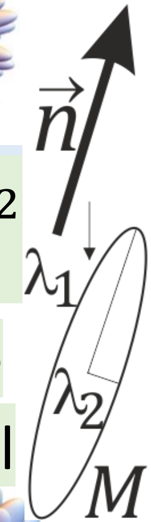






Standard liquid crystal models (review1, review2)





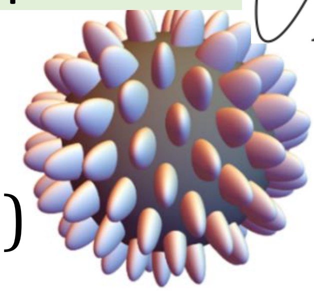
Uniaxial: $\vec{n} \in S^2$ (Higgs potential?), **Oseen-Frank** model (1958) :

$$E[\vec{n}] = \int_\Omega K_1 |\nabla \cdot \vec{n}|^2 + K_2 |\vec{n} \cdot (\nabla \times \vec{n})|^2 + K_3 |\vec{n} \times (\nabla \times \vec{n})|^2 + (|\vec{n}|^2 - 1)^2$$

Biaxial: **Landau-de Gennes** (1974, Nobel Prize in 1991) of **preferred shape**

$(Q \equiv)$ $M = \sum_{i=1}^3 \lambda_i \vec{n}\, \vec{n}^T$ **preferred** $(\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}, \boldsymbol{\lambda_3})$: $V(M)$ Higgs-like potential

$$E[M] = \int_\Omega |\nabla M|^2 + \frac{A}{2}\mathrm{Tr}(M^2) - \frac{B}{3}\mathrm{Tr}(M^3) + \frac{C}{4}\left(\mathrm{Tr}(M^2)\right)^2$$



(or $V(M) = \sum_i (\lambda_i - \Lambda_i)^2$ or $\sum_i \left(\mathrm{Tr}\left(M^i\right) - c_i\right)^2$ tough choice!)

→**particles?**: Higgs-like potential, e.g. as above, with **Coulomb, EM-like:**

$\boldsymbol{\mathcal{L}_{EM} = -F_{\mu\nu}F^{\mu\nu}}$ for $\boldsymbol{F_{\mu\nu} \propto R_{\mu\nu} = \Gamma_\mu \times \Gamma_\nu}$ $\boldsymbol{\Gamma_\nu = (\partial_\nu \vec{n}) \times \vec{n}}$

$\boldsymbol{R_{\mu\nu}}$ **curvature:** Gauss law counts (quantized) topological charge

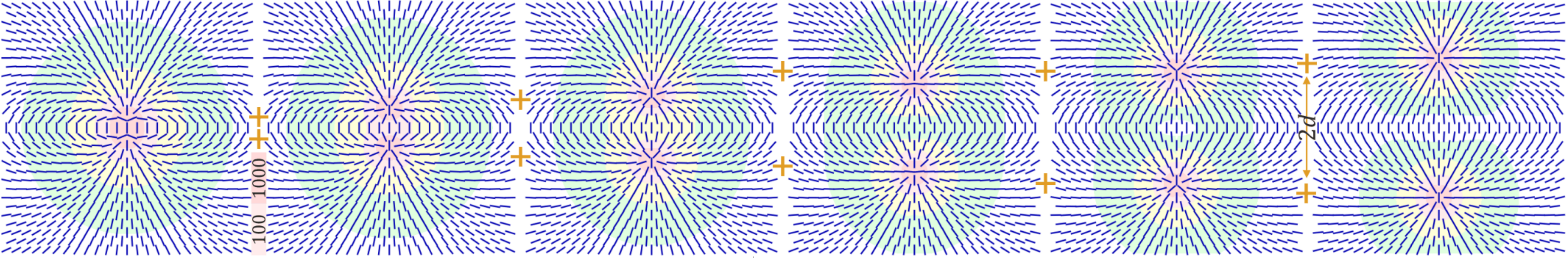
with **kinetic**: $\mathcal{L} \sim \sum_{i=1}^3 ||\partial_i \vec{n} \times \partial_0 \vec{n}||^2 - \sum_{1\le i<j\le 3} ||\partial_i \vec{n} \times \partial_j \vec{n}||^2 - V(\vec{n})$

Biaxial: $\boldsymbol{F_{\mu\nu\alpha\beta} = \left[\partial_\mu M, \partial_\nu M\right]_{\alpha\beta}}$ $\boldsymbol{\mathcal{L} = -F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} - V(M)}$

Lorentz-invariant Lagrangian mechanics leading to **electromagnetism**

plus ~**Klein-Gordon** for twist, **gravito-electromagnetism** for 0th axis

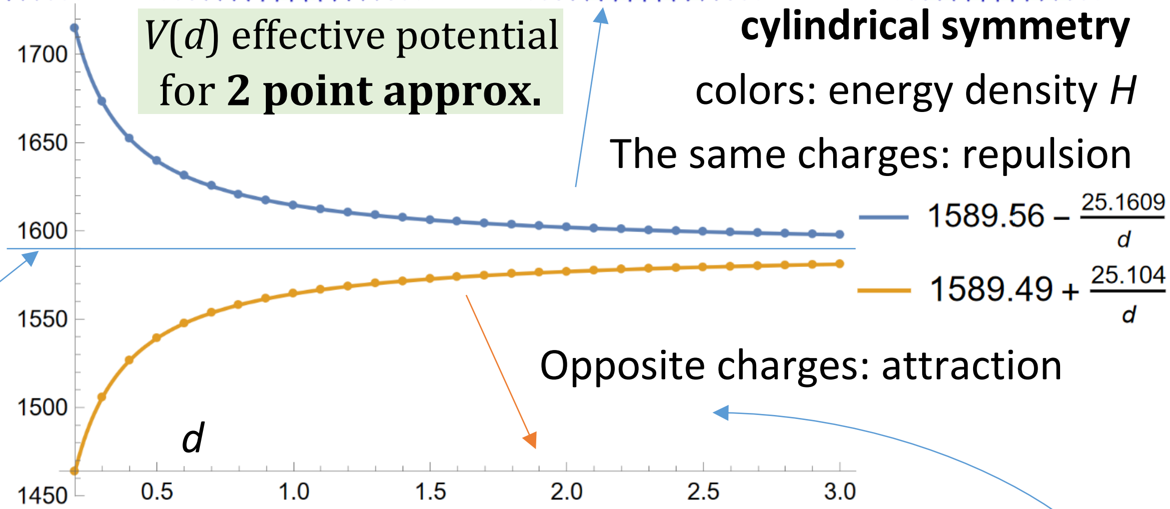
Einstein-Hilbert : $\mathcal{L} \sim R\sqrt{-g}$ spacetime curvature, here field curvature … Newton?



[Field energy: ~Coulomb potent.]

$$E \approx \frac{m_0}{\sqrt{1-v_1^2}} + \frac{m_0}{\sqrt{1-v_2^2}} + V(d)$$

cutoff ϵ around singularities:
to be **regularized** to rest masses
Lorentz inv.: SRT scaling, magnetism

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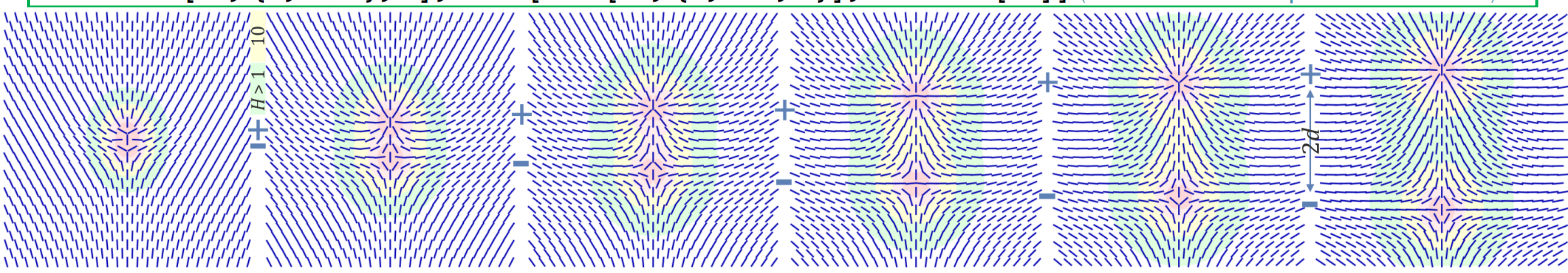
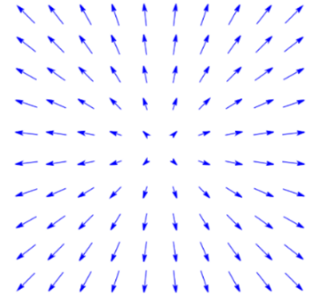
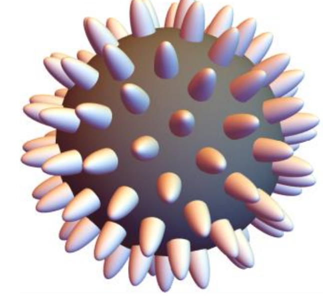
cos = 1 + (z - d) / Sqrt[(z - d)^2 + r^2] - (z + d) / Sqrt[(z + d)^2 + r^2];  (*Manfried Faber dipole ansatz*)
n = {Sqrt[1 - cos^2] x / r, Sqrt[1 - cos^2] y / r, cos} /. r → Sqrt[x^2 + y^2];  (* cylindrical symmetry *)
M = KroneckerProduct[n, n]; dM = {D[M, x], D[M, y], D[M, z]};  (*vector n → matrix M field*)
H = Simplify[Sum[Total[(dM[[i]].dM[[j]] - dM[[j]].dM[[i]])^2, 2], {i, 2}, {j, i + 1, 3}]]; (*Hamilitonian*)
Es = Table[{d, NIntegrate[4 Pi * x (H /. {y → 0}) * Boole[x^2 + (z - d)^2 > 0.001],  (* 0.001  cutoff *)
      {x, 0, Infinity}, {z, 0, Infinity}]}, {d, 0.1, 3, 0.1}]; (*integrate H: total field energy*)
ft = Fit[Es, {1, 1 / d}, d]; Show[Plot[ft, {d, 0.2, 3}], ListPlot[Es]] (*fit Coulomb potentential*)
```



Regularization to finite energy e.g. for "hedgehog":





Asymptotically (vacuum) $||\vec{n}|| \approx 1$, but $\vec{n}(0) = 0$

thanks to **Higgs**-like potential, e.g. $V(\vec{n}) = (\|\vec{n}\|^2 - 1)^2$

: $\vec{\Gamma}_i = (\partial_i \vec{n}) \times \vec{n} \qquad = \frac{1}{r} \quad ||\vec{n}||^2 \to \frac{1}{r}$ $\qquad \Gamma \propto r^{-1} \qquad R \propto r^{-2}$

$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu \quad \to \frac{1}{r^2}$ electric field in 3D

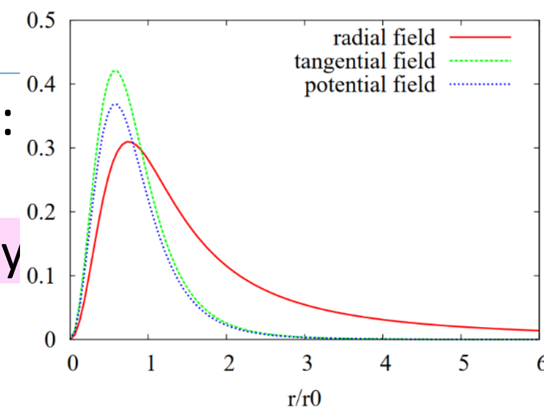
$$^*\vec{F}_{\mu\nu} = \frac{-e_0}{4\pi\epsilon_0 c}\vec{R}_{\mu\nu} \sim \begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

$\mathcal{L}_{EM} = -\frac{\alpha\hbar c}{16\pi}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} \quad \to$ electromagnetism in **vacuum**

Gauss law counting topological charge: $Q_{el}(\mathcal{S}) = \frac{e_0}{4\pi}\oint_{\mathcal{S}(u,v)} du\, dv\ (\partial_u \vec{n} \times \partial_v \vec{n}) \cdot \vec{n}$

Jacobian of closed surface $\to S^2$: $\det[\vec{n}, \vec{n}_\mu, \vec{n}_\nu] = \vec{n} \cdot (\vec{n}_\mu \times \vec{n}_\nu)$

No parton structure (!) for **electron**, Faber:



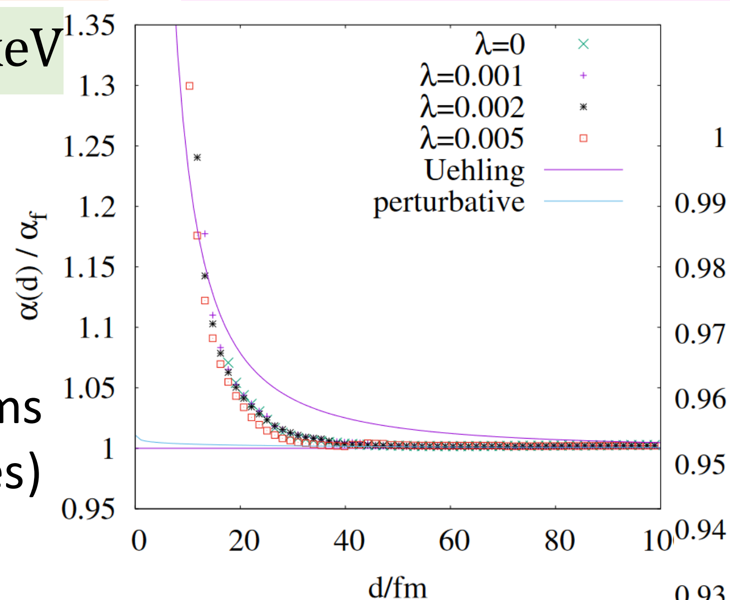
only field deformation not to exceed 511keVs, ??????

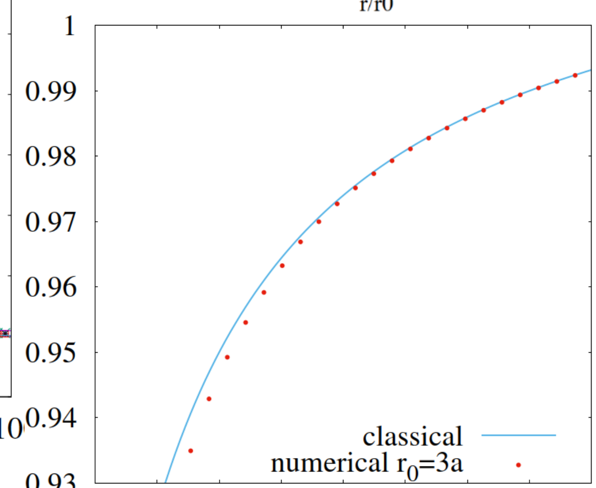
"infinity subtraction from **renormalization**" – subtracted energy density

$$\int_{\sim 1.4\text{fm}}^{\infty} \frac{1}{2}|E|^2 4\pi r^2 dr = 511\text{keV}$$

Experimental effects of finite size?

Coulomb deformation, **Faber**:





Running coupling:

$\alpha \approx \frac{1}{137} \to \approx \frac{1}{127}$ in 90 GeVs

To hide finite size in Feynman diagrams

(+ renormalization to remove infinities)

Experimental boundaries for size of electron?

Dehmelt 1988, Penning trap (Nobel in 1989): $R < 10^{-22} m$

extrapolating from g-factor: "(…) electron as formed by very tightly binding together **three smaller and much heavier new fermions** [Brodsky, Drell, 1980] (…)"

Neutron (udd): $g \approx -3.8 \qquad < r_n^2 > \approx -0.1\ \mathrm{fm}^2$

Classically: $g = \frac{2m}{q}\frac{\mu}{L} = \frac{2m}{q}\frac{\int A dI}{\omega I} = \frac{m}{q}\frac{\int \rho_q(r)\, r^2 dr}{\int \rho_m(r)\, r^2\, dr}$

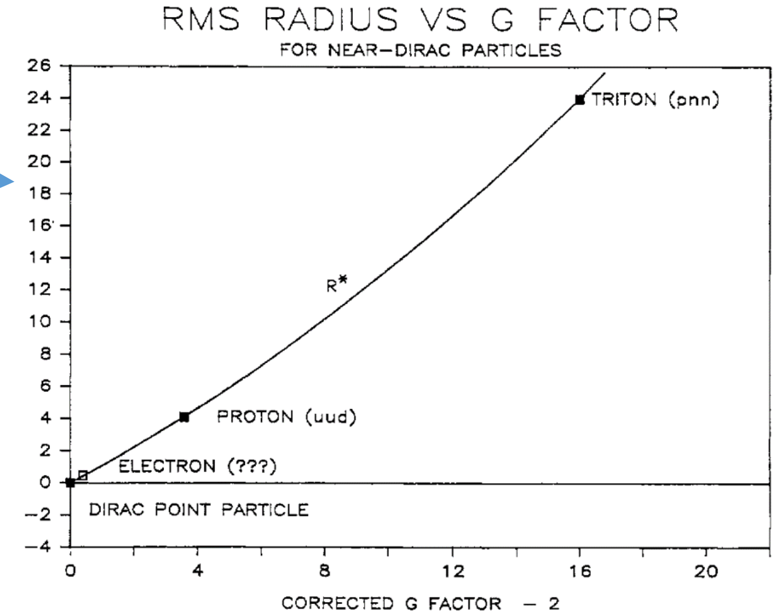


Fig. 8. Normalized RMS radius $R^* = R/\lambda_C$ vs. corrected g-factor minus 2 for near-Dirac particles [3]. A parabola has been fitted to the data points. Recent theories conjecture that the electron, similar to proton and triton, is composed of three smaller fermions. The data point at the origin represents a Dirac point particle of finite arbitrary charge and mass.

Electron-positron cross-section:

Which energy should we use??? (Lorentz contraction!),

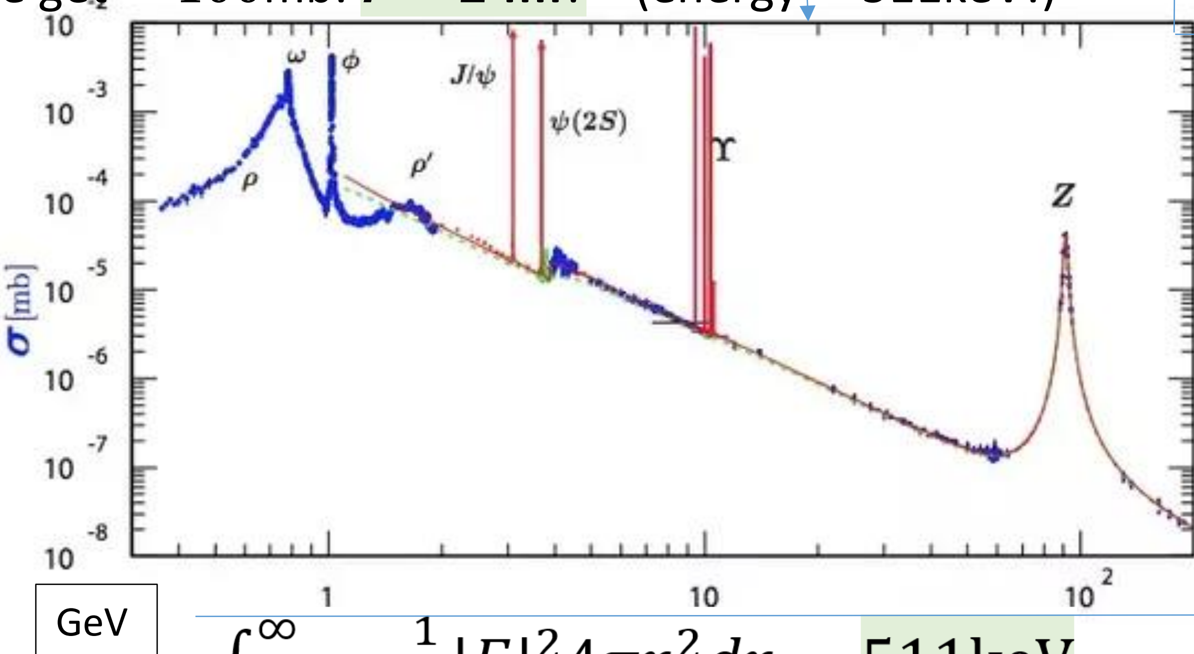
~**line** in log-log, $\approx 100\mathrm{nb}$ for 1GeV $\qquad \gamma \approx 1000$

Extrapolating to resting: $\gamma = 1 \qquad \sigma \propto \gamma^{-2}$

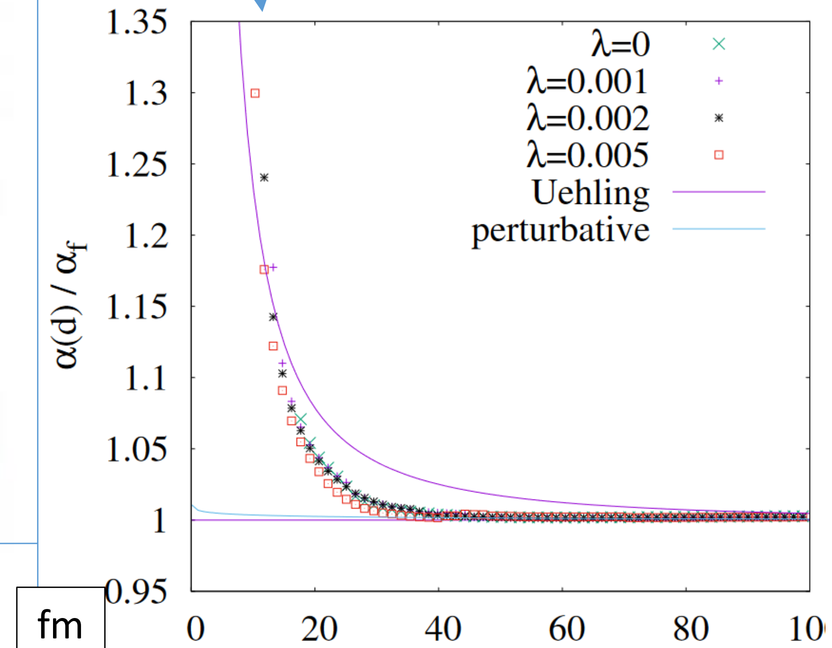
we get $\approx 100\mathrm{mb}$: $\boldsymbol{r \approx 2}$ **fm**? (energy < 511keV!)

Running coupling – deformation of α

(Coulomb) **proper in Faber's model**



Coulomb parameter of dipole



$$\int_{\sim 1.4\mathrm{fm}}^{\infty} \frac{1}{2}|E|^2 4\pi r^2 dr = 511\mathrm{keV}$$

What about quantum phenomena for (topological) solitons? E.g. fluxons:

Experimental demonstration of Aharonov-Casher **interference** in a Josephson junction circuit, PRB 2012 – for **fluxons**, Aharonov-Casher: for magnetic dipole in electric field

Tunneling and resonant tunneling of **fluxons** in a long Josephson junction, PRB 1997

Aharonov-Bohm type forces between magnetic **fluxons**, PRA 1997 **short-range**

Hydrodynamical classical wave-particle duality object "walking droplets":

Single-Particle **Diffraction** and **Interference** at a Macroscopic Scale, **PRL** 2006

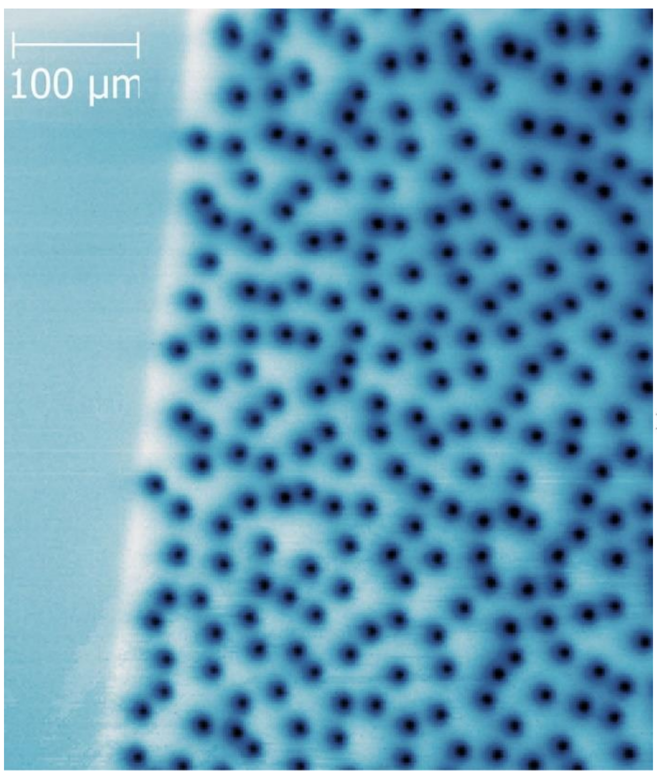
Unpredictable **Tunneling** of a Classical Wave-Particle Association, **PRL** 2009

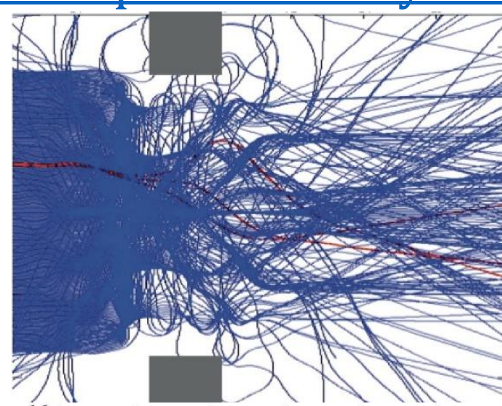
Path-memory induced quantization of classical orbits, PNAS 2010

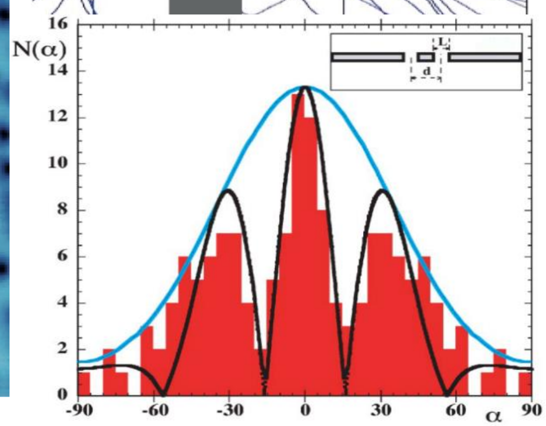
Level Splitting at Macroscopic Scale, PRL 2012 – **Zeeman splitting**

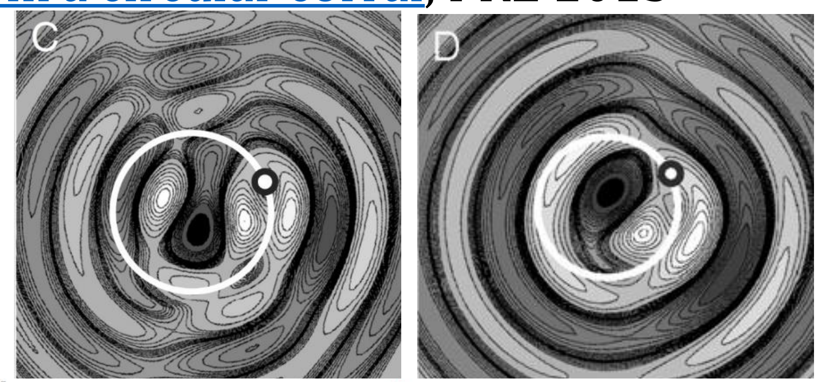
Self-organization into **quantized eigenstates** of a classical wave-driven particle, **Nature** 2014

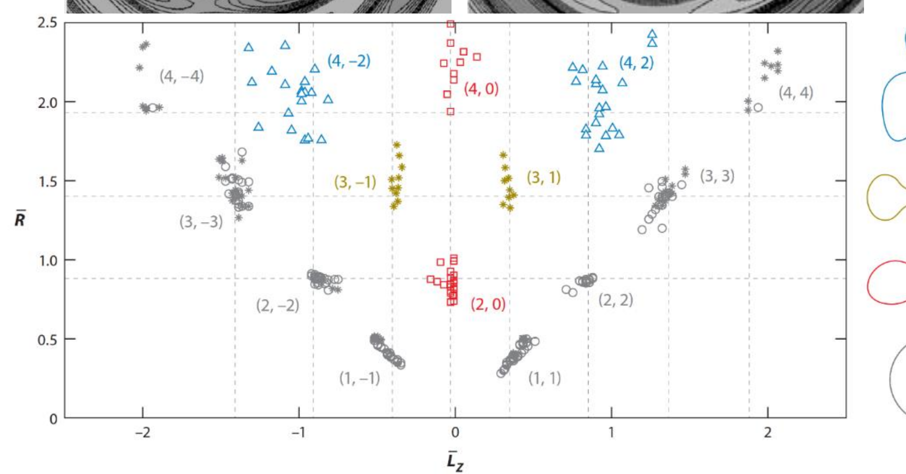
Wavelike statistics from pilot-wave dynamics in a circular corral, PRE 2013



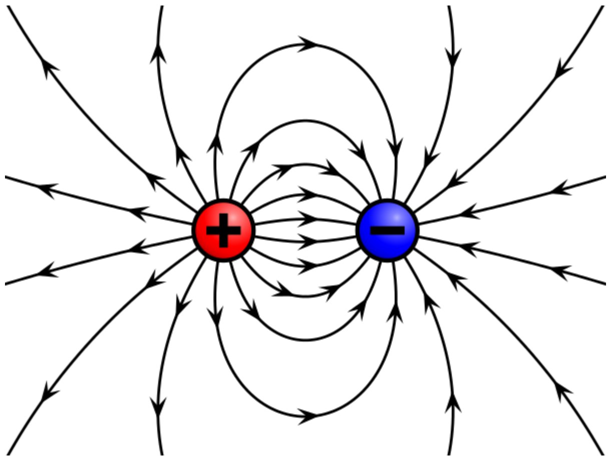


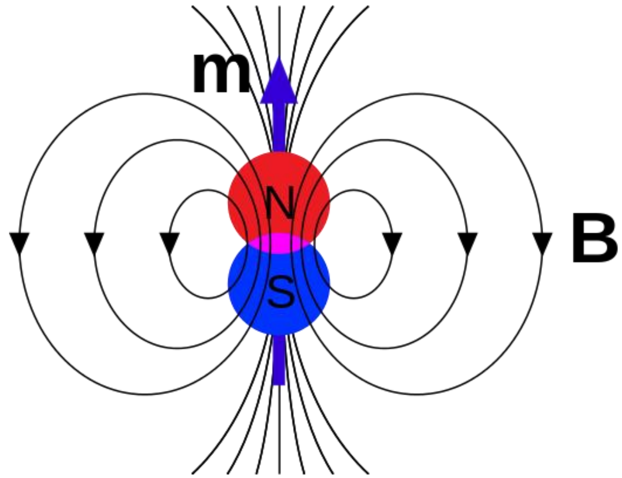


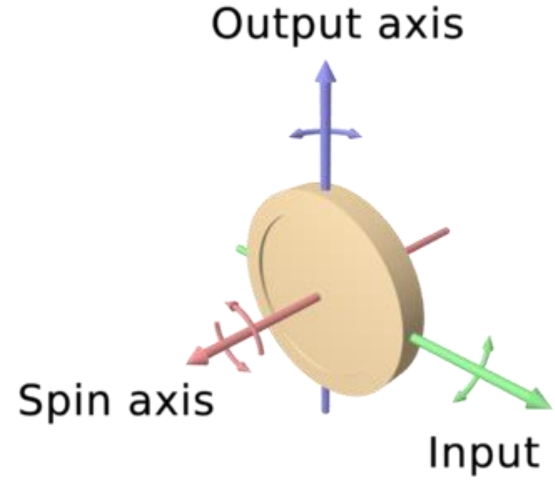


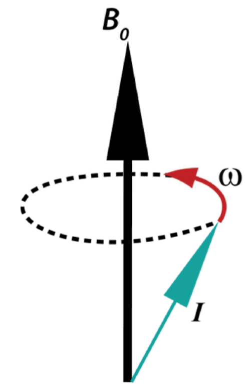


Electron – (at least) a complex configuration of electromagnetic field … Larmor



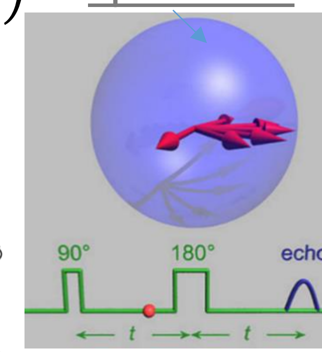






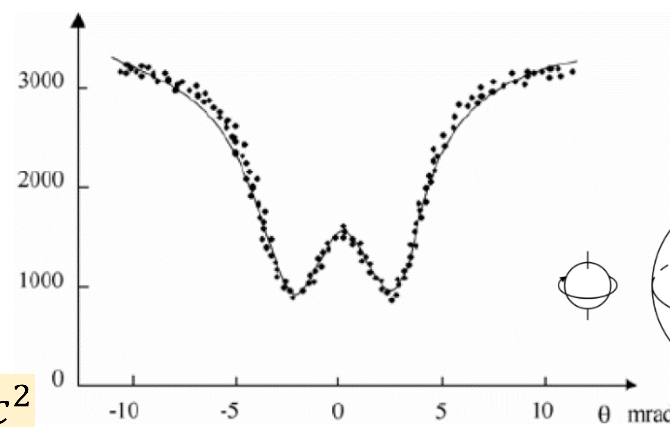
Electric charge $\left(E \propto \frac{1}{r^2}\right)$ + magnetic dipole $\left(B \propto \frac{1}{r^3}\right.$, magnets$\left.\right)$ + "gyroscope" $\left(L = \frac{\hbar}{2}\right)$ spin echo

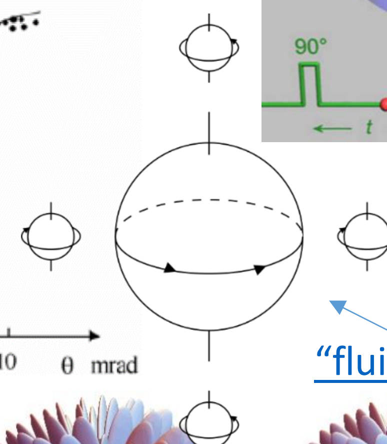
+ $\approx 10^{21} Hz$ zitterbewegung (observ.)/de Broglie's clock $(E = mc^2 = \hbar\omega)$:



some internal periodic process

Along <110> axis of silicon crystal atomic spacing correspond to 'internal clock' period for $E \approx 81 MeV$ electrons – observed resonance:



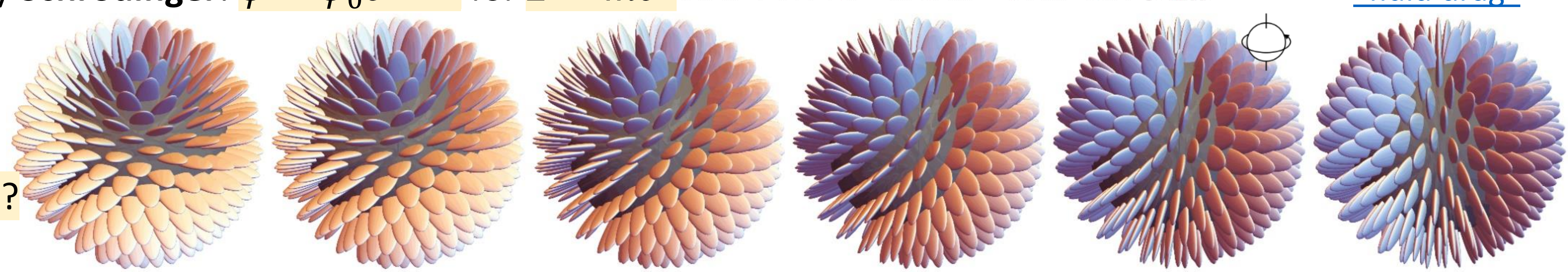


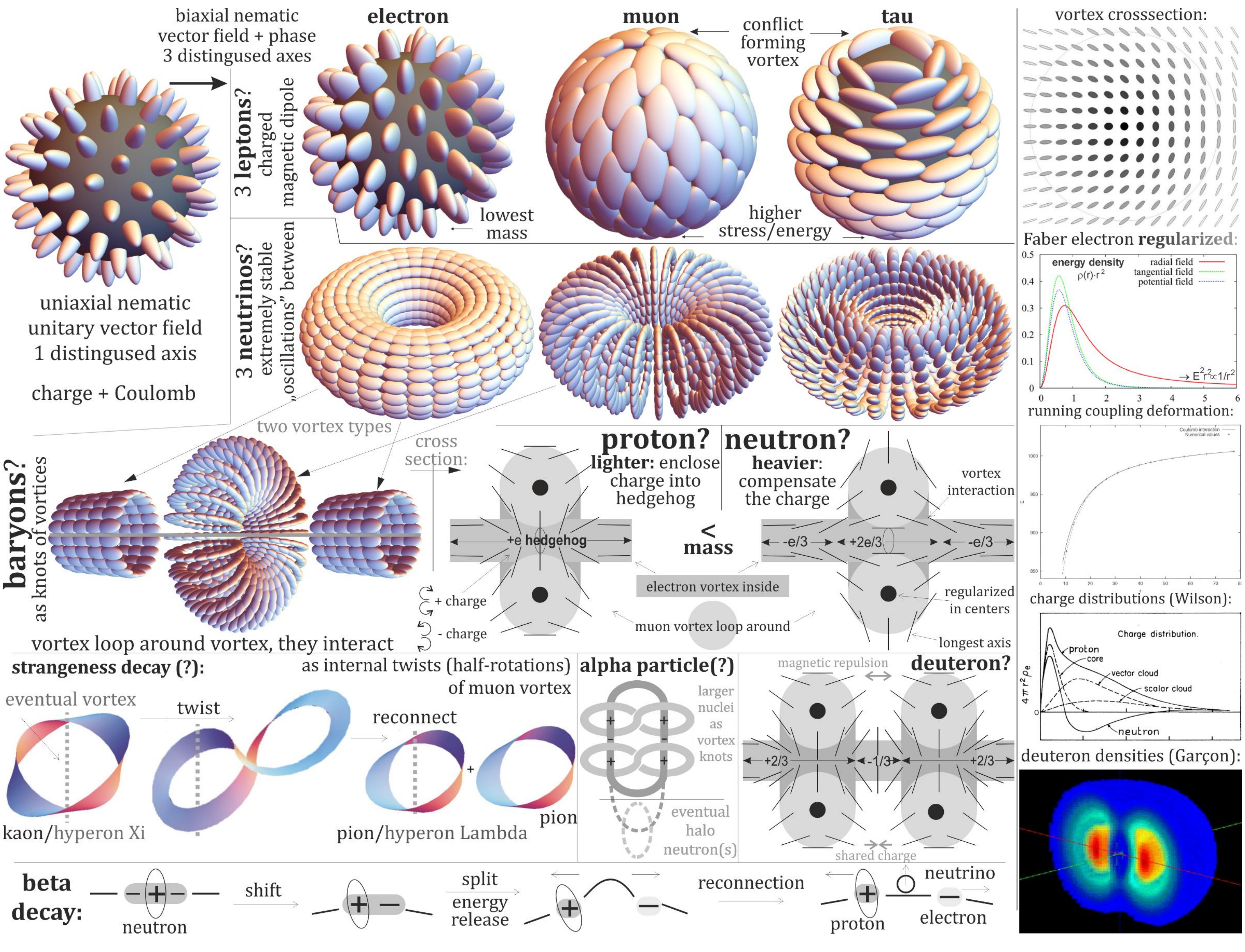
P. Catillon, N. Cue, M.J. Gaillard, R. Genre, M. Gouanère, R.G. Kirsch, J.-C. Poizat, J. Remillieux, L. Roussel, M. Spighel, *A Search for the de Broglie Particle Internal Clock by Means of Electron Channeling*, Found Phys (2008) 38: 659–664

"fluid drag"

stationary **Schrödinger**: $\psi = \psi_0 e^{iEt/\hbar}$ for $E = mc^2$

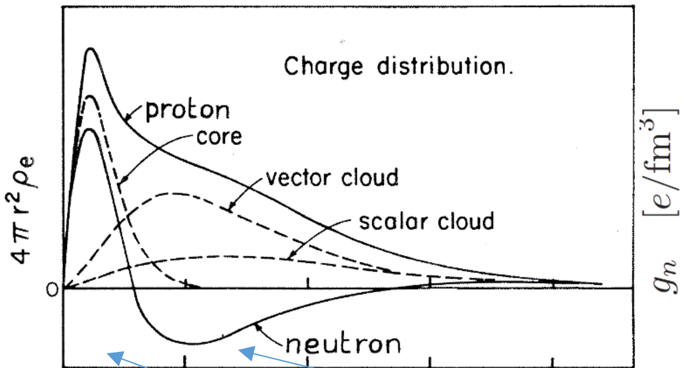
twist as phase ψ evolution?

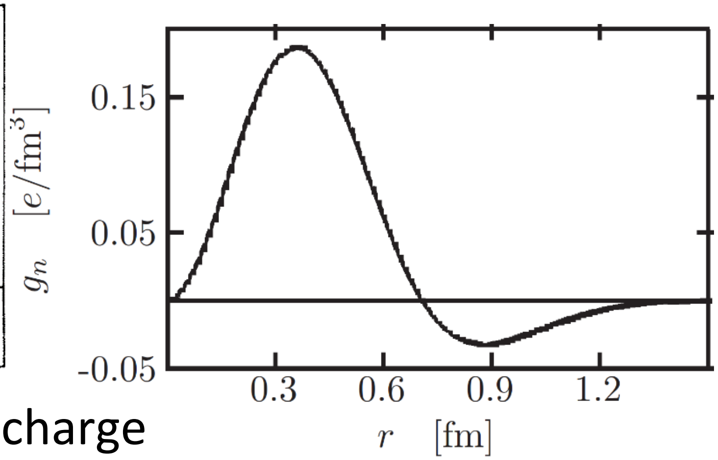


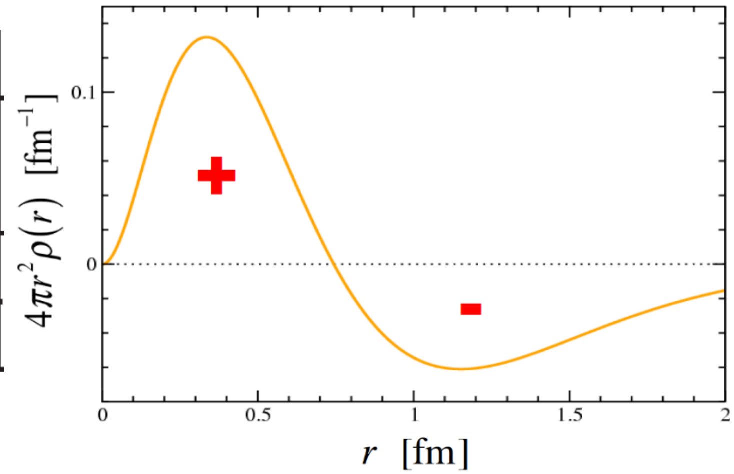


biaxial nematic
vector field + phase
3 distingused axes
electron
muon
conflict
forming
vortex
tau
vortex crosssection:
3 leptons?
charged
magnetic dipole
lowest
mass
higher
stress/energy
Faber electron regularized:
energy density
ρ(r)·r²
radial field
tangential field
potential field
→ E²r²∝1/r²
uniaxial nematic
unitary vector field
1 distingused axis
charge + Coulomb
3 neutrinos?
extremely stable
„oscillations" between
running coupling deformation:
two vortex types
baryons?
as knots of vortices
cross
section:
proton?
lighter: enclose
charge into
hedgehog
neutron?
heavier:
compensate
the charge
vortex
interaction
+e hedgehog
<
mass
-e/3
+2e/3
-e/3
electron vortex inside
muon vortex loop around
+ charge
- charge
regularized
in centers
longest axis
charge distributions (Wilson):
Charge distribution.
proton
core
vector cloud
scalar cloud
neutron
vortex loop around vortex, they interact
strangeness decay (?):
as internal twists (half-rotations)
of muon vortex
eventual vortex
twist
reconnect
kaon/hyperon Xi
pion/hyperon Lambda
pion
alpha particle(?)
larger
nuclei
as
vortex
knots
eventual
halo
neutron(s)
magnetic repulsion
deuteron?
+2/3
-1/3
+2/3
shared charge
deuteron densities (Garçon):
beta
decay:
neutron
shift
split
energy
release
reconnection
neutrino
proton
electron

Baryons: Wilson 1962 Acta. Phys. Pol. 1999: Greene 2015







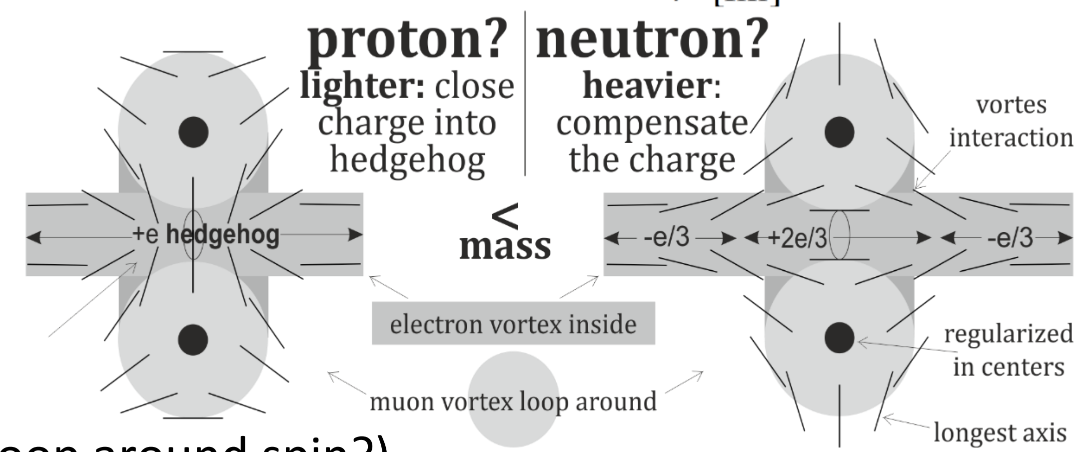
Neutron: "+" core, "-" shell charge

Deuteron: **large electric quadrupole moment**

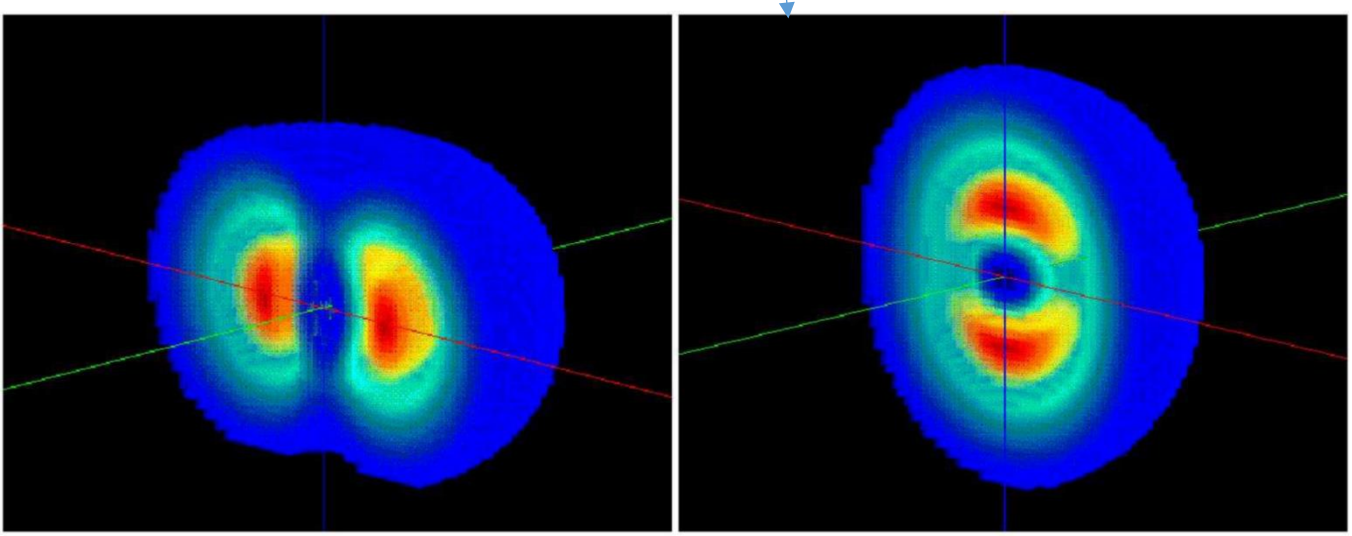
like "**+ - +**" (how to get it for p+n ???)

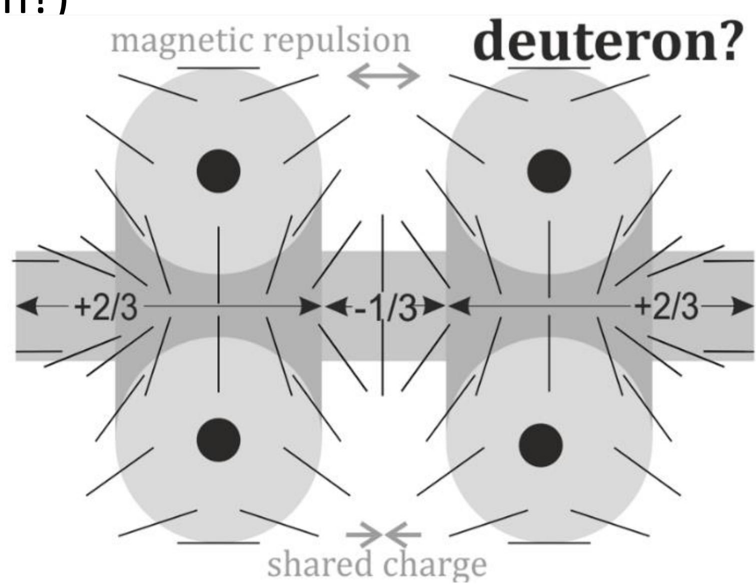
$\mu_d \approx \mu_p + \mu_n$ – **aligned spins?**

0.857 vs 0.879 μ_N magnetic dipole moments



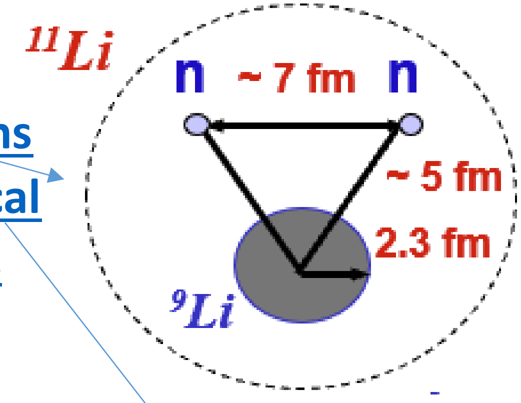
The deuteron: structure and form factors, Advances in Nuclear Physics, 2001 (energy in loop around spin?)



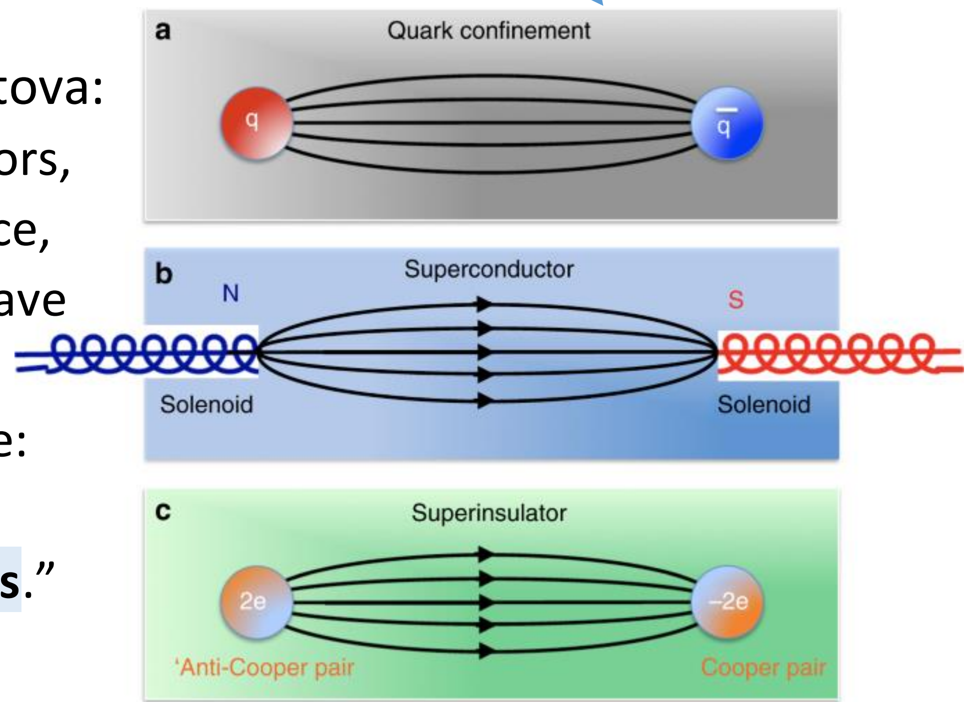


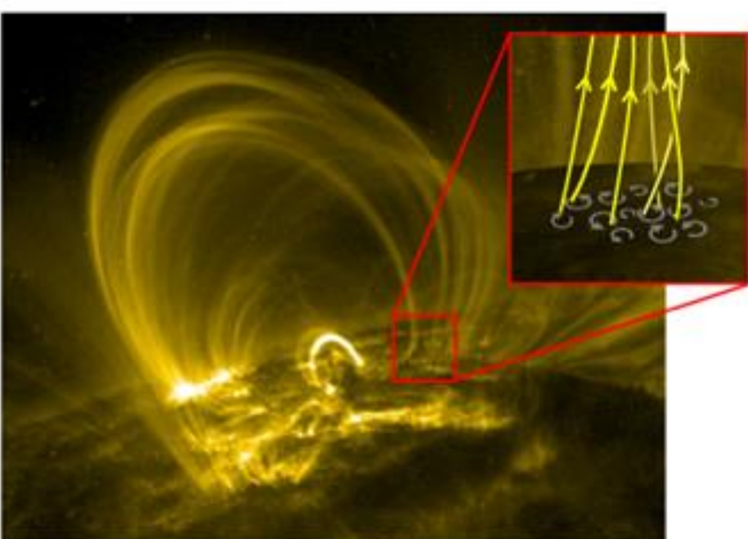
Required: **"fluxons" in vacuum** – magnetic field lines with energy density:

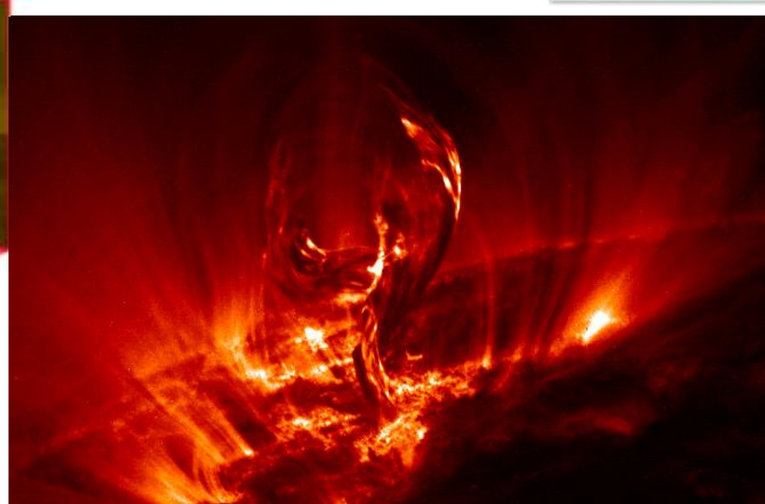
- **Holding nucleus together against Coulomb repulsion (?), halo neutrons**
- **popular quark string model, Nature article suggesting being topological**
- **coronal** heating problem (surface 6000K, corona 10^7K), reconnections
- **electrons in anti-parallel alignment –** in **100-1000nm for Cooper pairs**
- Brawley et. al., "Electron-like scattering of positronium", Science 2010

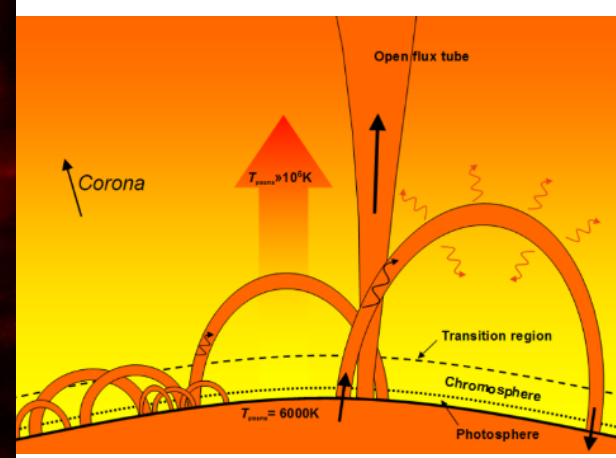


"Physics of **Magnetic Flux Tubes**" book by Ryutova: "Vortices in superfluid Helium and superconductors, magnetic flux tubes in solar atmosphere and space, filamentation process in biology and chemistry have probably a common ground, which is to be yet established. One conclusion can be made for sure: **formation of filamentary structures in nature is energetically favorable and fundamental process**."









Intermediate step: transform unitary "**director**" field → **rotation** matrix field O

Affine **connection**: $O \to O(I + \epsilon\Gamma_\mu)$ for $\boldsymbol{\Gamma_\mu = O^T O_\mu} = O^T \partial_\mu O$ anti-symmetric

Denote its coordinates with 2 vectors: (boosts +)

EM: $\vec{\Gamma}_\mu := (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$

GEM: $\vec{\Gamma}^g_\mu := (\Gamma_{\mu,01}, \Gamma_{\mu,02}, \Gamma_{\mu,03})$ tiny boosts of 0th axis

$$\Gamma_\mu = O^T O_\mu = \begin{pmatrix} 0 & \vec{\Gamma}^g_{\mu,1} & \vec{\Gamma}^g_{\mu,2} & \vec{\Gamma}^g_{\mu,3} \\ -\vec{\Gamma}^g_{\mu,1} & 0 & -\vec{\Gamma}_{\mu,3} & \vec{\Gamma}_{\mu,2} \\ -\vec{\Gamma}^g_{\mu,2} & \vec{\Gamma}_{\mu,3} & 0 & -\vec{\Gamma}_{\mu,1} \\ -\vec{\Gamma}^g_{\mu,3} & -\vec{\Gamma}_{\mu,2} & \vec{\Gamma}_{\mu,1} & 0 \end{pmatrix}$$

3D 1st axis **curvature**: $\vec{R}_{\mu\nu} \equiv \vec{R}^{ee}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ - **electromagnetism** we will focus on

4D 0th axis curvature: $\vec{R}^{gg}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}^g_\nu$ - **GEM** approximation of general relativity

4D EM-GEM interaction: $\vec{R}^{eg}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}^g_\nu$, $\vec{R}^{ge}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}_\nu = -\vec{R}^{eg}_{\nu\mu}$

Fermat principle: Sun's light bend

e.g. **GEM slowing EM**: time dilation

$F_{\mu\nu} = [\Gamma_\mu, \Gamma_\nu]$ in place of curvature for EM with topologically quantized charge?

$$[\Gamma_\mu, \Gamma_\nu] = \begin{pmatrix} 0 & -\vec{R}^{eg}_{\mu\nu} + \vec{R}^{eg}_{\nu\mu} \\ \vec{R}^{eg}_{\mu\nu} - \vec{R}^{eg}_{\nu\mu} & \vec{R}^{ee}_{\mu\nu} + \vec{R}^{gg}_{\mu\nu} \end{pmatrix} := \begin{pmatrix} 0 & -\vec{R}^{eg}_{\mu\nu,1} + \vec{R}^{eg}_{\nu\mu,1} & -\vec{R}^{eg}_{\mu\nu,2} + \vec{R}^{eg}_{\nu\mu,2} & -\vec{R}^{eg}_{\mu\nu,3} + \vec{R}^{eg}_{\nu\mu,3} \\ \vec{R}^{eg}_{\mu\nu,1} - \vec{R}^{eg}_{\nu\mu,1} & 0 & -\vec{R}^{ee}_{\mu\nu,3} - \vec{R}^{gg}_{\mu\nu,3} & \vec{R}^{ee}_{\mu\nu,2} + \vec{R}^{gg}_{\mu\nu,2} \\ \vec{R}^{eg}_{\mu\nu,2} - \vec{R}^{eg}_{\nu\mu,2} & \vec{R}^{ee}_{\mu\nu,3} + \vec{R}^{gg}_{\mu\nu,3} & 0 & -\vec{R}^{ee}_{\mu\nu,1} - \vec{R}^{gg}_{\mu\nu,1} \\ \vec{R}^{eg}_{\mu\nu,3} - \vec{R}^{eg}_{\nu\mu,3} & -\vec{R}^{ee}_{\mu\nu,2} - \vec{R}^{gg}_{\mu\nu,2} & \vec{R}^{ee}_{\mu\nu,1} + \vec{R}^{gg}_{\mu\nu,1} & 0 \end{pmatrix}$$

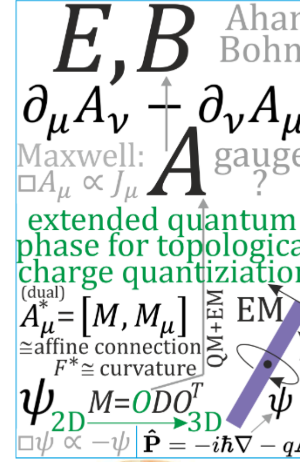
GEM – confirmed by Gravity Probe B approximation of **general relativity:** https://en.wikipedia.org/wiki/Gravitoelectromagnetism:

GEM equations	Maxwell's equations
$\nabla \cdot \mathbf{E}_g = -4\pi G \rho_g$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
$\nabla \cdot \mathbf{B}_g = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_g = -\frac{4\pi G}{c^2}\mathbf{J}_g + \frac{1}{c^2}\frac{\partial \mathbf{E}_g}{\partial t}$	$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

$[\Gamma_\mu, \Gamma_\nu]$ vanishes in flat spacetime

GEM causes spatial curvature:

$$0 = \partial_\mu \partial_\nu O - \partial_\nu \partial_\mu O = \partial_\mu(O\Gamma_\nu) - \partial_\nu(O\Gamma_\mu) = O[\Gamma_\mu, \Gamma_\nu]$$

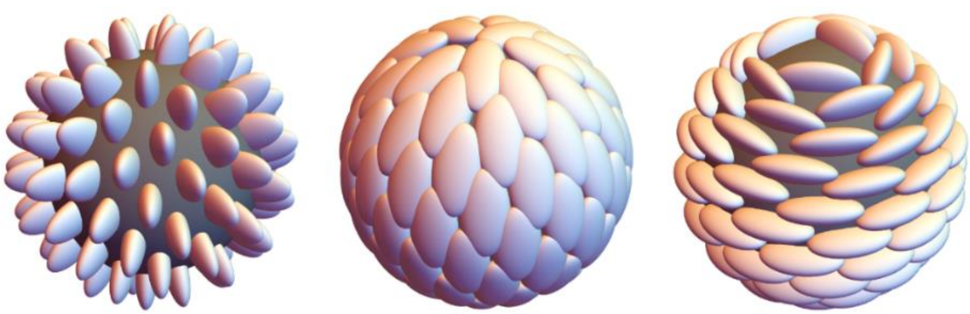


Let O matrix **rotate some objects**, e.g. **ellipsoid** in biaxial nematic

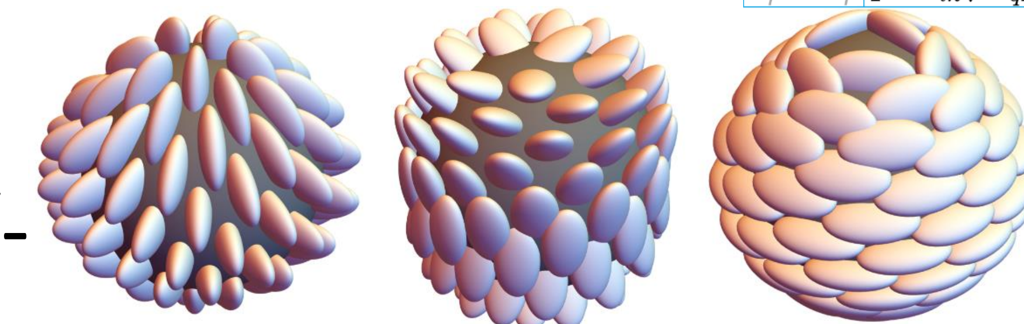
$M(x) \equiv \boldsymbol{M = ODO^T}$ field for $OO^T = I$ rotation

$D = \mathrm{diag}(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ with **Higgs-like e.g.** $\boldsymbol{V(M) = \sum_i (\lambda_i - \Lambda_i)^2}$

For $\Lambda_0 > \Lambda_1 > \Lambda_2 > \Lambda_3$ fixed preferred shape – **vacuum state**



+/-



(3D) F tensor containing curvature (so Gauss law gives topological charge)

Let us postulate: $\boldsymbol{F_{\mu\nu} = [M_\mu, M_\nu]} = \partial_\mu M\, \partial_\nu M - \partial_\nu M\, \partial_\mu M$, getting **vacuum**:

Curvatures:

Gauss law ∝ topological charge

$\Lambda_1 \gg \Lambda_2 \approx \Lambda_3$

High energy: **EM**

Low energy: **QM**

QED, ψ Lorentz group

$$O^T F_{\mu\nu} O = O^T [M_\mu, M_\nu] O \approx [\Gamma_\mu D - D\Gamma_\mu, \Gamma_\nu D - D\Gamma_\nu] =$$

$$(\Lambda_1 - \Lambda_2)(\Lambda_3 - \Lambda_1)(\Lambda_2 - \Lambda_3) \begin{pmatrix} 0 & \frac{-\vec{R}_{\mu\nu,3}}{\Lambda_1 - \Lambda_2} & \frac{\vec{R}_{\mu\nu,2}}{\Lambda_3 - \Lambda_1} \\ \frac{\vec{R}_{\mu\nu,3}}{\Lambda_1 - \Lambda_2} & 0 & \frac{-\vec{R}_{\mu\nu,1}}{\Lambda_2 - \Lambda_3} \\ \frac{-\vec{R}_{\mu\nu,2}}{\Lambda_3 - \Lambda_1} & \frac{\vec{R}_{\mu\nu,1}}{\Lambda_2 - \Lambda_3} & 0 \end{pmatrix}$$

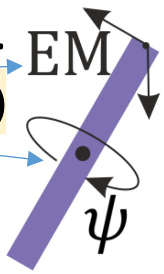
$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

for $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ and $\boldsymbol{\Gamma_\mu = O^T O_\mu}$, $\vec{\Gamma}_\mu := (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$ as previously

Vacuum (long distance) dynamics:

1) EM: **quantized** electric charge
with **Coulomb** in $V(r) \propto r^{-1}$
(+ magnetism from Lorentz invariance)
S^2: **Gauss law counts topological charge**

2) S^1 **quantum phase**: Berry, pilot wave
e.g. for Mach-Zehnder interfer.
Unify EM S^2 + QM S^1 → $SO(3)$
"extended phase"
governed by **wave equation**:
Maxwell $\square A_\mu \propto J_\mu$+~Klein-Gordon $\square\psi$



Momentum operator:

$$\hat{P} = -qA - i\hbar\nabla$$

suggests: A also hides derivative,
describes local rotation

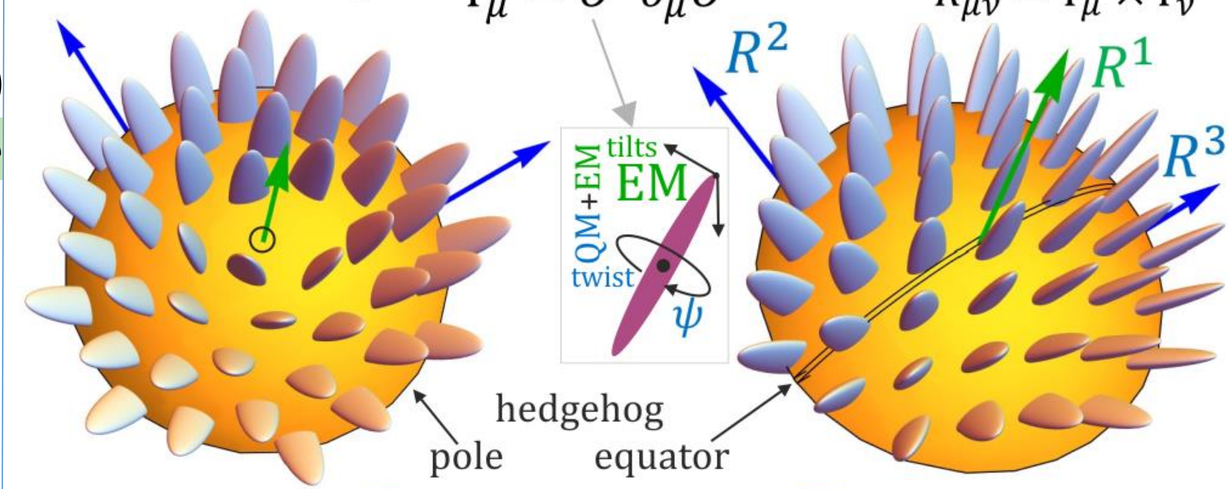
$$2\pi k = \Delta\varphi = \frac{q}{\hbar}\oint_{\partial S} A\cdot dl = \frac{q}{\hbar}\oint_S B\cdot dS$$

fluxon, GL order parameter

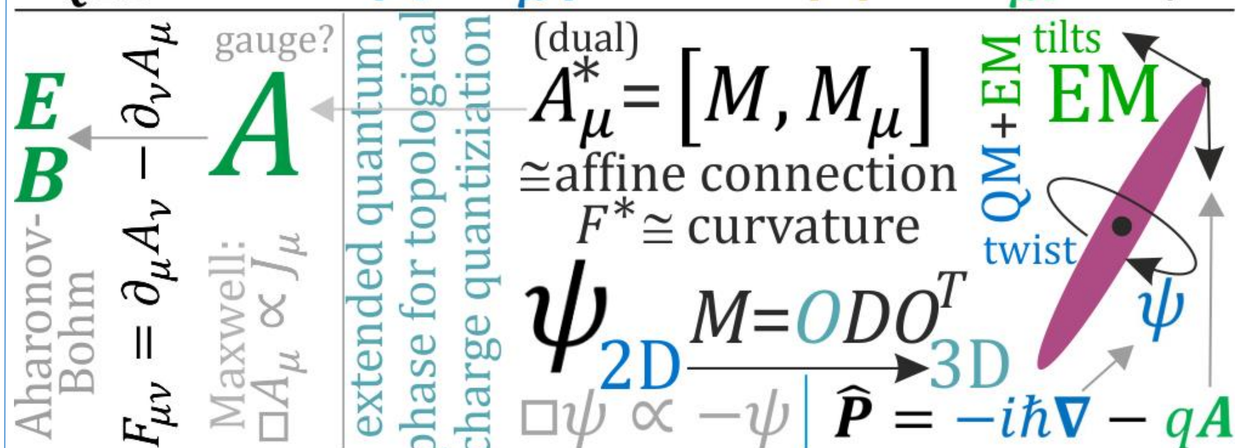
3 × 3 in 3D → 4 × 4 in 4D spacetime:
3) + gravity starting with GEM required
for Newton + B_g for Lorentz invariance

green: **tilt-tilt** of $\vec{n}$ main axis EM high energy **main curvature**
blue: **tilt-twist** QM phase low energy curvatures

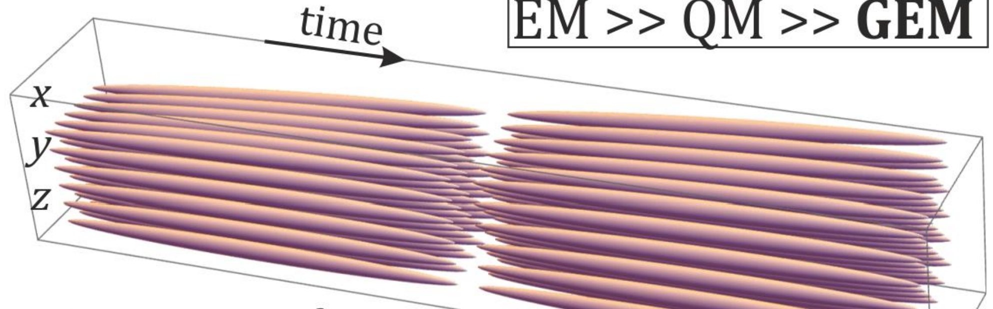
$\Lambda = (1, \delta, 0) \Rightarrow \vec{A}_\mu \approx (\delta^2\Gamma^1_\mu, \Gamma^2_\mu, \Gamma^3_\mu)$, $\vec{F}_{\mu\nu} \approx (R^1_{\mu\nu}, \delta R^2_{\mu\nu}, \delta R^3_{\mu\nu})$
≈fixed far from charge
$\Gamma_\mu = O^T\partial_\mu O$ $\quad \vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$



$$\mathcal{L}_{QED} = -\hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi - mc^2\bar{\psi}\psi - F_{\mu\nu}F^{\mu\nu}/4$$



EM >> QM >> **GEM**



+ GEM approx. of GR
from tiny perturbations of 4th axis

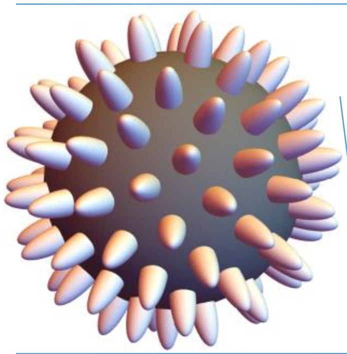
Postulate **Lagrangian** as EM: $\mathcal{L} = \sum_{\mu=1}^{3} \|F_{\mu 0}\|_F^2 - \sum_{1\le\mu<\nu\le 3} \|F_{\mu\nu}\|_F^2 - V$

and **four-potential** $\boldsymbol{A_\mu}$:

$2F_{\mu\nu} = 2[\boldsymbol{M_\mu}, \boldsymbol{M_\nu}] = \partial_\mu A_\nu - \partial_\nu A_\mu$ for $A_\mu = M\, M_\mu - M_\mu\, M \approx$ (**3D** vacuum)

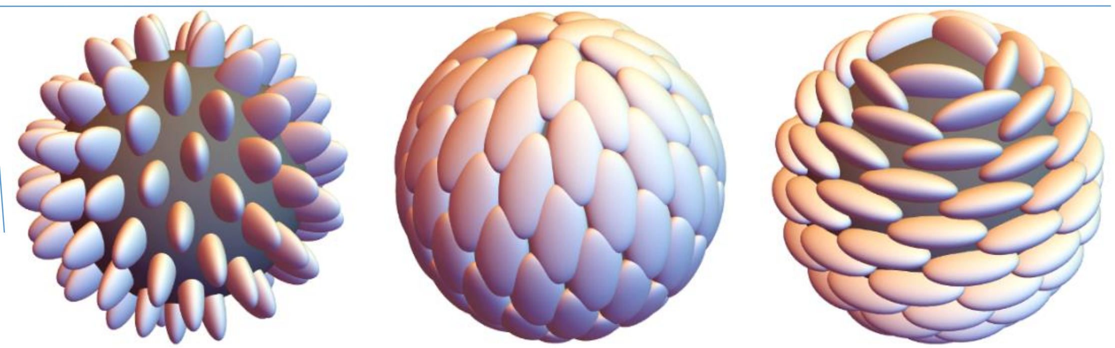
$\Lambda_1 \gg \Lambda_2 \approx \Lambda_3$
High energy: **EM**
Low energy: **QM**
$P = -i\hbar\nabla - qA$

$$\approx O \begin{pmatrix} 0 & \vec{\Gamma}_{\mu,3}(\Lambda_1-\Lambda_2)^2 & -\vec{\Gamma}_{\mu,2}(\Lambda_1-\Lambda_3)^2 \\ -\vec{\Gamma}_{\mu,3}(\Lambda_1-\Lambda_2)^2 & 0 & \vec{\Gamma}_{\mu,1}(\Lambda_2-\Lambda_3)^2 \\ \vec{\Gamma}_{\mu,2}(\Lambda_1-\Lambda_3)^2 & -\vec{\Gamma}_{\mu,1}(\Lambda_2-\Lambda_3)^2 & 0 \end{pmatrix} O^T$$



shape Λ dependence

perturbation $\Lambda_2 > 0$



In **uniaxial nematic case** e.g. simplest $(\Lambda_1 = 1, \Lambda_2 = 0, \Lambda_3 = 0)$: $\boldsymbol{M = \vec{n}\,\vec{n}^T}$

$$A_\mu = [M, M_\mu] = \|\vec{n}\|^2 \begin{pmatrix} 0 & (\vec{n}\times\vec{n}_\mu)_3 & -(\vec{n}\times\vec{n}_\mu)_2 \\ -(\vec{n}\times\vec{n}_\mu)_3 & 0 & (\vec{n}\times\vec{n}_\mu)_1 \\ (\vec{n}\times\vec{n}_\mu)_2 & -(\vec{n}\times\vec{n}_\mu)_3 & 0 \end{pmatrix}$$

leading to EM $F_{\mu\nu}$ curvature as Faber: $\partial_\mu(\vec{n}\times\vec{n}_\nu) - \partial_\nu(\vec{n}\times\vec{n}_\mu) = 2\,\vec{n}_\mu \times \vec{n}_\nu$

General: small perturbation $\Lambda_2 > 0$, **shape Λ dependence** in $\Gamma_\mu \xrightarrow{\Lambda} A_\mu,\ R_{\mu\nu} \xrightarrow{\Lambda} F_{\mu\nu}$

$$\frac{\partial \|F_{\mu\nu}\|_F^2}{\partial(\partial_\alpha A_\beta)} = \frac{1}{2}\frac{\partial \|\partial_\mu A_\nu - \partial_\nu A_\mu\|_F^2}{\partial(\partial_\alpha A_\beta)} = (\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha})F_{\alpha\beta}$$

Euler-Lagrange equation: extended EM with **topological charge quantization**

(integration by parts - last term should vanish as in Lorentz gauge condition):

Maxwell's equations:

F: E, B fields

EM: $V \sim AJ$

\+ Klein-Gordon: $V \sim m^2 A^2$

\+ GEM: $V \sim AJ^g$

J, J^g: four-currents

$$\frac{\partial \mathcal{L}}{\partial A_\alpha} = \frac{d}{dx_0}\frac{\partial \mathcal{L}}{\partial(\partial_0 A_\alpha)} + \sum_{i=1}^{3}\frac{d}{dx_i}\frac{\partial \mathcal{L}}{\partial(\partial_i A_\alpha)}$$

$$\frac{\partial V}{\partial A_\alpha} = \partial_0 F_{0\alpha} - \sum_{i=1}^{3}\partial_i F_{i\alpha} = \Box A_\alpha - \partial_\alpha\left(\partial_0 A_0 - \sum_i \partial_i A_i\right)$$

All 3x3 or 4x4 anti-symmetric matrices, $\Box = \partial_{00} - \partial_{11} - \partial_{22} - \partial_{33}$

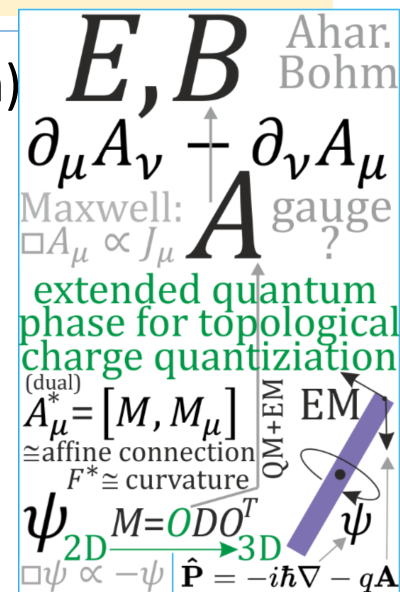
How to choose $V(M)$ or $V(A)$ $\sum_i(\lambda_i - \Lambda_i)^2$? $\sum_\mu\left(||A_\mu||_F^2 - 1\right)^2$? $\det M = 1$?

$A_\mu = [M, \partial_\mu M] \approx \left(\epsilon_{ijk}(\Lambda_i - \Lambda_j)\,\vec{\Gamma}_{\mu,k}\right)_{ij}$ contains velocity – **Higgs $V(A)$ would enforce M clock**

Energy density/Hamiltonian ($||A||_F^2 = \mathrm{Tr}(AA^T)$, last term should vanish)

$$\mathcal{H} = \sum_{\mu=1}^{3}\frac{\partial \mathcal{L}}{\partial(\partial_0 A_\mu)}\partial_0 A_\mu - \mathcal{L} = \sum_{\mu=1}^{3} F_{0\mu}\bullet(2F_{0\mu} + \partial_\mu A_0) - \mathcal{L}$$

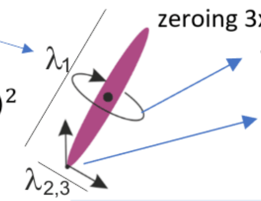
$$\mathcal{H} = \sum_{0\le\mu<\nu\le 3}\|F_{\mu\nu}\|_F^2 + V + \sum_{\mu=1}^{3} F_{0\mu}\bullet\partial_\mu A_0$$



Model: field $M(t,x,y,z) \equiv \boldsymbol{M = ODO^T}$ of real symmetric 3x3 matrices,
$OO^T = I$ describes **local rotation**, $D = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ **shape** of "molecule"
(to be extended to 4x4 tensor field – adding gravitoelectromagnetism)
$(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ is **shape** preferring (Λ_i) shape fixed by model, e.g. $V = \sum_i (\lambda_i - \Lambda_i)^2$

With **Lagrangian**: $\mathcal{L} = \sum_{\mu=1}^{3} ||F_{\mu 0}||_F^2 - \sum_{1 \le \mu < \nu \le 3} ||F_{\mu\nu}||_F^2 - V$

for $F_{\mu\nu} = [\partial_\mu M, \partial_\nu M] = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\qquad A_\mu = [M, \partial_\mu M]$



$(V \approx 0)$ Assume **vacuum case:** $(\lambda_i) \approx (\Lambda_i) = (1, \delta, 0)$ for tiny δ related with Planck $\hbar$
zeroing 3x3 M **variations: 3 rotations**, and 3 axis elongations – zeroing the lowest δ:

twist: ~Klein-Gordon $\quad X^2 \cdot \Gamma^3 = X^3 \cdot \Gamma^2$

tilt1: $X^1 \cdot \Gamma^3 = 0$

tilt2: $X^1 \cdot \Gamma^2 = 0$

all 3 **elongations**: $|B^1| = |E^1|$ (dominated in 4D)

for $X^i := \left(-\nabla \cdot B^i,\ \overrightarrow{\partial_0 B^i + \nabla \times E^i}\right)$ as in **Maxwell equations**

$\Gamma_\mu = O^T O_\mu$

$\vec{\Gamma}_\mu = (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$

$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$

$\vec{B}_i = \vec{R}_{0i}$

$\vec{E}_{1,2,3} = (\vec{R}_{32}, \vec{R}_{13}, \vec{R}_{23})$

$$\begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

Variating Lagrangian (vacuum $V = 0$) leads to evolution equation:

$$\boxed{0 = \sum_{\mu\nu} d_{\mu\nu} \mathrm{Tr}\left(\overline{F}_{\mu\nu}\left([\Gamma_\nu, [\overline{M}_\mu, G']] - [\Gamma_\mu, [\overline{M}_\nu, G']]\right) + \overline{F}_{\mu\nu,\nu}[\overline{M}_\mu, G'] - \overline{F}_{\mu\nu,\mu}[\overline{M}_\nu, G']\right)}$$

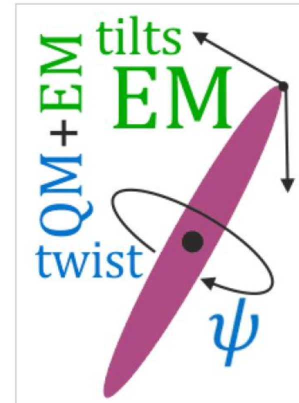
Which for **3 rotation generators** give
~Maxwell for 2 tilts (high energy) as:
~Klein-Gordon for twist (low energy) as:
for hedgehog:

$$X^i := \left(-\nabla \cdot B^i, \overrightarrow{\partial_0 B^i + \nabla \times E^i}\right)$$

$$X^1 \cdot \Gamma^3 = 0 = X^1 \cdot \Gamma^2$$

$$X^2 \cdot \Gamma^3 = X^3 \cdot \Gamma^2$$

$$2\partial_{tt}\psi = \left(\left(\nabla - \vec{A}^{hedg}\right)^2 + \left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|} \cdot \nabla\right)^2\right)\psi$$




```
d = DiagonalMatrix[{1, δ, 0}];                       (* ellipsoid shape, δ ~ ℏ *)
Gx = {{0, 0, 0}, {0, 0, -1}, {0, 1, 0}};   (* twist generator *)
Gy = {{0, 0, 1}, {0, 0, 0}, {-1, 0, 0}};   (* tilt1 generator *)
Gz = {{0, -1, 0}, {1, 0, 0}, {0, 0, 0}};   (* tilt2 generator *)
Ga = {{1, 0, 0}, {0, 0, 0}, {0, 0, 0}}; (* 3 elongation generators *)
Gb = {{0, 0, 0}, {0, 1, 0}, {0, 0, 0}};
Gc = {{0, 0, 0}, {0, 0, 0}, {0, 0, 1}}; (* Gpt of G'=Gd + dGᵀ *)
Gpt = Join[Table[G.d + d.Transpose[G], {G, {Gx, Gy, Gz}}], {Ga, Gb, Gc}];
com[A_, B_] := A.B - B.A;                          (* commutator *)
cd = {{3, 2}, {1, 3}, {2, 1}};                            (* ∈ijk*)
vect[m_] := Table[m⟦cd⟦i, 1⟧, cd⟦i, 2⟧⟧, {i, 3}]; (* → rotation vector *)
Γμ = {{0, -Γμ^3, Γμ^2}, {Γμ^3, 0, -Γμ^1}, {-Γμ^2, Γμ^1, 0}};    (* its matrix form *)
sub = Table[Cross[{Γμ^1, Γμ^2, Γμ^3}, {Γν^1, Γν^2, Γν^3}]⟦i⟧ == R^i_{μ,ν}, {i, 3}];
Mμ = com[Γμ, d]; Γν = Γμ /. μ → ν; Mν = com[Γν, d]; Fμν = Simplify[com[Mμ, Mν], sub];
vrip = Table[Simplify[Tr[Fμν.(com[Γν, com[Mμ, Gp]] - com[Γμ, com[Mν, Gp]]) +
        (Fμν /. Table[R^i_{μ,ν} → R^i_{μ,ν,ν}, {i, 3}]).com[Mμ, Gp] - (*integrate by parts*)
        (Fμν /. Table[R^i_{μ,ν} → R^i_{μ,ν,μ}, {i, 3}]).com[Mν, Gp]]], {Gp, Gpt}];
vr = Simplify[Series[vrip / {2 δ^2, 2, 2, -4, 2, 2}, {δ, 0, 0}] // Normal, sub]
```

$$\{R^3_{\{\mu,\nu,\nu\}}\Gamma^2_\mu - R^2_{\{\mu,\nu,\nu\}}\Gamma^3_\mu - R^3_{\{\mu,\nu,\mu\}}\Gamma^2_\nu + R^2_{\{\mu,\nu,\mu\}}\Gamma^3_\nu,$$
$$R^1_{\{\mu,\nu,\nu\}}\Gamma^3_\mu - R^1_{\{\mu,\nu,\mu\}}\Gamma^3_\nu,\ -R^1_{\{\mu,\nu,\nu\}}\Gamma^2_\mu + R^1_{\{\mu,\nu,\mu\}}\Gamma^2_\nu,\ (R^1_{\{\mu,\nu\}})^2,\ (R^1_{\{\mu,\nu\}})^2,\ (R^1_{\{\mu,\nu\}})^2\}$$

```
fin = Table[Sum[v /. μ → 0, {ν, 1, 3}] - Sum[v /. {μ → cd⟦i, 1⟧, ν → cd⟦i, 2⟧},
     {i, 3}], {v, vr}];                       (* Lagrangian = Σμν ±‖Fμν‖² *)
sub1 =                                     (* rename R curvatures as BE fields *)
 Flatten[Table[{R^i_{0,j} → B^i_j, R^i_{cd⟦j,1⟧,cd⟦j,2⟧} → E^i_j,
     Table[{R^i_{0,j,k} → B^i_{j,k}, R^i_{cd⟦j,1⟧,cd⟦j,2⟧,k} → E^i_{j,k}}, {k, 0, 3}]}, {i, 3}, {j, 3}]];
Column[FullSimplify[fn = fin /. sub1], Dividers → All]
```

$(B^3_{\{1,1\}} + B^3_{\{2,2\}} + B^3_{\{3,3\}})\Gamma^2_0 - (B^2_{\{1,1\}} + B^2_{\{2,2\}} + B^2_{\{3,3\}})\Gamma^3_0 + \Gamma^3_3(B^2_{\{3,0\}} + E^2_{\{1,2\}} - E^2_{\{2,1\}}) - \Gamma^2_3(B^3_{\{3,0\}} + E^3_{\{1,2\}} - E^3_{\{2,1\}}) + \Gamma^3_2(B^2_{\{2,0\}} - E^2_{\{1,3\}} + E^2_{\{3,1\}}) - \Gamma^2_2(B^3_{\{2,0\}} - E^3_{\{1,3\}} + E^3_{\{3,1\}}) + \Gamma^3_1(B^2_{\{1,0\}} + E^2_{\{2,3\}} - E^2_{\{3,2\}}) - \Gamma^2_1(B^3_{\{1,0\}} + E^3_{\{2,3\}} - E^3_{\{3,2\}})$	**~Klein-Gordon**
$(B^1_{\{1,1\}} + B^1_{\{2,2\}} + B^1_{\{3,3\}})\Gamma^3_0 - \Gamma^3_3(B^1_{\{3,0\}} + E^1_{\{1,2\}} - E^1_{\{2,1\}}) - \Gamma^3_2(B^1_{\{2,0\}} - E^1_{\{1,3\}} + E^1_{\{3,1\}}) - \Gamma^3_1(B^1_{\{1,0\}} + E^1_{\{2,3\}} - E^1_{\{3,2\}})$	**~Maxwell1**
$-((B^1_{\{1,1\}} + B^1_{\{2,2\}} + B^1_{\{3,3\}})\Gamma^2_0) + \Gamma^2_3(B^1_{\{3,0\}} + E^1_{\{1,2\}} - E^1_{\{2,1\}}) + \Gamma^2_2(B^1_{\{2,0\}} - E^1_{\{1,3\}} + E^1_{\{3,1\}}) + \Gamma^2_1(B^1_{\{1,0\}} + E^1_{\{2,3\}} - E^1_{\{3,2\}})$	**~Maxwell2**
$(B^1_1)^2 + (B^1_2)^2 + (B^1_3)^2 - (E^1_1)^2 - (E^1_2)^2 - (E^1_3)^2$	electric field enforces magnetic (?)
$(B^1_1)^2 + (B^1_2)^2 + (B^1_3)^2 - (E^1_1)^2 - (E^1_2)^2 - (E^1_3)^2$	-> de Broglie clock/zitterbewegung?
$(B^1_1)^2 + (B^1_2)^2 + (B^1_3)^2 - (E^1_1)^2 - (E^1_2)^2 - (E^1_3)^2$	in 4D dominated by 0-th: gravity?

$E^{2}_{\{1,3\}}$ - type: 1 - high energy (EM tilt-tilt), 2,3 low energy (QM tilt-twist)
- spatial coordinate (1,2,3), derivative (0,1,2,3)

Hedgehog ansatz for longest axis (± 1 charge), with $\psi(t,x,y,z)$ phase/twist function

In vacuum ($A=0$) leads to **Klein-Gordon-like**: $(\hat{E}-q\phi)^2\psi = (\hat{p}-qA)^2\psi + m^2\psi$

dual formulation ($E \leftrightarrow B$): $A^{hedg} = \frac{1}{r^2}(x,y,z)$ $\qquad$ $(\hat{E} = i\hbar\partial_t \quad , \quad \hat{p} = -i\hbar\nabla)$

$\Psi = \exp(i\psi)$ $\qquad$ $\hat{p}\Psi = -i\nabla\Psi = \Psi\nabla\psi$ $\qquad$ here from: $X^2\cdot\Gamma^3 = X^3\cdot\Gamma^2$

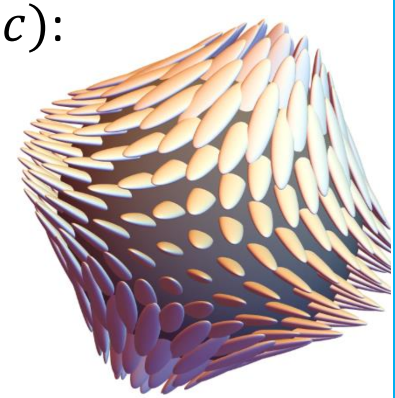
Dirac equation? (also zitterbewegung)

Bispinor for **electron** (up), **positron** (down) with **spin direction** (a,b,c):

$$\frac{1}{4}\begin{bmatrix} 1+c & a-ib & \pm(1+c) & \pm(a-ib) \\ a+ib & 1-c & \pm(a+ib) & \pm(1-c) \\ \pm(1+c) & \pm(a-ib) & 1+c & a-ib \\ \pm(a+ib) & \pm(1-c) & a+ib & 1-c \end{bmatrix}$$

$$S[\Lambda_{\text{boost}}] = \begin{pmatrix} e^{+\chi\cdot\sigma/2} & 0 \\ 0 & e^{-\chi\cdot\sigma/2} \end{pmatrix}$$

$$S[\Lambda_{\text{rot}}] = \begin{pmatrix} e^{+i\phi\cdot\sigma/2} & 0 \\ 0 & e^{+i\phi\cdot\sigma/2} \end{pmatrix}$$



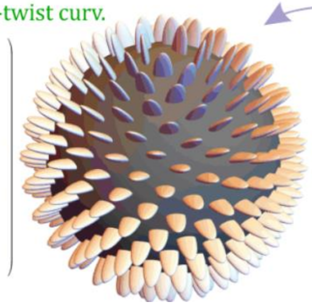
we get wave-like: $2\partial_{tt}\psi = \left(\left(\nabla - \vec{A}^{hedg}\right)^2 + \left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|}\cdot\nabla\right)^2\right)\psi$

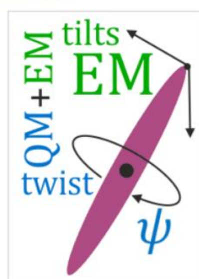
```
sph = {x → r * Cos[θ] * Cos[ϕ], y → r * Cos[θ] * Sin[ϕ], z → r * Sin[θ]}; (*spherical*)
Q0 = FullSimplify[MatrixExp[ϕ * Gz].MatrixExp[θ * Gy].MatrixExp[ψ * Gx] /.
    {ϕ → ArcTan[x, y], θ → -ArcTan[Sqrt[x^2 + y^2], z]}]; (* hedgehog *)
Q = Q0; tQ = Transpose[Q]; M = Simplify[Q.d.tQ];
fBE := Table[Simplify[vect[tQ.com[D[M, c[[1]]], D[M, c[[2]]]].Q]],
    {c, {{t, x}, {t, y}, {t, z}, {z, y}, {x, z}, {y, x}}}];
M0 = 0.001 * IdentityMatrix[3] + 0.05 * M /. {δ → 0.1, ψ → 0}; (* shape to draw *)
points = SpherePoints[300];
Row[{Column[{"B"1, "B"2, "B"3, "E"1, "E"2, "E"3}], "=", fBE /. ψ → 0 // MatrixForm,
  Graphics3D[{Table[Ellipsoid[p, M0 /. {x → p[[1]], y → p[[2]], z → p[[3]]}], {p, points}],
    Gray, Sphere[{0, 0, 0}, 1]}, Boxed → False, ImageSize → Small]}]
```

type: EM: high energy tilt-tilt curvature $\qquad$ QM: low energy, tilt-twist curv.

$$\begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ E_1 \\ E_2 \\ E_3 \end{array} = \begin{array}{ccc} 1 & 2 & 3 \end{array}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{x(-1+\delta)}{(x^2+y^2+z^2)^{3/2}} & 0 & -\frac{xz\delta}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)^{3/2}} \\ -\frac{y(-1+\delta)}{(x^2+y^2+z^2)^{3/2}} & 0 & -\frac{yz\delta}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)^{3/2}} \\ -\frac{z(-1+\delta)}{(x^2+y^2+z^2)^{3/2}} & 0 & -\frac{z^2\delta}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)^{3/2}} \end{pmatrix}$$

coordinate






```
Q = Q0 /. {ψ → ψ[t, x, y, z]};                                    (* assume phase dependence *)
Γs = Simplify[Table[vect[Transpose[Q].D[Q, v]], {v, {t, x, y, z}}], r > 0];
BE = Simplify[Table[Cross[Γs[[c[[1]]]], Γs[[c[[2]]]]],           (* find BE fields *)
    {c, {{1, 2}, {1, 3}, {1, 4}, {4, 3}, {2, 4}, {3, 2}}}]];
BEd = Simplify[Table[D[BE, v], {v, {t, x, y, z}}]];              (* BE derivatives *)
sub2 = Flatten[Join[Table[Γ^j_(k-1) → Γs[[k, j]], {k, 4}, {j, 3}],
    Table[{B^j_i → BE[[i, j]], E^j_i → BE[[i + 3, j]],
      Table[{B^j_{i,k-1} → BEd[[k, i, j]], E^j_{i,k-1} → BEd[[k, i + 3, j]]}, {k, 4}]},
    {i, 3}, {j, 3}]]];
(fne = FullSimplify[fn[[1 ;; 3]] /. sub2] * (x^2 + y^2 + z^2)^2) // Column (*equations:*)
```

$-2z\,\psi^{(0,0,0,1)}[t,x,y,z] + (x^2+y^2+2z^2)\,\psi^{(0,0,0,2)}[t,x,y,z] -$
$2y\,\psi^{(0,0,1,0)}[t,x,y,z] + 2yz\,\psi^{(0,0,1,1)}[t,x,y,z] + x^2\,\psi^{(0,0,2,0)}[t,x,y,z] +$
$2y^2\,\psi^{(0,0,2,0)}[t,x,y,z] + z^2\,\psi^{(0,0,2,0)}[t,x,y,z] - 2x\,\psi^{(0,1,0,0)}[t,x,y,z] +$
$2xz\,\psi^{(0,1,0,1)}[t,x,y,z] + 2xy\,\psi^{(0,1,1,0)}[t,x,y,z] + 2x^2\,\psi^{(0,2,0,0)}[t,x,y,z] +$
$y^2\,\psi^{(0,2,0,0)}[t,x,y,z] + z^2\,\psi^{(0,2,0,0)}[t,x,y,z] - 2(x^2+y^2+z^2)\,\psi^{(2,0,0,0)}[t,x,y,z]$

0

0,0 for 2nd, 3rd equation - they are satified, the first equation equalized to 0:

which turns out Klein-Gordon-like: $2\partial_{tt}\psi = \left(\left(\nabla - \vec{A}^{hedg}\right)^2 + \left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|}\cdot\nabla\right)^2\right)\psi$

for dual: $\vec{A}^{hedg}(x,y,z) = (x,y,z)/r^2$ $\qquad$ $\Psi = \exp(i\psi)$ $\qquad$ $\hat{p}\Psi = -i\nabla\Psi = \Psi\nabla\psi$

```
r = Sqrt[x^2 + y^2 + z^2]; A = {x, y, z} / r^2;
gmA[f_] := Grad[f, {x, y, z}] - A * f; Adg[f_] := (A * r).Grad[f, {x, y, z}];
Simplify[fne[[1]] / r^2 - Sum[gmA[gmA[ψ[t, x, y, z]]][[i, i]], {i, 3}] -
  Adg[Adg[ψ[t, x, y, z]]]]
```

$-2\,\psi^{(2,0,0,0)}[t,x,y,z]$

Derivation of Maxwell-like equations for gravity (GEM), in $E = B = 0$ case

$$\text{boost+}\quad \begin{pmatrix} 0 & \tilde\Gamma^1_\mu & \tilde\Gamma^2_\mu & \tilde\Gamma^3_\mu \\ -\tilde\Gamma^1_\mu & 0 & -\Gamma^3_\mu & \Gamma^2_\mu \\ -\tilde\Gamma^2_\mu & \Gamma^3_\mu & 0 & -\Gamma^1_\mu \\ -\tilde\Gamma^3_\mu & -\Gamma^2_\mu & \Gamma^1_\mu & 0 \end{pmatrix}$$

```
d = DiagonalMatrix[{g, 1, δ, 0}]; cd = {{3, 2}, {1, 3}, {2, 1}}; com[A_, B_] := A.B - B.A;
ξ = DiagonalMatrix[{-1, 1, 1, 1}]; (* signature *)   coms[A_, B_] := A.ξ.B - B.ξ.A;
Γμ = {{0, Γ̃μ^1, Γ̃μ^2, Γ̃μ^3}, {Γ̃μ^1, 0, -Γμ^3, Γμ^2}, {Γ̃μ^2, Γμ^3, 0, -Γμ^1}, {Γ̃μ^3, -Γμ^2, Γμ^1, 0}};   (*3D rot + boost*)
G4 = Table[Coefficient[Γμ, v], {v, {Γμ^1, Γμ^2, Γμ^3, Γ̃μ^1, Γ̃μ^2, Γ̃μ^3}}];   (* rotation + boost generators *)
dg = Table[tm = Table[0, 4, 4]; tm[[i, i]] = 1; tm, {i, 4}];   (* elongation generators *)
Gpt = Join[Table[coms[G, d], {G, G4}], dg];   (* G' size 3+3+4=10 tables *)
sub = Flatten[Table[{Cross[{Γμ^1, Γμ^2, Γμ^3}, {Γν^1, Γν^2, Γν^3}][[i]] == R^i_{μ,ν},   (* EM curvatures *)
      Cross[{Γ̃μ^1, Γ̃μ^2, Γ̃μ^3}, {Γ̃ν^1, Γ̃ν^2, Γ̃ν^3}][[i]] == R̃^i_{μ,ν}}, {i, 3}]];
ds[a_] := Flatten[Table[{R^i_{μ,ν} → R^i_{μ,ν,a}, R̃^i_{μ,ν} → R̃^i_{μ,ν,a}}, {i, 3}]];   (* derivatives *)
cs = Flatten[Table[{Γμ^i → 0, Γν^i → 0}, {i, 3}]];   (* assume EM E=B=0 here *)
Mμ = coms[Γμ, d] /. cs; Γν = Γμ /. μ → ν; Mν = coms[Γν, d] /. cs; Fμν = Simplify[coms[Mμ, Mν], sub];
vr = Table[Tr[Fμν.ξ.(coms[Γν, coms[Mμ, Gp]] - coms[Γμ, coms[Mν, Gp]]).ξ +   (* evolution equations *)
      (Fμν /. ds[ν]).ξ.coms[Mμ, Gp].ξ - (Fμν /. ds[μ]).ξ.coms[Mν, Gp].ξ], {Gp, Gpt}] /. cs;
sub1 = Flatten[Table[{R̃^i_{0,j} → B̃^i_j, R̃^i_{cd[[j,1]],cd[[j,2]]} → Ẽ^i_j,   (* GEM EB fields *)
      Table[{R̃^i_{0,j,k} → B̃^i_{j,k}, R̃^i_{cd[[j,1]],cd[[j,2]],k} → Ẽ^i_{j,k}}, {k, 0, 3}]}, {i, 3}, {j, 3}]];
fin = Simplify[Table[Sum[v /. μ → 0, {v, 1, 3}] - Sum[v /. {μ → cd[[i, 1]], ν → cd[[i, 2]]}, {i, 3}], {v, vr}]];
Column[fnl = Limit[(fin[[4 ;; 6]] /. sub1) / 2 / g^4, g → Infinity] // FullSimplify, Dividers → All]
```

$$\begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

$\left(\tilde B^3_{\{1,1\}} + \tilde B^3_{\{2,2\}} + \tilde B^3_{\{3,3\}}\right)\tilde\Gamma^2_0 - \left(\tilde B^2_{\{1,1\}} + \tilde B^2_{\{2,2\}} + \tilde B^2_{\{3,3\}}\right)\tilde\Gamma^3_0 + \tilde\Gamma^3_3\left(\tilde B^2_{\{3,0\}} + \tilde E^2_{\{1,2\}} - \tilde E^2_{\{2,1\}}\right) - \tilde\Gamma^2_3\left(\tilde B^3_{\{3,0\}} + \tilde E^3_{\{1,2\}} - \tilde E^3_{\{2,1\}}\right) + \tilde\Gamma^3_2\left(\tilde B^2_{\{2,0\}} - \tilde E^2_{\{1,3\}} + \tilde E^2_{\{3,1\}}\right) - \tilde\Gamma^2_2\left(\tilde B^3_{\{2,0\}} - \tilde E^3_{\{1,3\}} + \tilde E^3_{\{3,1\}}\right) + \tilde\Gamma^3_1\left(\tilde B^2_{\{1,0\}} + \tilde E^2_{\{2,3\}} - \tilde E^2_{\{3,2\}}\right) - \tilde\Gamma^2_1\left(\tilde B^3_{\{1,0\}} + \tilde E^3_{\{2,3\}} - \tilde E^3_{\{3,2\}}\right)$
$-\left(\left(\tilde B^3_{\{1,1\}} + \tilde B^3_{\{2,2\}} + \tilde B^3_{\{3,3\}}\right)\tilde\Gamma^1_0\right) + \left(\tilde B^1_{\{1,1\}} + \tilde B^1_{\{2,2\}} + \tilde B^1_{\{3,3\}}\right)\tilde\Gamma^3_0 - \tilde\Gamma^3_3\left(\tilde B^1_{\{3,0\}} + \tilde E^1_{\{1,2\}} - \tilde E^1_{\{2,1\}}\right) + \tilde\Gamma^1_3\left(\tilde B^3_{\{3,0\}} + \tilde E^3_{\{1,2\}} - \tilde E^3_{\{2,1\}}\right) - \tilde\Gamma^3_2\left(\tilde B^1_{\{2,0\}} - \tilde E^1_{\{1,3\}} + \tilde E^1_{\{3,1\}}\right) + \tilde\Gamma^1_2\left(\tilde B^3_{\{2,0\}} - \tilde E^3_{\{1,3\}} + \tilde E^3_{\{3,1\}}\right) - \tilde\Gamma^3_1\left(\tilde B^1_{\{1,0\}} + \tilde E^1_{\{2,3\}} - \tilde E^1_{\{3,2\}}\right) + \tilde\Gamma^1_1\left(\tilde B^3_{\{1,0\}} + \tilde E^3_{\{2,3\}} - \tilde E^3_{\{3,2\}}\right)$
$\left(\tilde B^2_{\{1,1\}} + \tilde B^2_{\{2,2\}} + \tilde B^2_{\{3,3\}}\right)\tilde\Gamma^1_0 - \left(\tilde B^1_{\{1,1\}} + \tilde B^1_{\{2,2\}} + \tilde B^1_{\{3,3\}}\right)\tilde\Gamma^2_0 + \tilde\Gamma^2_3\left(\tilde B^1_{\{3,0\}} + \tilde E^1_{\{1,2\}} - \tilde E^1_{\{2,1\}}\right) - \tilde\Gamma^1_3\left(\tilde B^2_{\{3,0\}} + \tilde E^2_{\{1,2\}} - \tilde E^2_{\{2,1\}}\right) + \tilde\Gamma^2_2\left(\tilde B^1_{\{2,0\}} - \tilde E^1_{\{1,3\}} + \tilde E^1_{\{3,1\}}\right) - \tilde\Gamma^1_2\left(\tilde B^2_{\{2,0\}} - \tilde E^2_{\{1,3\}} + \tilde E^2_{\{3,1\}}\right) + \tilde\Gamma^2_1\left(\tilde B^1_{\{1,0\}} + \tilde E^1_{\{2,3\}} - \tilde E^1_{\{3,2\}}\right) - \tilde\Gamma^1_1\left(\tilde B^2_{\{1,0\}} + \tilde E^2_{\{2,3\}} - \tilde E^2_{\{3,2\}}\right)$

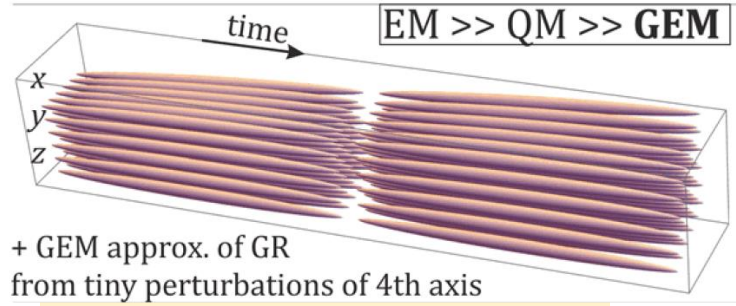
$\tilde X^i := \left(-\nabla\cdot\tilde B^i,\ \overrightarrow{\partial_0\tilde B^\iota + \nabla\times\tilde E^\iota}\right)$ **Maxwell:** $\tilde X^3\cdot\tilde\Gamma^2 = \tilde X^2\cdot\tilde\Gamma^3$, $\tilde X^3\cdot\tilde\Gamma^1 = \tilde X^1\cdot\tilde\Gamma^3$, $\tilde X^2\cdot\tilde\Gamma^1 = \tilde X^1\cdot\tilde\Gamma^2$

Newton attraction? $\sim 10^{-36} \times$ of Coulomb???

reduced energy for closer curvature sources???

reduced distance - increased curvature^2 in energy

- $r \to 0$ Coulomb, $r \to \infty$ Newton – maybe $r \to r^{-1}$, "**repulsion in infinity**"?
- maybe somehow **opposite curvatures**? E.g. for protons – neutrons?
- due to **spacetime signature**? (also "negative" energy to propel the clock?)



$\xi = \mathrm{diag}(-1,1,1,1)$ $\qquad AB \to A\xi B$ $\qquad A_\mu^\nu = \sum_\alpha A_{\mu\alpha}\xi^{\alpha\nu}$

time has opposite sign

Lorentz-invariant Lagrangian?

$$\mathcal{L} = -F_{\alpha\beta\mu\nu}F^{\alpha\beta\mu\nu} - V(M)$$

for $F_{\alpha\beta\mu\nu} = \left[\partial_\mu M, \partial_\nu M\right]_{\alpha\beta}$

$[A,B] \to A\xi B - B\xi A$

$\mathrm{Tr}(AA^T) \to \mathrm{Tr}(A\xi A^T\xi)$

General relativity Lagrangian

(Einstein-Hilbert action):

$$\mathcal{L} = R\sqrt{-g} \qquad \left(R = R_{\alpha\beta\mu\nu}\xi^{\alpha\beta}\xi^{\mu\nu}\right)$$

First power of (contracted) curvature,

so which is proper 1 vs 2? Mixture?

```
cos = 1 + (z - d) / Sqrt[(z - d)^2 + r^2] - (z + d) / Sqrt[(z + d)^2 + r^2]; (* Manfried Faber dipole ansatz *)
n = {Sqrt[1 - cos^2] x / r, Sqrt[1 - cos^2] y / r, cos} /. r → Sqrt[x^2 + y^2];     (* cylindrical symmetry *)
G = Table[t = Table[0, 4, 4]; t[[1, i]] = -1; t[[i, 1]] = 1; t, {i, 2, 4}];   (* 4D rotation generators *)
M0 = {{g, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}};           (* rest field for {g,1,0,0} axes *)
o = MatrixExp[m * {a, b, c}.G];            (* m is the mass - it is tiny, we focus on Taylor to m^1 *)
M = Series[o.M0.Transpose[o] /. {a → n[[1]], b → n[[2]], c → n[[3]]}, {m, 0, 1}] // Normal // Simplify;
dM = {D[M, x], D[M, y], D[M, z]};                         (* Hamiltonian taking the lowest m term: m^4 *)
H = Coefficient[Sum[Total[(dM[[i]].dM[[j]] - dM[[j]].dM[[i]])^2, 2], {i, 2}, {j, i + 1, 3}] /. y → 0, m^4];
HH = FullSimplify[CoefficientList[H, g][[-1]]];                      (* take the highest g term: g^4 *)
Es = Table[{d, NIntegrate[4 Pi * x * HH * Boole[x^2 + (z - d)^2 > 0.001],                (* 0.001  cutoff *)
      {x, 0, Infinity}, {z, 0, Infinity}]}, {d, 0.1, 3, 0.1}]; (*integrate H: total field energy*)
ft = Fit[Es, {1, 1 / d}, d]; Show[ListPlot[Es], Plot[ft, {d, 0.1, 3}]]   (* fit Newton potential *)
```

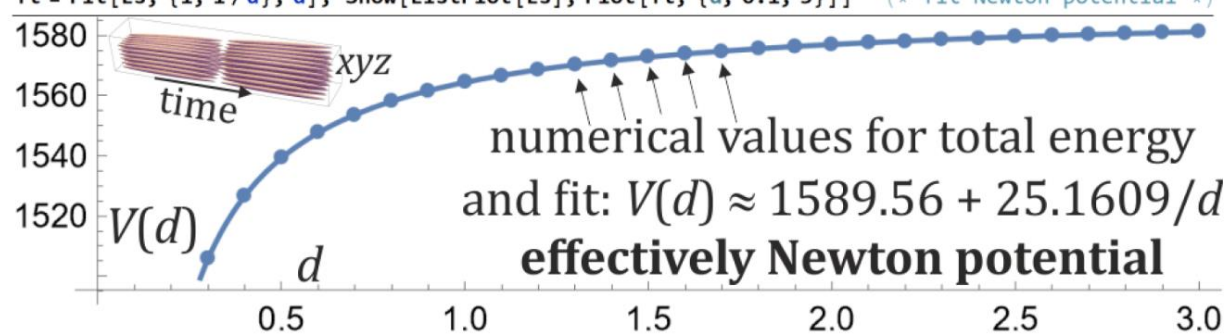


Figure 3. A simplified calculation of Newton effective potential (in GitHu
- analogously as in Fig. 2, but this time instead of large spatial rotations, usi
tiny tilts of 0th time axis for gravity (no mass quantization). Spherically sy

What propels $\sim 10^{21}$Hz electron's clock? $\psi = \psi_0 e^{iEt/\hbar}$ for $E = mc^2$ exper.

Hypothesized "time crystals" with **oscillations in the lowest energy state?**

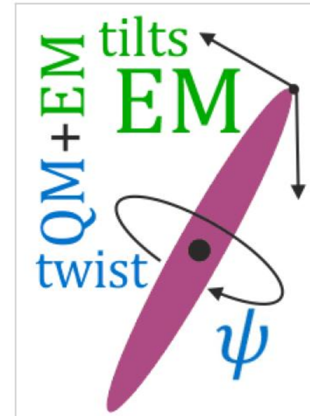
Naively such time derivative come with positive energy contribution …

EM Lagrangian $B^2 - E^2 = \frac{1}{2} F_{\mu\nu}F^{\mu\nu} \to \frac{1}{2}\sum_{\mu\nu}\left(F_{\mu\nu}\right)^2 = B^2 + E^2$ Hamiltonian

Full spacetime curvature: 4 indexes $F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} \to F_{\mu\nu\alpha\beta}F^{\alpha\beta}_{\mu\nu}$

(general relativity, SO(1,3) connection gen.: rotations+boosts)

Hamiltonian (energy density) gets **negative energy contributions** for $\Gamma^1\,\tilde{\Gamma}^2$ and $\Gamma^1\,\tilde{\Gamma}^3$ – quantum phase twist Γ^1, perpend. time boost



$$\mathcal{L} = -\sum_{\alpha\beta\mu\nu} F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} - V(M)$$

for $\quad F_{\mu\nu\alpha\beta} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$

$$\mathcal{H} = \sum_{0\le\mu<\nu\le 3} F_{\mu\nu\alpha\beta}F^{\alpha\beta}_{\mu\nu} + V(M) =$$

$$= 2\sum_{0\le\mu<\nu\le 3}\left(\boxed{\sum_{1\le\alpha<\beta\le 3}(F_{\mu\nu\alpha\beta})^2} - \boxed{\sum_{\alpha=1}^{3}(F_{\mu\nu\alpha 0})^2}\right) + V(M)$$

$M_\mu = \partial_\mu(ODO^T) = O\overline{M}_\mu O^T \quad$ for $\quad \overline{M}_\mu = [\Gamma_\mu, D]$

$A_\mu = [M, M_\mu] = O\overline{A}_\mu O^T \quad$ for $\quad \overline{A}_\mu = [D, \overline{M}_\mu]$

$F_{\mu\nu} = [M_\mu, M_\nu] = O\overline{F}_{\mu\nu}O^T \quad$ for $\quad \overline{F}_{\mu\nu} = [\overline{M}_\mu, \overline{M}_\nu]$

vacuum: $V(M) = 0$

```
sub = Flatten[Table[{Cross[{Γ¹μ, Γ²μ, Γ³μ}, {Γ¹ν, Γ²ν, Γ³ν}][[i]] == Rⁱ{μ,ν}, Cross[{Γ̃¹μ, Γ̃²μ, Γ̃³μ}, {Γ̃¹ν, Γ̃²ν, Γ̃³ν}][[i]] == R̃ⁱ{μ,ν}}, {i, 3}]];
Γμ = {{0, Γ̃¹μ, Γ̃²μ, Γ̃³μ}, {Γ̃¹μ, 0, -Γ³μ, Γ²μ}, {Γ̃²μ, Γ³μ, 0, -Γ¹μ}, {Γ̃³μ, -Γ²μ, Γ¹μ, 0}};    (*3D rotations + boosts *)
ξ = DiagonalMatrix[{-1, 1, 1, 1}]; coms[A_, B_] := A.ξ.B - B.ξ.A; d = DiagonalMatrix[{g, 1, δ, 0}]; M̄μ = coms[Γμ, d]
{{0, Γ̃¹μ + g Γ̃¹μ, g Γ̃²μ + δ Γ̃²μ, g Γ̃³μ}, {-Γ̃¹μ - g Γ̃¹μ, 0, Γ³μ - δ Γ³μ, -Γ²μ}, {-g Γ̃²μ - δ Γ̃²μ, Γ³μ - δ Γ³μ, 0, δ Γ¹μ}, {-g Γ̃³μ, -Γ²μ, δ Γ¹μ, 0}}
Āμ = coms[d, M̄μ] // FullSimplify        before g >> 1 >> δ > 0 approximation: removed [] terms
{{0, -(1 + g)² Γ̃¹μ, -(g + δ)² Γ̃²μ, -g² Γ̃³μ}, {-(1 + g)² Γ̃¹μ, 0, (-1 + δ)² Γ³μ, -Γ²μ}, {-(g + δ)² Γ̃²μ, -(-1 + δ)² Γ³μ, 0, δ² Γ¹μ}, {-g² Γ̃³μ, Γ²μ, -δ² Γ¹μ, 0}}
M̄μ = {{0, g Γ̃¹μ, g Γ̃²μ, g Γ̃³μ}, {-g Γ̃¹μ, 0, Γ³μ, -Γ²μ}, {-g Γ̃²μ, Γ³μ, 0, δ Γ¹μ}, {-g Γ̃³μ, -Γ²μ, δ Γ¹μ, 0}}; (* g >> 1 >> δ > 0 approximation *)
(F̄μν = FullSimplify[coms[M̄μ, M̄μ /. μ → ν], sub]) // MatrixForm
```

→ clock ? EM/QM - gravity interaction

$$\begin{pmatrix} 0 & g\left(\Gamma^3_\nu\tilde{\Gamma}^2_\mu - \Gamma^2_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^2_\nu + \Gamma^2_\mu\tilde{\Gamma}^3_\nu\right) & g\left(\Gamma^3_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^3_\nu\right) & g\left(-\Gamma^2_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^2_\mu + \Gamma^2_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^2_\nu\right) \\ g\left(\Gamma^3_\nu\tilde{\Gamma}^2_\mu - \Gamma^2_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^2_\nu + \Gamma^2_\mu\tilde{\Gamma}^3_\nu\right) & 0 & \delta R^3_{\{\mu,\nu\}} + g^2\tilde{R}^3_{\{\mu,\nu\}} & \delta R^2_{\{\mu,\nu\}} - g^2\tilde{R}^2_{\{\mu,\nu\}} \\ g\left(\Gamma^3_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^3_\nu\right) & -\delta R^3_{\{\mu,\nu\}} - g^2\tilde{R}^3_{\{\mu,\nu\}} & 0 & R^1_{\{\mu,\nu\}} + g^2\tilde{R}^1_{\{\mu,\nu\}} \\ g\left(-\Gamma^2_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^2_\mu + \Gamma^2_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^2_\nu\right) & -\delta R^2_{\{\mu,\nu\}} + g^2\tilde{R}^2_{\{\mu,\nu\}} & -R^1_{\{\mu,\nu\}} - g^2\tilde{R}^1_{\{\mu,\nu\}} & 0 \end{pmatrix}$$

+/- squared energy contributions

QM: tilt-twist

EM: tilt-tilt

$\tilde{R} \equiv R^g$ gravity

Figure 6. Gathered formulas for suggested 4D model, $M = ODO^T$, $D = \text{diag}(g, 1, \delta, 0)$ with calculated (in GitHub) vacuum $V(M) = 0$, $g >> 1 >> \delta > 0$ approximation for $\bar{F}_{\mu\nu}$. With tilde there are noted time axis/gravitational parts, $\tilde{R}_{\mu\nu} = \tilde{\Gamma}_\mu \times \tilde{\Gamma}_\nu$. Surprisingly, the Hamiltonian turns out having not only positive (red), but also negative (blue) contributions. Positive energy contributions (red) for separate EM and GEM are as expected ($\sim E^2 + B^2$), would lead to Maxwell equations for independent each of them. However, combined suggest tendency for opposite curvatures - nearly unsatisfied due to very rigid time axis direction (with tilde). The negative energy contributions (blue) give tendency to increase imbalance, such freedom is mostly in $\Gamma^1_0\tilde{\Gamma}_i$ for $i = 1, 2, 3$ terms (violet) for temporal derivative of twist as in de Broglie clock, with rotation of temporal axis in spatial directions.

Standard model – 26+ parameters, Lagrangian

Noncompatible with **gravity** (“too big infinity”)

Is it “Taylor **expansion” of some simpler model?**

Where to search for it? Predictability vs freedom

Maybe in liquid crystals? Available in physics …

charge quantization + Coulomb-like interaction …

Postulate **skrymion-like Lagrangian** for M:

$$\boldsymbol{\mathcal{L} = -F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} - V(M)} \qquad \text{(Higgs)}$$

$(R\ \sim)\ \ F_{\mu\nu\alpha\beta} = \left[\partial_\mu M, \partial_\nu M\right]_{\alpha\beta}$ **field curvature**

EM >> **quantum phase** >> **GEM vacuum dynamics**

EM with build in **charge quantization**, **regularization**

3 leptons: same charge – different mass, $\mu \neq 0$

3 neutrinos: very stable, oscillations, ↑beta decay

3 families of **baryons**: $m_n > m_p, m_d < m_p + m_n$

deuteron with quadrupole moment, $\mu_d \approx \mu_p + \mu_n$

nuclei as knots – also halo neutrons in e.g. ~5 fm

strangeness, decaying to mesons … effective?

Feynman ensemble of topological defects → QFT

epicycles…?

1

$$-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \tfrac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu$$

2

$$- \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H -$$
$$\tfrac{1}{2}m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} +$$
$$\tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$
$$\tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$
$$g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$
$$\tfrac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] -$$
$$gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$
$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$$
$$\phi^+\partial_\mu H)] + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$$
$$igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$
$$\tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^1 s_w^2 A_\mu A_\mu\phi^+\phi^-$$

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$$- \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j -$$
$$\bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] +$$
$$\tfrac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 -$$
$$1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) +$$
$$(\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 +$$
$$\gamma^5)u^\lambda_j)] +$$

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$$\tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] -$$
$$\tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) +$$
$$m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 -$$
$$\gamma^5)u^\kappa_j] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) -$$
$$\tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) +$$

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$$\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 -$$
$$\tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- -$$
$$\partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y -$$
$$\partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ -$$
$$\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H] +$$
$$\tfrac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-] + \tfrac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] +$$
$$igMs_w[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \tfrac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]$$