Spontaneous Hopf Fibration in the 2HDM

Steven Cotterill

University of Manchester

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The Two Higgs Doublet Model (2HDM)

- Adds an additional complex scalar doublet to the SM with the general potential.

$$V = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+ \Big[- m_{12}^2(\Phi_1^\dagger\Phi_2) + \lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \text{h.c.}\Big]$$

- 5 scalar particles: h, H, $H_\pm$ and A.
- Neutral vacuum ($f_+ = 0$) for a massless photon.

$$\Phi = \begin{pmatrix}\Phi_1 \\ \Phi_2\end{pmatrix} = \frac{v_{\rm SM}}{\sqrt{2}} e^{\frac{1}{2}i\chi}(\sigma^0 \otimes U_L)\begin{pmatrix}0 \\ f_1 \\ f_+ \\ f_2 e^{i\xi}\end{pmatrix}$$

- Used in MSSM and DFSZ axion model.
- Experimental constraints from Higgs boson, masses of new particles, FCNCs, etc…

Bilinear forms

- Rewrite the potential in terms of $R^\mu = \Phi^\dagger(\sigma^\mu \otimes \sigma^0)\Phi$,

$$V = -\frac{1}{2}M_\mu R^\mu + \frac{1}{4}L_{\mu\nu}R^\mu R^\nu$$

- There is also $n^a = -\Phi^\dagger(\sigma^0 \otimes \sigma^a)\Phi$ associated with isospin rotations.
- Vectors associated with the map SU(2) → SO(3): $U^\dagger \sigma^a U = \mathcal{R}^{ab}\sigma^b$.
- $R_+ = R_\mu R^\mu$ tracks the neutral vacuum condition (needs to be zero for a massless photon).
- Topology is more easily seen with R^a than Φ.

Accidental symmetries of the 2HDM

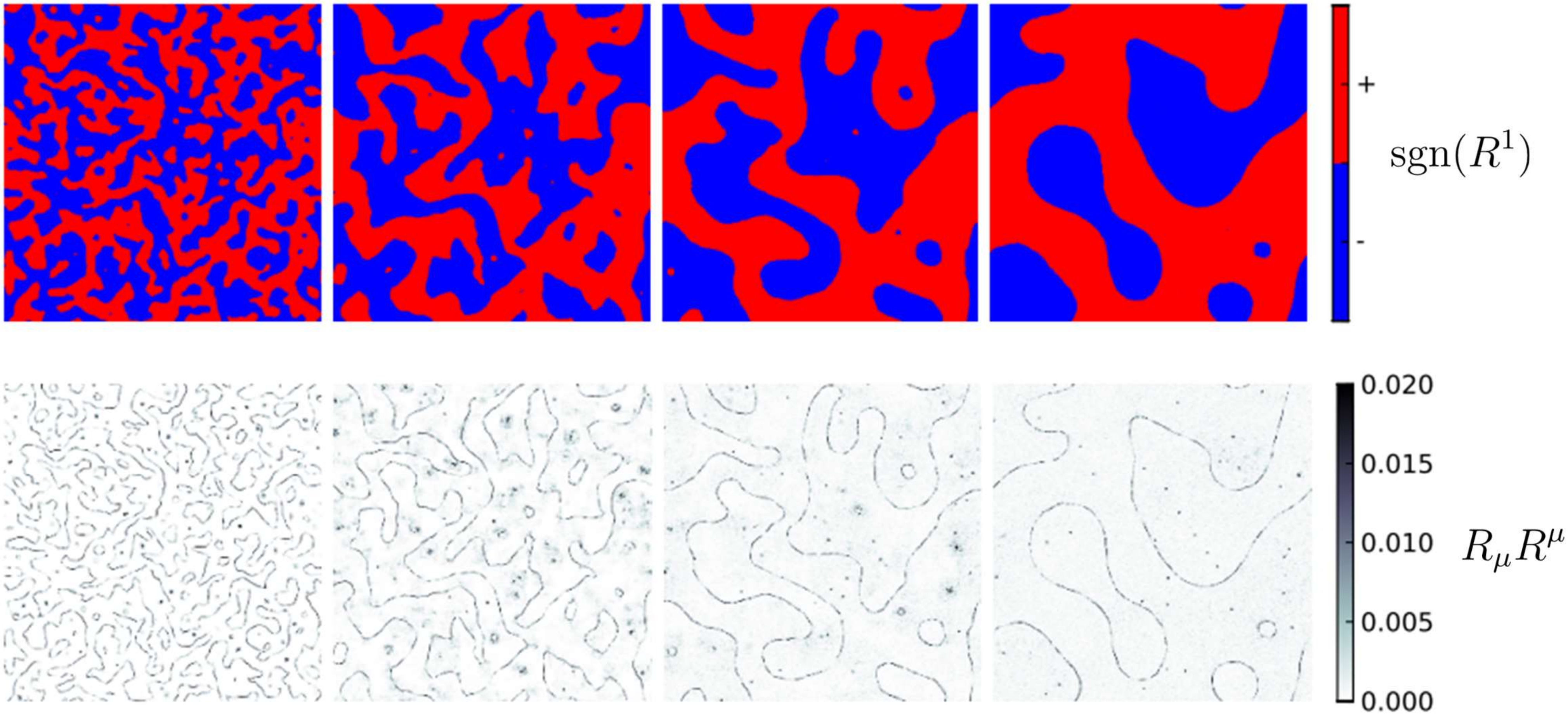
- Parameter choices → additional "accidental" symmetries.

Symmetry	μ_1^2	μ_2^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
Z_2	–	–	0	–	–	–	–	Real	0	0
$U(1)_{PQ}$	–	–	0	–	–	–	–	0	0	0
$SO(3)_{HF}$	–	μ_1^2	0	–	λ_1	–	$2\lambda_1 - \lambda_3$	0	0	0

"Vacuum Topology of the Two Higgs Doublet Model" – Battye, Brawn & Pilaftsis JHEP **08** 020 (2011)

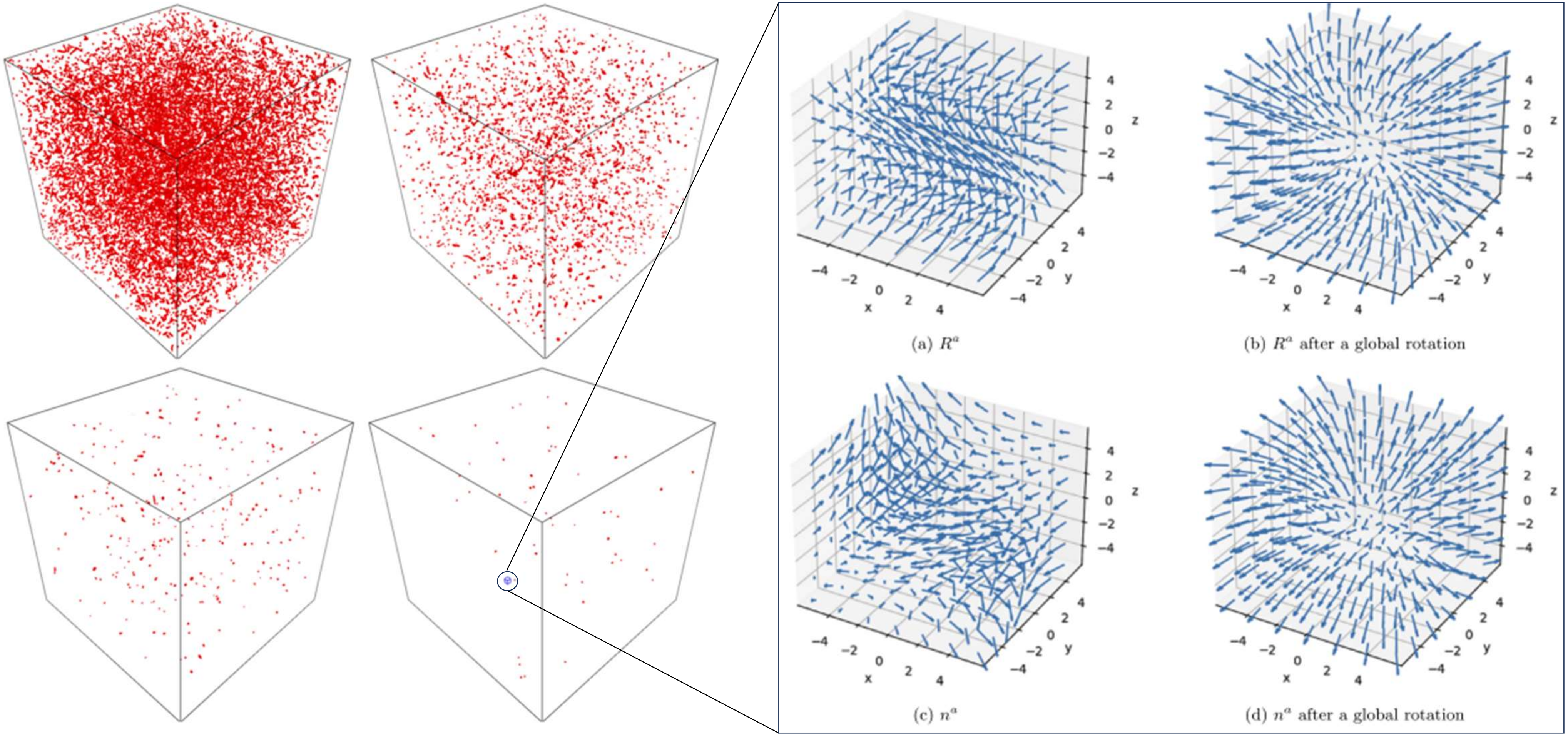
$$\text{Topological solitons:} \begin{cases} \mathbb{Z}_2, & \pi_0(S^0 \times S^3) = \mathbb{Z}_2 \Longrightarrow \text{Domain walls} & R^1 \to -R^1 \\ U(1)_{PQ}, & \pi_1(S^1 \times S^3) = \mathbb{Z} \Longrightarrow \text{Strings} & R^1, R^2 \text{ rotations} \\ SO(3)_{HF}, & \pi_2(S^2 \times S^3) = \mathbb{Z} \Longrightarrow \text{Monopoles} & R^a \text{ rotations} \end{cases}$$

Random simulations in the $\mathbb{Z}_2$ case



"Simulations of Domain Walls In Two Higgs Doublet Models" – Battye, Pilaftsis & Viatic JHEP **01** 105 (2021)

Global monopoles



"Global monopoles in the two-Higgs-doublet-model" – Battye, Cotterill & Viatic Phys. Lett B **844** 138091 (2023)

Initial attempts for monopoles

$$\Phi = \frac{v_{\rm SM}}{\sqrt{2}} \begin{pmatrix} 0 \\ k(r)\cos\theta \\ 0 \\ k(r)\sin\theta e^{i\phi} \end{pmatrix} \quad \text{but} \quad \hat{R}^a = \begin{pmatrix} \sin 2\theta \cos\phi \\ \sin 2\theta \sin\phi \\ \cos 2\theta \end{pmatrix} \longrightarrow \Phi = \frac{v_{\rm SM}}{\sqrt{2}} \begin{pmatrix} 0 \\ k(r)\cos\frac{1}{2}\theta \\ 0 \\ k(r)\sin\frac{1}{2}\theta e^{i\phi} \end{pmatrix}$$

Nambu monopole

Can improve with the SM degrees of freedom!

$$D_\mu \Phi = \left[(\sigma^0 \otimes \sigma^0)\partial_\mu + \frac{1}{2} i g (\sigma^0 \otimes \sigma^a) W_\mu^a + \frac{1}{2} i g' (\sigma^0 \otimes \sigma^0) + \frac{1}{2} i g'' (\sigma^a \otimes \sigma^0) V_\mu^a \right] \Phi$$

Cancel divergent terms with the coupling between W_μ^a and V_μ^a.
Both assume that f_+ is zero everywhere, not just in the vacuum.

Gauged monopole ansatz

- Full ansatz:

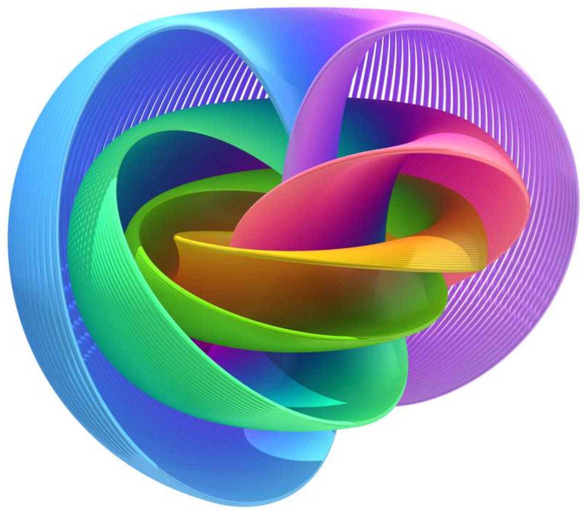
$$\Phi = \frac{v_{\rm SM}}{2\sqrt{2}} \begin{pmatrix} -(k+k_+)\sin\theta e^{-i\phi} \\ (k-k_+)+(k+k_+)\cos\theta \\ -(k-k_+)+(k+k_+)\cos\theta \\ (k+k_+)\sin\theta e^{i\phi} \end{pmatrix}$$

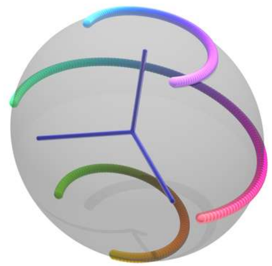
$$gW_i^a = -\frac{1}{r}h(r)\epsilon_{ij}^a\hat{r}^j, \quad g'Y_i = 0 \quad \text{and} \quad g''V_i^a = -\frac{1}{r}H(r)\epsilon_{ij}^a\hat{r}^j$$

$$R^a = n^a = \frac{1}{2}v_{\rm SM}^2(k^2 - k_+^2)\hat{r}^a$$

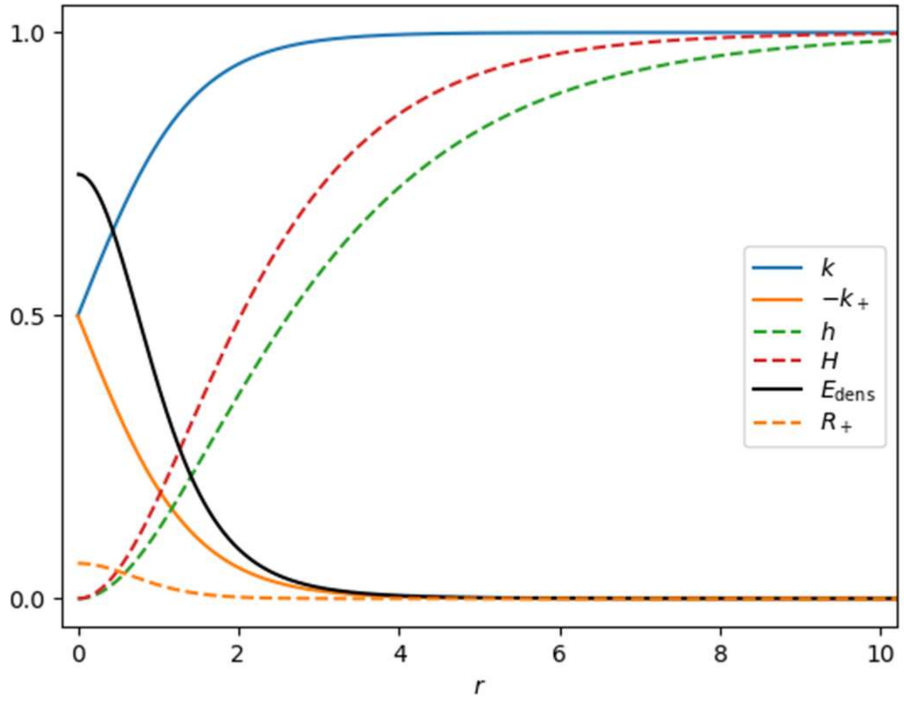
- Gradient energy causes a “Spontaneous Hopf Fibration” with winding in S^2 but not S^1.

$S^3 \to S^2$

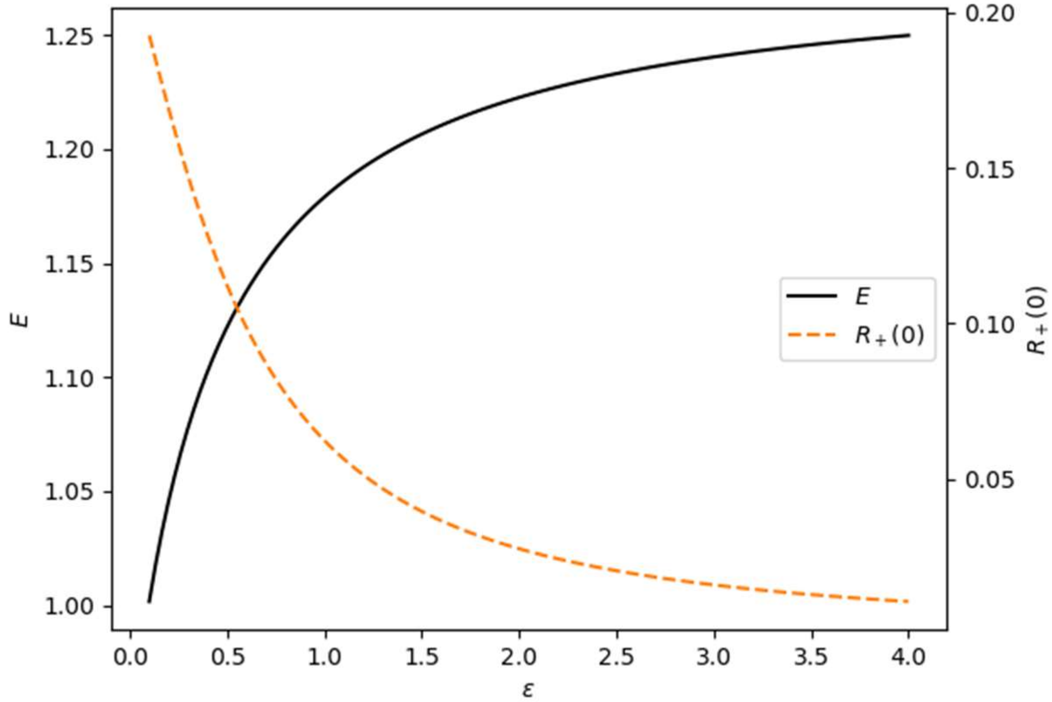




Gauged monopole solution



$$\tilde{\lambda}_1 = \lambda_1/g^2 = 1, \quad \tilde{g}^2 = (g''/g)^2 = 2$$



$$(M_H = M_A = 0) \quad \epsilon = M_{H\pm}/M_h$$

Gauged string solution

$$\Phi = \frac{v_{\rm SM}}{\sqrt{2}} \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 e^{i\theta} \end{pmatrix} \qquad \hat{R}^a = \hat{n}^a = \hat{r}^a \text{ , where } a \in [1,2]$$

$$gW_i^a = -\frac{1}{r}\Big[h_1(r)\hat{x}^a + (1 - h_3(r))\hat{z}^a\Big]\hat{\theta}_i + h_2(r)\hat{y}^a\hat{r}_i,$$

$$g'Y_i = 0 \quad \text{and} \quad g''V_i^3 = -\frac{1}{r}(1 - H(r))\hat{\theta}_i$$

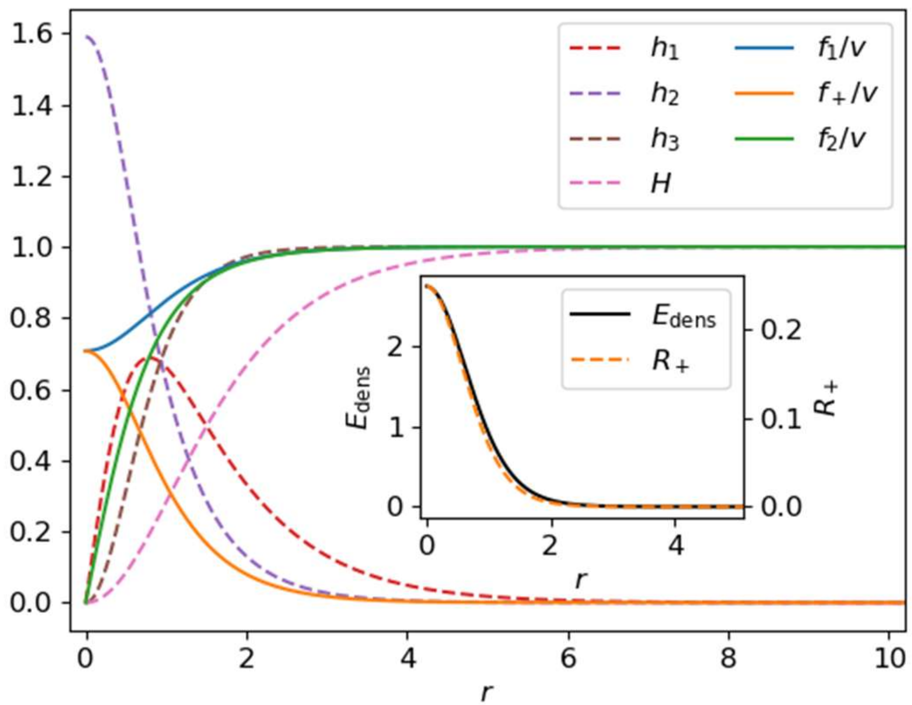
Parameters

$$\tilde{\lambda}_1 = \lambda_1/g^2 = 1, \quad \tilde{g}^2 = (g''/g)^2 = 1$$

$\alpha = \beta = \pi/4$ "Alignment limit"

$$\delta = M_H/M_h, \quad \epsilon = M_{H\pm}/M_h$$

$(M_A = 0)$

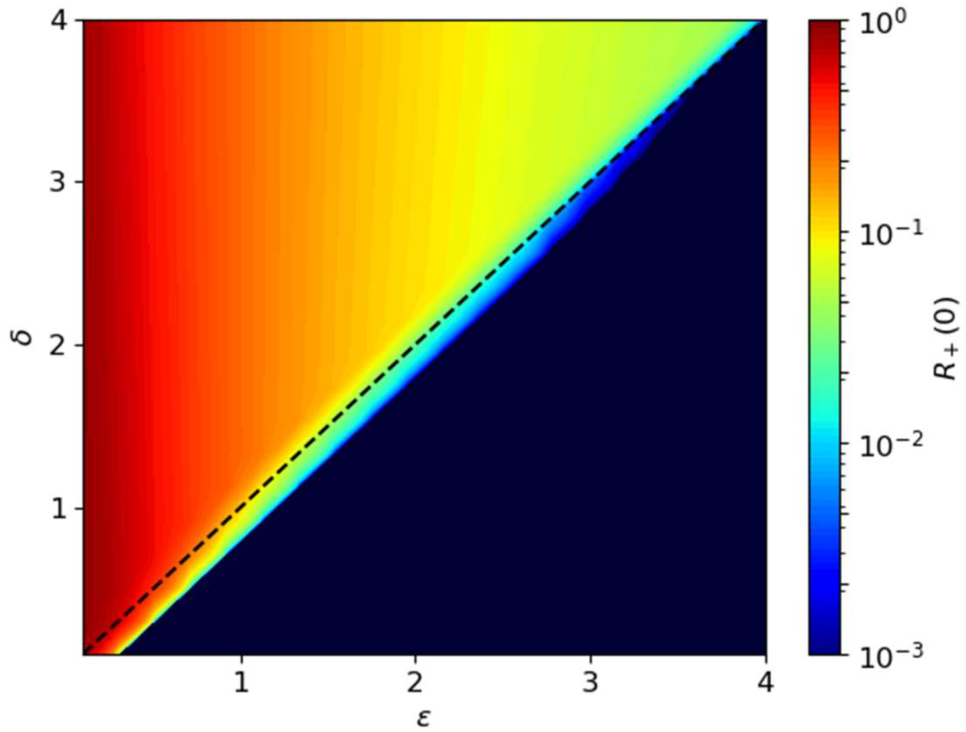


Neutral vacuum violation mass analysis

- Hopf fibration removes gradient energy divergences. What about local neutral vacuum violation?
- Neglect gradient energy to estimate effective mass.

- Monopoles: $m_{k_+}^2(0) = -\frac{1}{2}\tilde{\lambda}_1$

- Strings: $m_{f_+}^2(0) = \tilde{\lambda}_1\left(\frac{\epsilon^2 - \delta^2}{1+\delta^2}\right)$
  (when $\alpha = \beta = \pi/4$)



Conclusions

- Spontaneous Hopf fibration mechanism is not necessarily specific to the 2HDM
- Localised neutral vacuum violation → new interactions
- Phenomenology?
- Changes to the standard cosmology for these types of solitons?

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"Global monopoles in the two-Higgs-doublet-model" – Battye, Cotterill & Viatic Phys. Lett B **844** 138091 (2023)
"Spontaneous Hopf Fibration" – Battye & Cotterill *in prep.*