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Elastic Wave Model of Magnetic Flux & Electric Charge

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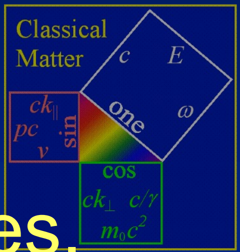
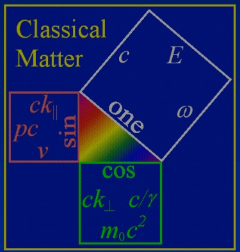
History of Solid Aether



1817: Thomas Young: polarization of light waves.
1839: James MacCullagh: equation for light waves.
1868: Joseph Boussinesq: light & matter interactions.
1873: James Maxwell: electromagnetic equations.
1921: Harold Wilson: gravity as refractive index.
2009: Interpretation of 1st order Dirac equation as 2nd order vector wave equation for spin density ($\mathbf{s}$) with intrinsic momentum ($\mathbf{p}$) density: $\mathbf{p} = \frac{1}{2}\nabla\times\boldsymbol{s}$

"The modern concept of the vacuum of space, confirmed by everyday experiment, is a relativistic ether. But we do not call it this because it is taboo."
(Robert Laughlin, *A Different Universe, 2005*)

Quantum Mechanics



1924: Louis de Broglie: wave nature of matter.

1926: Klein & Gordon: 2nd order Klein-Gordon equation

1928: Paul Dirac: 1st order Dirac equation.

1965: Tomonaga, Schwinger & Feynman: QED.

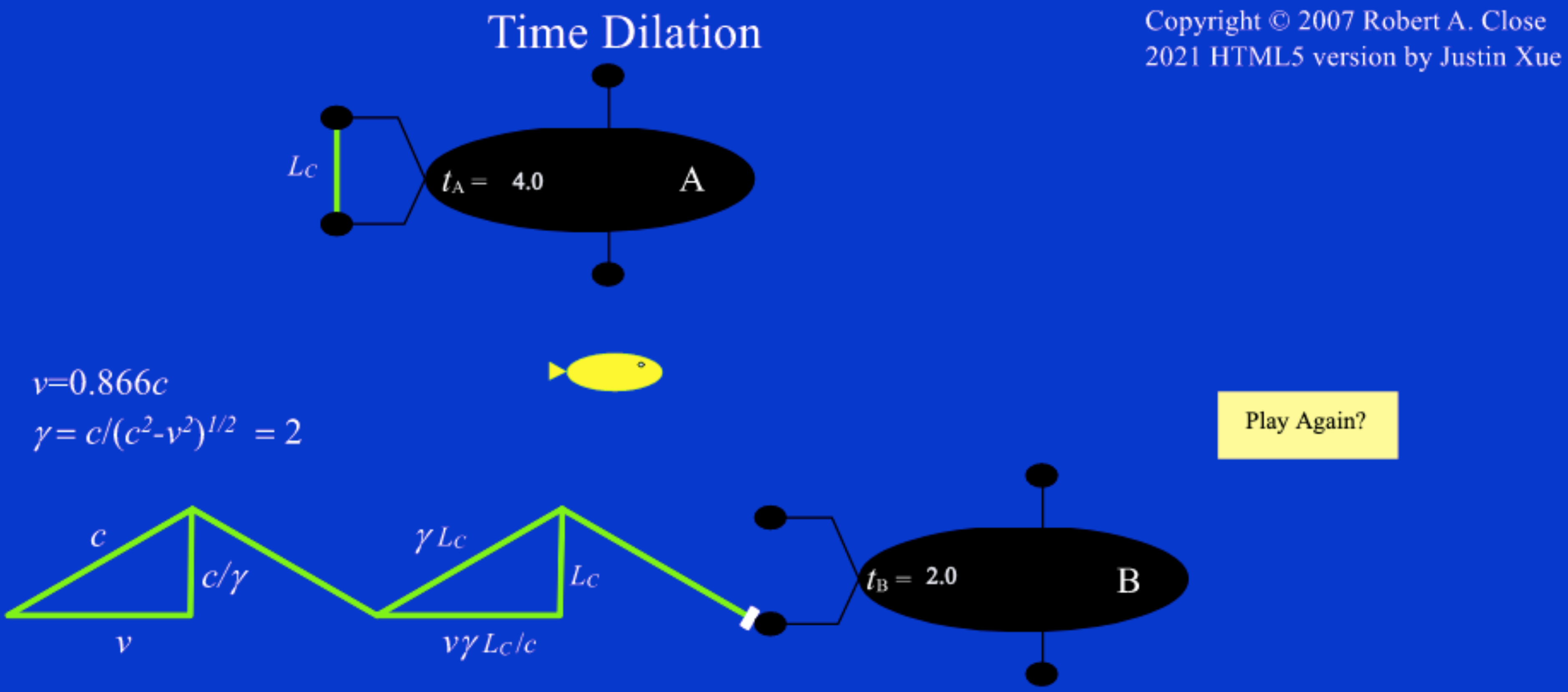
1986: Hans Ohanian: QM spin & intrinsic momentum density $\mathbf{p} = \frac{1}{2}\nabla\times\boldsymbol{s}$ (same as for elastic solid)

Question:

Is Standard Model simply a decomposition of elastic spin density waves into so-called "particles"?

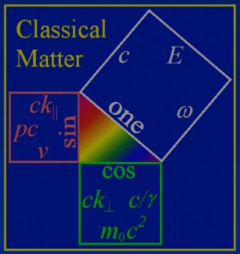
Special Relativity

Wave measurements are related by Lorentz transforms.



https://www.classicalmatter.org/UnderwaterRelativity.htm

Gravity is Refraction



Refractive index changes w/ distortion:

$$n = \left(\frac{\rho}{\rho_1}\right)^{2/3} = \left(\frac{\mu_1}{\mu}\right)^2$$

(Consistent with SCG shear modulus model w/ const. pressure)

Equivalent to GR with:

$$g_{tt} = \frac{1}{g^{tt}} = \frac{\mu}{\mu_1} = \frac{1}{\sqrt{n}}$$

$$g_{xx} = \frac{1}{g^{xx}} = -\left(\frac{\rho}{\rho_1}\right)^{1/3} = -\sqrt{n}$$

Cf. Evans, et al., Am. J. Phys., Vol. 69, No. 10, October 2001, Eq. 14

Classical Spin Density

Incompressible Helmholtz decomposition:

$$\mathbf{p} = \rho \mathbf{u} = \frac{1}{2} \nabla \times \mathbf{s}; \;\; \mathbf{w} = \frac{1}{2} \nabla \times \mathbf{u}$$

$$\mathbf{S} = \int \mathbf{r} \times \mathbf{p} \, d^3 r = \int \mathbf{s} \, d^3 r$$ ~~+ b.t.~~

$$K = \int \frac{1}{2} \rho u^2 \, d^3 r = \int \frac{1}{2} \mathbf{w} \cdot \mathbf{s} \, d^3 r$$ ~~+ b.t.~~

Same total angular momentum and K.E.

Spin density is the momentum conjugate to angular velocity ($\mathfrak{L} \sim K$; $\frac{\partial \mathfrak{L}}{\partial \mathbf{w}} = \frac{\mathbf{s}}{2} + \frac{\mathbf{s}}{2} = \mathbf{s}$).

Equation of Evolution

Momentum density: $\partial_t \mathbf{p} + \mathbf{u} \cdot \nabla \mathbf{p} = \mathbf{f}$

Changes due to convection & force.

Spin density: $\partial_t \mathbf{s} + \mathbf{u} \cdot \nabla \mathbf{s} - \mathbf{w} \times \mathbf{s} = \boldsymbol{\tau}$

Changes ~ convection, rotation, & torque.

Nonlinear wave equation ($\mathbf{s} \equiv \partial_t \mathbf{Q}$):

$\partial_t^2 \mathbf{Q} + \mathbf{u} \cdot \nabla \partial_t \mathbf{Q} - \mathbf{w} \times \partial_t \mathbf{Q} = c^2 \nabla^2 \mathbf{Q}$

Nonlinearity $\Rightarrow$ solitons

Factor the Wave Equation

- 1st -order equation required to compute evolution.
- 2nd -order linear wave equation in 1-D:

$$\partial_t^2 a - c^2 \partial_z^2 a = 0$$

- Solution is Forward + Backward waves:

$$a(z,t) = a_F(ct - z) + a_B(ct + z)$$

- Independent solutions are 180° apart. Wave solutions form a spin one-half system (3-D requires bispinors).

Backward Forward

Factor the Wave Equation

$$\dot{a}(z,t) = \dot{a}_F(ct - z) + \dot{a}_B(ct + z)$$

- The second-order wave equations are:

$$\partial_t^2 a_B - c^2 \partial_z^2 a_B = 0$$
$$\partial_t^2 a_F - c^2 \partial_z^2 a_F = 0$$

- Equivalent to first-order matrix equation:

$$\partial_t \begin{bmatrix} \dot{a}_B \\ \dot{a}_F \end{bmatrix} - c \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \partial_z \begin{bmatrix} \dot{a}_B \\ \dot{a}_F \end{bmatrix} = 0$$

- The Dirac equation of quantum mechanics simply extends the matrix equation to vectors in 3D.

Factor the Wave Equation

- To accommodate rotation, separate positive and negative time derivative components:

$$\begin{aligned}\partial_t a(z,t) = & \ \dot{a}_{F+}(z-ct) \\ & - \dot{a}_{F-}(z-ct) \\ & + \dot{a}_{B+}(z+ct) \\ & - \dot{a}_{B-}(z+ct)\end{aligned}$$

Define the wave function (chiral representation):

$$\psi(z,t) = \begin{bmatrix} (\dot{a}_{B+})^{1/2} \\ (\dot{a}_{F-})^{1/2} \\ (\dot{a}_{F+})^{1/2} \\ (\dot{a}_{B-})^{1/2} \end{bmatrix}$$

Factor the Wave Equation

- The first-order 1-D wave equation is:

$$\partial_t(\psi^T\sigma_{\boldsymbol{z}}\psi) + c\partial_z(\psi^T\gamma^5\psi) = 0$$

where:

$$\psi^T\sigma_{\boldsymbol{z}}\psi = \begin{bmatrix}(\dot{a}_{B+})^{1/2}\\(\dot{a}_{F-})^{1/2}\\(\dot{a}_{F+})^{1/2}\\(\dot{a}_{B-})^{1/2}\end{bmatrix}^T \begin{pmatrix}1 & 0 & 0 & 0\\0 & -1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & -1\end{pmatrix} \begin{bmatrix}(\dot{a}_{B+})^{1/2}\\(\dot{a}_{F-})^{1/2}\\(\dot{a}_{F+})^{1/2}\\(\dot{a}_{B-})^{1/2}\end{bmatrix} = \partial_t a$$

$$\psi^T\gamma^5\psi = \begin{bmatrix}(\dot{a}_{B+})^{1/2}\\(\dot{a}_{F-})^{1/2}\\(\dot{a}_{F+})^{1/2}\\(\dot{a}_{B-})^{1/2}\end{bmatrix}^T \begin{pmatrix}-1 & 0 & 0 & 0\\0 & -1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{pmatrix} \begin{bmatrix}(\dot{a}_{B+})^{1/2}\\(\dot{a}_{F-})^{1/2}\\(\dot{a}_{F+})^{1/2}\\(\dot{a}_{B-})^{1/2}\end{bmatrix} = -c\partial_z a$$

Dirac Equation

1-D wave equation:

$$0 = \psi^T \sigma_{\boldsymbol{z}} \{\partial_t \psi + \ \psi^T c \gamma^5 \sigma_{\boldsymbol{z}} \partial_z \psi\} + \text{Transp.}$$

$$0 = \partial_t^2 Q_z - c^2 \partial_z^2 Q_z$$

3-D wave equation (replace z with i or j):

$$\psi^\dagger \sigma_{\boldsymbol{i}} [\partial_t \psi + c\, \gamma^5 \sigma_{\boldsymbol{j}} \partial_j \psi] + adj. = 0$$

<u>Same as electron!</u> Mass term drops out.

$$\psi^\dagger \sigma_{\boldsymbol{i}} [\partial_t \psi + c\, \gamma^5 \sigma_{\boldsymbol{j}} \partial_j \psi + iM\gamma^0 \psi] + adj. = 0$$

Wave function describes spin density.

Dirac Equation

Linear vector wave equations:

$$0 = \partial_t[\psi^\dagger \boldsymbol{\sigma}\psi] + c\nabla[\psi^\dagger\gamma^5\psi]$$
$$-ic[\nabla\psi^\dagger \times \gamma^5\boldsymbol{\sigma}\psi + \psi^\dagger\gamma^5\boldsymbol{\sigma} \times \nabla\psi]$$

$$0 = \partial_t^2\mathbf{Q} - c^2\nabla(\nabla \cdot \mathbf{Q}) + c^2\nabla \times \nabla \times \mathbf{Q}$$
$$= \partial_t^2\mathbf{Q} - c^2\nabla^2\mathbf{Q}$$

$$\mathbf{s} = \partial_t\mathbf{Q} = (1/2)\psi^\dagger\boldsymbol{\sigma}\psi$$

$$c\nabla \cdot \mathbf{Q} = -(1/2)\psi^\dagger\gamma^5\psi$$

$$c^2\nabla \times \nabla \times \mathbf{Q} = \frac{-ic}{2}[\nabla\psi^\dagger \times \boldsymbol{\sigma}\psi + \psi^\dagger\gamma^5\boldsymbol{\sigma} \times \nabla\psi]$$

Equation of Evolution

Nonlinear Dirac eqn. $\boldsymbol{s} \equiv \partial_t \mathbf{Q} = \left[\psi^\dagger \frac{\boldsymbol{\sigma}}{2} \psi\right]$:

$$\partial_t \psi =$$

$$-c\,\gamma^5 \boldsymbol{\sigma} \cdot \nabla\psi + \frac{\mathrm{i}}{2} \Delta w\,\gamma^0 \psi - \mathbf{u} \cdot \nabla\psi - \frac{\mathrm{i}}{2} \mathbf{w} \cdot \boldsymbol{\sigma}\psi$$

Propagation, rotation of wave velocity, convection, rotation of the solid medium.

Mass represents rotation of wave velocity.

c.f. David Hestenes, *Found. Phys.* 20,(10):1213-32,1990

Each term has a simple physical interpretation.

For plane waves, velocity rotation cancels rotation of the medium.

Lagrangian Density

Lagrangian density is the real part:
(w & Δw terms contribute twice to E-L eqn.)

$$\mathfrak{L} = \psi^\dagger i\,\partial_t\psi + c\psi^\dagger\gamma^5\boldsymbol{\sigma}\cdot i\nabla\psi$$
$$+\frac{1}{4}\Delta w\psi^\dagger\,\gamma^0\psi + \mathbf{u}\cdot\psi^\dagger i\nabla\psi - \frac{1}{4}\mathbf{w}\cdot\psi^\dagger\boldsymbol{\sigma}\psi$$

$$\mathbf{u} = \frac{1}{2\rho}\nabla\times\ \mathbf{s} = \frac{1}{2\rho}\nabla\times\left(\psi^\dagger\frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right);$$

$$\mathbf{w} = \frac{1}{2}\nabla\times\ \mathbf{u}\ = \frac{1}{4\rho}\nabla\times\nabla\times\left(\psi^\dagger\frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right)$$

$$\Delta w = -\frac{1}{2\rho c}f = -\ \frac{1}{4\rho}\nabla^2\left(\psi^\dagger\frac{\gamma^0}{\mathbf{2}}\psi\right)$$

Hamiltonian Density

$$\mathcal{H} =$$
$$-c\psi^\dagger\gamma^5\boldsymbol{\sigma}\cdot i\nabla\psi - \frac{\Delta w}{4}\psi^\dagger\gamma^0\psi - \mathbf{u}\cdot\psi^\dagger i\nabla\psi + \frac{\mathbf{w}}{4}\cdot\psi^\dagger\boldsymbol{\sigma}\psi$$
$$\xrightarrow{\text{plane wave}} \quad E \quad + \quad \frac{1}{2}\mathbf{f}\cdot\boldsymbol{\xi} \quad - \quad 0 \quad + \quad \frac{1}{2}\mathbf{w}\cdot\mathbf{s}$$
$$= U_R + K_R$$

Rotational potential energy: $U_R = E + \dfrac{1}{2}\mathbf{f}\cdot\boldsymbol{\xi}$

Rotational kinetic energy: $K_R = \frac{1}{2}\mathbf{w}\cdot\mathbf{s} = E - \frac{1}{2}\rho u^2$

Equal integrals for vectors & bispinors:

Vectors: Wave velocity × wave momentum = $2U$

Bispinors: Wave velocity × wave momentum = E

Dirac Equation

Dynamical quantities:

$$\mathbf{P} = -\left[\psi^\dagger i\nabla\psi\right] + \frac{1}{2}\nabla\times\left[\psi^\dagger\frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right]$$

$$\mathbf{J} = -\mathbf{r}\times\left[\psi^\dagger i\nabla\psi\right] + \left[\psi^\dagger\frac{\boldsymbol{\sigma}}{\mathbf{2}}\psi\right]$$

- Momentum consistent with symmetric energy-momentum tensor of GR [Ohanian 1986].
- Wave (orbital) and spin angular momenta.
- If the vacuum were not a (solid) medium, there would be no spin angular momentum.

Exclusion Principle & Potentials

- Wave superposition of "particles" A and B:

$$[\psi_A + \psi_B]^\dagger \boldsymbol{\sigma} [\psi_A + \psi_B] = \psi_A^\dagger \boldsymbol{\sigma} \psi_A + \psi_B^\dagger \boldsymbol{\sigma} \psi_B + \psi_A^\dagger \boldsymbol{\sigma} \psi_B + \psi_B^\dagger \boldsymbol{\sigma} \psi_A$$

- Interference terms cancel for "independent" particles. Eigenfunctions $\Longrightarrow$ exclusion principle:

$$\psi_A^\dagger \psi_B + \psi_B^\dagger \psi_A = 0$$

- Potentials derive from phase shifts introduced to maintain zero interference.

$$\psi_A^\dagger e^{-i\varphi_A} e^{i\varphi_B} \psi_B + c.c. = 0$$

Exclusion Principle & Potentials

Neglect nonlinear terms.

Transformation of momentum operator:

$$\psi_A^\dagger(-i\nabla)\psi_A \rightarrow \psi_A^\dagger e^{-i\varphi_A}(-i\nabla)e^{i\varphi_A}\psi_A$$
$$= \psi_A^\dagger(-i\nabla + \nabla\varphi_A)\psi_A$$

Transformation of Hamiltonian:

$$\psi_A^\dagger(\hbar\partial_t + \mathrm{i}H)\psi_A \rightarrow \psi_A^\dagger e^{-i\varphi_A}(\hbar\partial_t + \mathrm{i}H)e^{i\varphi_A}\psi_A$$
$$= \psi_A^\dagger(\hbar\partial_t + \mathrm{i}H + \mathrm{i}\hbar\partial_t\varphi_A + \mathrm{i}\hbar c\gamma^5\boldsymbol{\sigma}\cdot\nabla\varphi_A)\psi_A$$
$$\equiv \psi_A^\dagger(\hbar\partial_t + \mathrm{i}H + \mathrm{i}e\Phi - \mathrm{i}\gamma^5\boldsymbol{\sigma}\cdot e\mathbf{A})\psi_A$$

Electromagnetic potentials:

$$e\mathbf{A} = -\hbar\nabla\varphi_A; \quad e\Phi = \hbar\partial_t\varphi_A$$

Exclusion Principle & Potentials

Change of wave momentum (force): $\frac{d}{dt}P_i =$

$$\psi_A^\dagger\left((\partial_t\partial_i - \partial_i\partial_t)\varphi_A - c\gamma^5\sigma_j(\partial_i\partial_j - \partial_j\partial_i)\varphi_A\right)\psi_A$$

Multivalued phase $\Longrightarrow$ Non-commuting!

Electromagnetic variables:

$$e\mathbf{A} = -\hbar\nabla\varphi_A; \quad e\Phi = \hbar\partial_t\varphi_A$$

$$\rho_e = e\psi_A^\dagger\psi_A; \quad \mathbf{J} = e\psi_A^\dagger c\gamma^5\boldsymbol{\sigma}\psi_A$$

$$\mathbf{E} = -\frac{\hbar}{e}\nabla(\partial_t\varphi_A) + \frac{\hbar}{e}\partial_t(\nabla\varphi_A) = -\nabla\Phi - \partial_t\mathbf{A}$$

$$B_k = \epsilon_{kij}\partial_i A_j = -\frac{\hbar}{e}\epsilon_{kij}\partial_i\partial_j\varphi_A$$

$$\frac{d}{dt}\mathbf{P} = \rho_e\mathbf{E} + \mathbf{J}\times\mathbf{B}$$

Exclusion Principle & Potentials

Stokes' Law: $\oint \mathbf{A} \cdot d\boldsymbol{\ell} = \iint (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$

Macroscopic sytem:

Pure phase shift $\Rightarrow$ magnetic flux quantized:

$$\varphi_A = \left(m_\phi \phi - \omega t\right)$$

$$\mathbf{A} = -\frac{\hbar}{e} \nabla \varphi_A = -\frac{\hbar m_\phi}{er \ \sin\theta}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{\hbar m_\phi}{e} 2\pi \delta^{(2)}(x, y)$$

$$\oint \mathbf{A} \cdot d\boldsymbol{\ell} = -\frac{\hbar}{e} 2\pi m_\phi \xrightarrow{m_\phi = 1/2} \frac{h}{2e}$$

c.f. Hagen Kleinert, *Multivalued Fields in Condensed Matter, Electromagnetism, and Gravitation* (Chapter 4).

Exclusion Principle & Potentials

Radially weighted phase shift from particle:

$$\varphi_A = (m_\phi \phi - \omega t)\left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega r}$$

$$e\mathbf{A} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega r}\left\{\left(\frac{m_\phi}{r\ \sin\theta}\right)\hat{\phi} - \frac{m_\phi \phi - \omega t}{r^2}\hat{r}\right\}$$

Neglect radial term (no flux). Note $\hbar\omega = m_e c^2$

$$e\mathbf{B} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega}\left(\frac{m_\phi}{r^3\ \sin\theta}\right)\hat{\boldsymbol{\theta}}$$

Same flux as *e*⁻ dipole moment: $\mu_0 M / 2r$:

$$\oint \mathbf{A}\cdot d\boldsymbol{\ell} = -\frac{m_\phi e}{2\epsilon_0 \omega r} \xrightarrow{m_\phi = 1/2} \frac{\mu_0 \hbar e}{4 m_e r}$$

Exclusion Principle & Potentials

Radially weighted phase shift:

$$\varphi_A = (m_\phi \phi - \omega t)\left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)\frac{c}{\omega r}$$

Phase velocity: $v_\phi = \frac{\omega}{m_\phi}\, r \sin\theta \;\to\; 2\omega\, r \sin\theta$

Electric field (Note $\partial_t \varphi_A = -\mathbf{v}\cdot\nabla\varphi_A$):

$$eE_i = \hbar\left(\partial_i v_j\right)\partial_j \varphi_A + \hbar v_j\left(\partial_i \partial_j - \partial_j \partial_i\right)\varphi_A$$

Only first term contributes ($v_\phi = 0$ at $r = 0$)

$$\mathbf{E} = \left(\frac{e}{4\pi\epsilon_0 r^2}\right)\hat{\mathbf{r}}$$

Exclusion Principle & Potentials

“…the source’s Coulomb field results from a spinning of the quantized flux loop (about the center of the source) with an angular velocity equal to the Zitterbewegung frequency $\frac{2m_e c^2}{\hbar}$ *”*

- Herbert Jehle, Phys. Rev. 3(2):306-345, 1971

The factor of 2 comes from dividing $m_\phi = 1/2$.

The sign of the charge depends on the relative signs of s_z and v_ϕ.

Matter & Antimatter

Assume vector spherical harmonic "particles".
Bispinor angular quantum number is half of vector angular quantum number.

Vector: $\ell = 2N + 1 \Longrightarrow$ Bispinor: $\ell = \frac{2N+1}{2}$

Odd parity $\Longrightarrow$ Distinct mirror image $\Longrightarrow$ Fermion

Vector: $\ell = 2N \Longrightarrow$ Bispinor: $\ell = N$

Even parity $\Longrightarrow$ Same mirror image $\Longrightarrow$ Boson

(bosons = antiparticles except W+/-)

Assumption valid except for W+/-.

Quantum Electrodynamics

2-Particle Lagrangian:

$$\mathfrak{L} = \bar{\psi}_A[\gamma^\mu(\mathrm{i}\,\partial_\mu - A_\mu) - m_A]\psi_A + (A \to B)$$

Apply Dirac equation to B: $\mathfrak{L} =$

$$\bar{\psi}_A[\gamma^\mu(\mathrm{i}\,\partial_\mu - A_\mu) - m_A]\psi_A + J^\mu A_\mu - J^\mu A_\mu$$

Green's 1st identity, vector identities, etc.:

$$\int J^\mu A_\mu = \int (E^2 - B^2) = -\frac{1}{2}\int F^{\mu\nu}F_{\mu\nu}$$

Factor of ½ due to counting variations twice:

$$\mathfrak{L} = \bar{\psi}_A[\gamma^\mu(\mathrm{i}\,\partial_\mu - A_\mu) - m_A]\psi_A + \frac{1}{2}(E^2 - B^2) - J^\mu A_\mu$$

Solid Aether Model

- Lorentz-invariant wave equation (SR).
- “Gravity” is wave refraction (GR).
- Spin is the angular momentum of the medium.
- Mass $\Rightarrow$ spatial localization, velocity rotation.
- Spin & orbital angular momentum.
- Odd/even spherical harmonics: fermions/bosons
- Spatial reflection yields “antiparticles”.
- Exclusion principle and interaction potentials.
- Interpretation of electromagnetism.
- Relation between magnetic flux & electric charge.
- Wave uncertainty principle.

Publications

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- *The Wave Basis of Special Relativity* (Verum Versa, 2014)
- "Spin Angular Momentum and the Dirac Equation," **Electr. J. Theor. Phys. 12**(33):43-60, 2015.

- More at: www.ClassicalMatter.org

Related Publications

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