ADHM/Nahm construction of instanton/monopole (Part I)

Masashi Hamanaka
(Tagen)

2019.10.04
Intensive lecture at Chiba univ

1. Introduction

- **Non-Commutative (NC) spaces are defined by noncommutativity of spatial coordinates:**

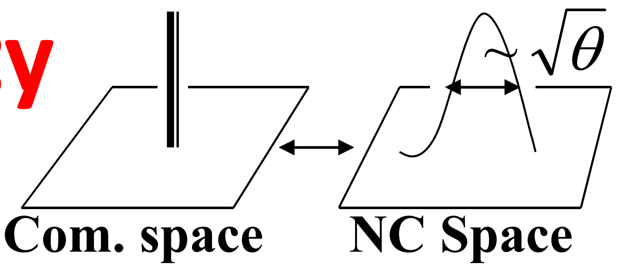
$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$ $\theta^{\mu\nu}$: NC parameter (real const.)

(cf. CCR in QM : $[q, p] = i\hbar$)

(→ ``space-space uncertainty relation'' →)

Resolution of singularity

(→ new physical objects)

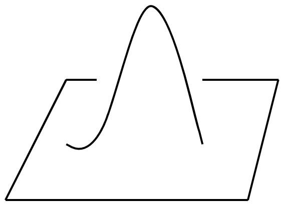


Ex) Resolution of small instanton singularity

(→ U(1) instantons) [Nekrasov-Schwarz]

Memo on instantons

- Instanton = instant + on ＝瞬間子



localized configuration in 4-dim.

- Descendant of the Twistor correspondence

  Anti-Self-Dual connection

  Holomorphic vector bdl.

- Instantons play important roles in geometry and physics. e.g. Donaldson invariant, Nekrasov partition fcn., AGT corresp. …

ASDYM eq. in 4-dim. with G=U(N)

- **ASDYM eq. (real rep.)**

$\mu,\nu = 1,2,3,4$

$$F_{12} = -F_{34}, \quad F_{13} = -F_{42}, \quad F_{14} = -F_{23}.$$

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu A_\nu - A_\nu A_\mu$$

Field strength

A_μ: **Gauge field (N × N anti-Hermitian)**

$$(\Leftrightarrow F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0, \quad F_{z_1 z_2} = 0 \quad (cpx.\,rep.))$$

- **There are two descripitions of NC extension:**
  - **Moyal-product formalism** (deformation quantization)
  - **Operator formalism** (Connes' theory)

NC ASDYM eq. with G=U(N) in Moyal

- NC ASDYM eq. (real rep.)

$$F^*_{01} = -F^*_{23}, \quad F^*_{02} = -F^*_{31}, \quad F^*_{03} = -F^*_{12}$$

$$(F^*_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu * A_\nu - A_\nu * A_\mu)$$

$$\theta^{\mu\nu} = \left[\begin{array}{cc|cc} 0 & \theta^1 & \multicolumn{2}{c}{O} \\ -\theta^1 & 0 & & \\ \hline \multicolumn{2}{c|}{O} & 0 & \theta^2 \\ & & -\theta^2 & 0 \end{array}\right]$$

(Spell: All products are Moyal products.)

$$f(x) * g(x) := f(x) \exp\left(\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu\right) g(x)$$

$$= f(x) \cdot g(x) + i\frac{\theta^{\mu\nu}}{2}\partial_\mu f(x) \partial_\nu g(x) + O(\theta^2)$$

Note: Coordinates and functions themselves are c-number-valued usual ones

Under the spell, we can calculate :

$$[x^\mu, x^\nu]_* := x^\mu * x^\nu - x^\nu * x^\mu$$

NC ASDYM eq. with G=U(N) in Moyal

- **NC ASDYM eq. (real rep.)**

$$F^*_{01} = -F^*_{23}\,,\qquad F^*_{02} = -F^*_{31}\,,\qquad F^*_{03} = -F^*_{12}$$

$$(F^*_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu * A_\nu - A_\nu * A_\mu)$$

$$\theta^{\mu\nu} = \left[\begin{array}{cc|cc} 0 & \theta^1 & \multicolumn{2}{c}{O} \\ -\theta^1 & 0 & & \\ \hline \multicolumn{2}{c|}{O} & 0 & \theta^2 \\ & & -\theta^2 & 0 \end{array}\right]$$

(Spell: All products are Moyal products.)

$$f(x) * g(x) := f(x)\exp\left(\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu\right) g(x)$$

$$= f(x)\cdot g(x) + i\frac{\theta^{\mu\nu}}{2}\partial_\mu f(x)\partial_\nu g(x) + O(\theta^2)$$

Under the spell, we can calculate :

$$[x^\mu, x^\nu]_* := x^\mu \cancel{\cdot} x^\nu + \frac{i}{2}\theta^{\mu\nu} - (x^\nu \cancel{\cdot} x^\mu - \frac{i}{2}\theta^{\mu\nu})$$

NC ASDYM eq. with G=U(N) in Moyal

- **NC ASDYM eq. (real rep.)**

$$F^{*}_{01} = -F^{*}_{23}, \quad F^{*}_{02} = -F^{*}_{31}, \quad F^{*}_{03} = -F^{*}_{12}$$

$$(F^{*}_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu * A_\nu - A_\nu * A_\mu)$$

$$\theta^{\mu\nu} = \left[\begin{array}{cc|cc} 0 & \theta^1 & & \\ -\theta^1 & 0 & \multicolumn{2}{c}{O} \\ \hline & & 0 & \theta^2 \\ \multicolumn{2}{c|}{O} & -\theta^2 & 0 \end{array}\right]$$

(Spell: All products are Moyal products.)

$$f(x) * g(x) := f(x) \exp\left(\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu\right) g(x)$$

$$= f(x)g(x) + i\frac{\theta^{\mu\nu}}{2}\partial_\mu f(x)\partial_\nu g(x) + O(\theta^2)$$

Under the spell, we get a theory on NC spaces:

$$[x^\mu, x^\nu]_* = i\theta^{\mu\nu}$$

Under the spell, the solution is deformed:

$$A(x,\theta) = A^{(0)}(x) + \theta A^{(1)}(x) + \theta^2 A^{(2)}(x) + \cdots,$$

2. Atiyah-Drinfeld-Hitchin-Manin Construction

based on duality for the instanton moduli space

4dim. ASDYang-Mills eq. (Difficult)

$$F_{z_1\bar{z}_1}+F_{z_2\bar{z}_2}=0$$

$$F_{z_1z_2}=0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM) (Easy)

$$[B_1,B_1^+]+[B_2,B_2^+]+II^+-J^+J=0$$

$$[B_1,B_2]+IJ=0$$

k × k Matrix eqs.

1:1

Sol.= instantons (G=U(N), C_2 =k) ⟷ Sol.=ADHM data (G=`U(k)’)

$$A_\mu : N\times N$$

$$B_{1,2}:k\times k,\quad I:k\times N,\quad J:N\times k$$

Gauge trf.:

$$A_\mu \mapsto g^{-1}A_\mu g+g^{-1}\partial_\mu g$$

$$g\in U(N)$$

Gauge trf.:

$$B_{1,2}\mapsto \tilde{g}^{-1}B_{1,2}\tilde{g},\quad \tilde{g}\in U(k)$$

$$I\mapsto \tilde{g}^{-1}I,\quad J\mapsto J\tilde{g}$$

Fourier-Mukai-Nahm transformation

Beautiful duality between instanton moduli on 4-tori and instanton moduli on the dual tori

4dim. ASD Yang-Mills eq. on a 4-torus

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

$\xrightarrow{G}$ / $\xleftarrow{F}$

4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$
$$\widetilde{F}_{\xi_1\xi_2} = 0$$

k × k PDE

Sol.=instantons (G=U(N), C_2 =k)

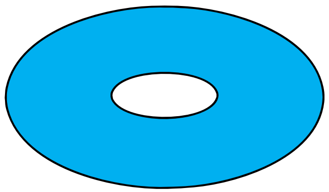
1:1

Sol.=the dual instantons (G=U(k), C_2=N)

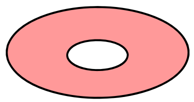
$$A_\mu : N \times N$$

$\Updownarrow$

$$\widetilde{A}_\mu : k \times k$$



Define the maps F & G, & G∘F=id. & F∘G=id.



On a 4-torus

On the dual 4-torus

Fourier-Mukai-Nahm transformation

Beautiful duality between instanton moduli on 4-tori and instanton moduli on the dual tori

4dim. ASDYang-Mills eq. on a 4-torus

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

1:1

4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$
$$\widetilde{F}_{\xi_1\xi_2} = 0$$

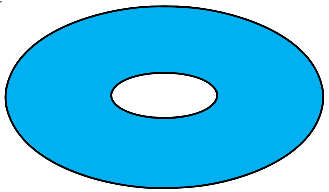
k × k PDE

Sol.=instantons (G=U(N), C_2 =k) $\longleftrightarrow$ Sol.=the dual instantons (G=U(k), C_2=N)

map F (Dirac eq.)

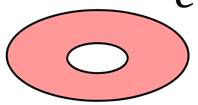
$$A_\mu(x) = \langle V, \partial_\mu V \rangle_\xi \longleftarrow \widetilde{A}_\mu(\xi) : k \times k$$

N × N



$$\nabla^+ V = \bar{e}^\mu \otimes \left(\frac{\partial}{\partial \xi^\mu} + \widetilde{A}_\mu - i x_\mu\right) V = 0$$

$$\bar{e}^\mu := (i\sigma_a, 1_2)$$



$$V : 2k \times \underline{N}$$

On a 4-torus $: x_\mu$

Family index thm.

On the dual 4-torus $: \xi_\mu$

Fourier-Mukai-Nahm transformation

Beautiful duality between instanton moduli on 4-tori and instanton moduli on the dual tori

4dim. ASDYang-Mills eq. on a 4-torus

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$

$$\widetilde{F}_{\xi_1 \xi_2} = 0$$

k × k PDE

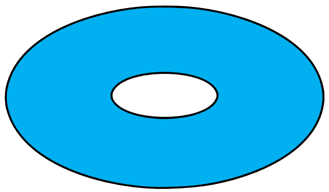
1:1

Sol.=instantons (G=U(N), C_2 =k) ⟷ Sol.=the dual instantons (G=U(k), C_2=N)

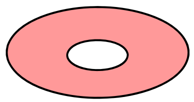
map G (Dirac eq.)

$$A_\mu(x) : N \times N \longrightarrow \widetilde{A}_\mu(\xi) = \left\langle \psi, \widetilde{\partial}_\mu \psi \right\rangle_\xi$$

k × k



$$\bar{e}_\mu D_\mu \psi = \bar{e}^\mu \otimes \left(\frac{\partial}{\partial x^\mu} + A_\mu - i\xi_\mu\right)\psi = 0$$



$$\psi : 2N \times \underline{k}$$

On a 4-torus $: x_\mu$

Family index thm.

On the dual 4-torus $: \xi_\mu$

Fourier-Mukai-Nahm transformation

Beautiful reciprocity between instanton moduli on 4-tori and instanton moduli on the dual tori

4dim. ASDYang-Mills eq. on a 4-torus

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

Dirac eq.

$$\xrightarrow{G}$$
$$\xleftarrow[F]{}$$

Dirac eq.

4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$
$$\widetilde{F}_{\xi_1\xi_2} = 0$$

k × k PDE

Sol.=instantons (G=U(N), C_2 =k)

1:1 (reciprocity)

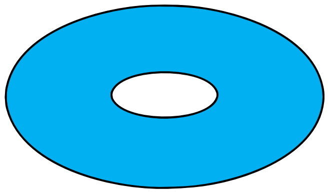
⇕

Define the maps F & G, & G∘F=id. & F∘G=id.

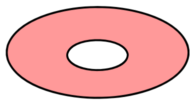
Sol.=the dual instantons (G=U(k), C_2=N)

$A_\mu : N \times N$

$\widetilde{A}_\mu : k \times k$



On a 4-torus



On the dual 4-torus

Fourier-Mukai-Nahm trf. (radii of the torus→∞)

reciprocity between instanton moduli on R^4 and instanton moduli on ``1pt.''

[cf. van Baal, hep-th/9512223]

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

0dim. ASD Yang-Mills eq.

$$\tilde{F}_{\mu\nu} := \cancel{\partial_\mu \tilde{A}_\nu} - \cancel{\partial_\nu \tilde{A}_\mu} + [\tilde{A}_\mu, \tilde{A}_\nu]$$

$$\tilde{F}_{\xi_1\bar{\xi}_1} + \tilde{F}_{\xi_2\bar{\xi}_2} = 0$$
$$\tilde{F}_{\xi_1\xi_2} = 0$$

Matrix eq. !

k × k ~~PDE~~

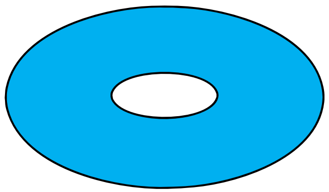
1:1

Sol.=instantons (G=U(N), C_2 =k) ⟷ Sol.=``dual instantons'' (G=U(k), ``C_2=N'')

$$A_\mu = V^+ \partial_\mu V$$

map F (0dim Dirac eq.)

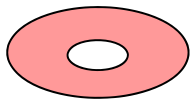
$$\tilde{A}_\mu : k \times k$$



$$\nabla^+ V = \bar{e}^\mu \otimes \left(\cancel{\frac{\partial}{\partial \xi^\mu}} + \tilde{A}_\mu - i x_\mu\right) V = 0$$

Matrix eq. !

$$V : 2k \times \underline{N}$$



On a 4-torus→R^4

Linear alg.

On the dual 4-torus→1 pt.

Atiyah-Drinfeld-Hitchin-Manin (ADHM) Construction

based on the following reciprocity

4dim. ASDYang-Mills eq.

ADHM eq. (≒0dim. ASDYM)

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

$$\xrightarrow{\text{G(4dim D.eq.)}}$$
$$\xleftarrow[\text{F(0dim D.eq.)}]{}$$

$$[B_1, B_1^+] + [B_2, B_2^+] + I\,I^+ - J^+J = 0$$
$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

Sol.=instantons
(G=U(N), C_2 =k)

1:1

$$\longleftrightarrow$$

Sol.=ADHM data
(G=`U(k)')

$$A_\mu : N \times N$$

Proved in the same way as the Nahm trf.

$$B_{1,2} : k \times k,$$
$$I : k \times N, \quad J : N \times k$$

Other limits and related works

- 3radii→∞&1radius→0 : [Nahm, Hitchin, Nakajima,…]

  monopole on 3-dim ←→ Nahm data on 1-dim
- 2radii→∞&2radii→0 :

  Hitchin system on 2dim ←→ Hitchin system 2dim
- 3radii→∞&finite1radius : [Hurtubise-Murray,…]

  caloron on R^2x S^1 ←→ Nahm data on S^1
- 2radii→∞&finite2radii : : [Jardim, Mochizuki, …]

  instanton on R^2x T^2 ←→ Hitchin system on T^2
- 1radii→∞&finite3radii : : [Charbonneau, Yoshino,…]

  instanton on R x T^3 ←→ monopole on T^3

Atiyah-Drinfeld-Hitchin-Manin (ADHM) Construction

based on the following reciprocity

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

G(4dim D.eq.)

F(0dim D.eq.)

ADHM eq.. (≒0dim. ASDYM)

RHS is in fact $[z_1, \bar{z}_1] + [z_2, \bar{z}_2]$

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$

$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

Sol.=instantons
(G=U(N), C_2 =k)

$$A_\mu : N \times N$$

1:1

Proved in the same way as the Nahm trf.

Sol.=ADHM data
(G=`U(k)')

$$B_{1,2} : k \times k,$$

$$I : k \times N, \quad J : N \times k$$

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$

$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

1:1

BPST instanton
(G=U(2), C_2 =1)

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta_{\mu\nu}^{ASD}$$

Sol.=ADHM data
(G=`U(1)’)

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0), \; J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton (N=2, k=1)

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$
$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

1:1

BPST instanton (G=U(2), C_2 =1) ⟷ Sol.=ADHM data (G=`U(1)')

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta_{\mu\nu}^{ASD}$$

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0), \ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

``The first time abstract modern mathematics had been of any use!" (Polyakov)

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^+] + [B_2, B_2^+] + I\,I^+ - J^+ J = 0$$

$$[B_1, B_2] + IJ = 0$$

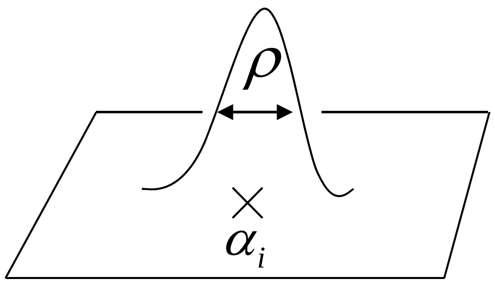
k × k matrix eq.

1:1

BPST instanton (G=U(2), C2 =1) ⟷ **Sol.=ADHM data (G=`U(1)’)**

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta_{\mu\nu}^{ASD}$$



position

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0), \; J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^{+}] + [B_2, B_2^{+}] + I\,I^{+} - J^{+}J = 0$$
$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

1:1

$$\longleftrightarrow$$

BPST instanton (G=U(2), C_2 =1)

Sol.=ADHM data (G=`U(1)’)

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

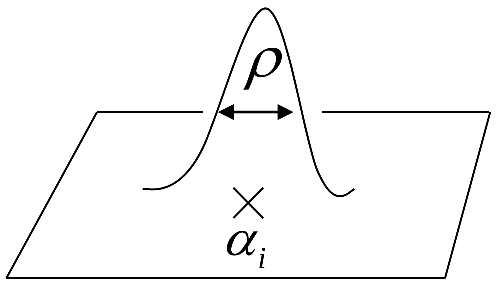
$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta_{\mu\nu}^{ASD}$$

position

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size



$\rho \to 0$: singular

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

ADHM eq. (≒0dim. ASDYM)

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

$$\mu_R = [B_1, B_1^+] + [B_2, B_2^+] + I\,I^+ - J^+ J = 0$$

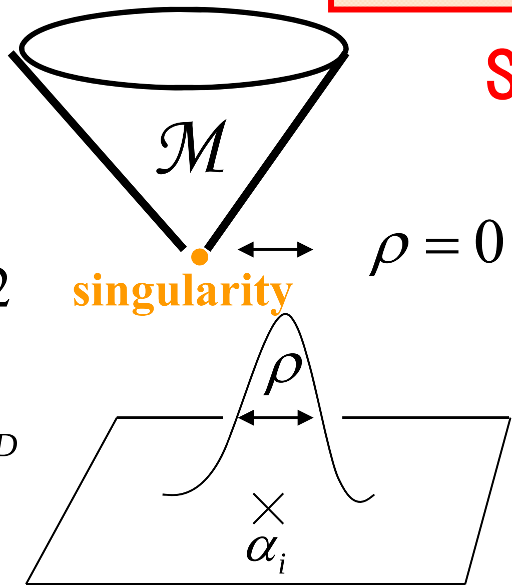
$$\mu_C = [B_1, B_2] + IJ = 0$$

k × k matrix eq.

BPST instanton
(G=U(2), C_2 =1)

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta_{\mu\nu}^{ASD}$$



$\rho = 0$

$\rho \to 0$: singular

Sol.=ADHM data
(G=`U(1)')

position

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

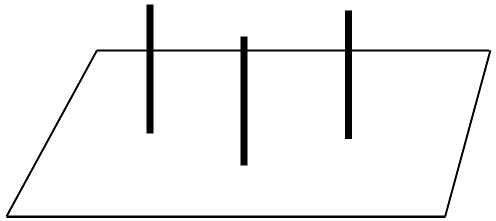
size

The instanton moduli space

- **in commutative spaces**

$$\overline{\mathrm{M}}_{k,N} = \mathrm{M}_{k,N} \cup (\mathrm{M}_{k-1,N} \times R^4) \cup (\mathrm{M}_{k-2,N} \times Sym^2 R^4) \cup \ldots$$
$$\cup (\mathrm{M}_{1,N} \times Sym^{k-1} R^4) \cup \underbrace{Sym^k R^4}_{\text{Symmetric Product}}$$

k size-zero instantons



- **In noncommutative spaces**

$$\overline{\mathrm{M}}_{k,N} = \mathrm{M}_{k,N} \cup (\mathrm{M}_{k-1,N} \times R^4) \cup (\mathrm{M}_{k-2,N} \times \widetilde{Sym^2 R^4}) \cup \ldots$$
$$\cup (\mathrm{M}_{1,N} \times \widetilde{Sym^{k-1} R^4}) \cup \underbrace{\widetilde{Sym^k R^4}}_{\text{Hilbert Scheme}} \longleftrightarrow \text{k U(1) instantons}$$

- **dim** $\mathrm{M}_{k,N} = 2 \cdot 2k^2 + 2 \cdot 2Nk - 3k^2 - k^2 = 4Nk$ 算数

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) NC BPST instanton（N=2, k=1）

NC ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

NC ADHM eq.

$$\mu_R = [B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = \zeta$$

$$\mu_C = [B_1, B_2] + IJ = 0$$

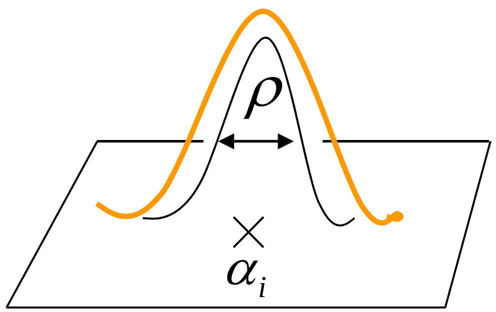
k × k matrix eq.

NC BPST instanton
(G=U(2), C_2=1)

By calculation of TrFΛF

$A_\mu, F_{\mu\nu}$: exact sol.

Do k×k ADHM data give Instanton number k in general ? (We prove this.)



$\rho \to 0$: regular!

Sol.=ADHM data
(G=\`U(1)’)

position

$$B_{1,2} = \alpha_{1,2},$$

$$I = (\sqrt{\rho^2 + \zeta}, 0),\quad J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size

Fat by ζ !

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) NC BPST instanton（N=2, k=1）

NC ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

NC ADHM eq.

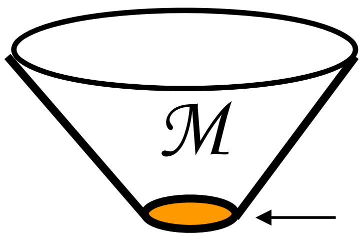
$$\mu_R = [B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = \zeta$$

$$\mu_C = [B_1, B_2] + IJ = 0$$

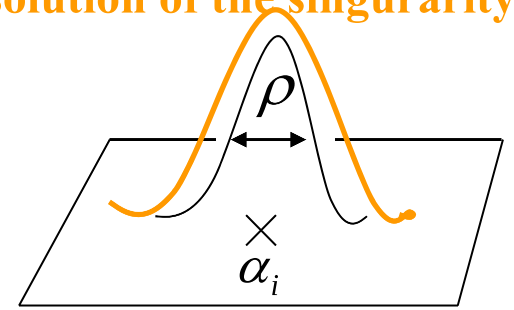
k × k marix eq.

NC BPST instanton
(G=U(2), C_2 =1)

$A_\mu , F_{\mu\nu}$ **: exact sol.**



Resolution of the singurarity!



$\rho \to 0$: regular!

Sol.：ADHM data
(G=`U(1)’)

position

$$B_{1,2} = \alpha_{1,2},$$

$$I = (\sqrt{\rho^2 + \zeta}, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size Fat by ζ !