Overview of Classical and Quantum Integrability in Four Dimensions

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Overview

Over the last few years there's been substantial progress by a number of groups in understanding classical and quantum integrable systems in four dimensions. This has been achieved by exploiting twistor methods.

It was initiated by the paper 'Quantizing Holomorphic Theories on Twistor Space' by Kevin Costello. **[Costello, 21]**

Therein, he studies two examples of classically integrable theories in four dimensions:

- Self-dual Yang-Mills theory (sdYM)
- 4d Wess-Zumino-Witten model (WZW_4)

We will meet these later on.

There have been a number of related developments in different directions:

- Hidden symmetries of 4d integrable theories can be identified with celestial chiral algebras **[Adamo et al., 21; Costello, Paquette, 22;. . .]** discovered through the celestial holography program. **[Guevara et al. 21; Strominger, 21;. . .]**
- These hidden symmetries can be leveraged to compute Yang-Mills loop amplitudes. **[Costello, 23]**
- Many of the results for self-dual Yang-Mills extend to self-dual Einstein gravity. **[RB et al., 22; RB, 22]** Much more on this tomorrow!

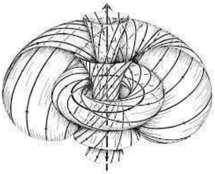
- The Ward conjecture relates the sdYM equations to a range of lower dimensional integrable systems. Performing this reduction on twistor space gives 4d Chern-Simons theory. **[RB, Skinner, 20]**.
- Some results have been extended to non-trivial gauge **[Garner, Paquette, 23]** and gravitational backgrounds **[Crawley et al., 23; RB et al., 23]**.
- A remarkable holographic duality has been obtained between an exotic theory know as Mabuchi gravity coupled to WZW_4 on Burns space, and a holomorphic theory with target the ADHM quiver. **[Costello et al., 22, 23]**

In this talk I will review many of these developments.

Twistor Theory

Twistors were first introduced by Roger Penrose as a means of encoding the conformal structure of flat space-time in a three dimensional complex manifold, now known as twistor space.

Massless free fields on space-time are described by holomorphic data on twistor space. **[Penrose, 67; Penrose, MacCallum, 72]**



Picture credit: **[Atiyah et al., 2017]**

The twistor space $\mathbb{PT}$ of Euclidean $\mathbb{R}^4$ is the total space of the rank two holomorphic vector bundle

$$\mathcal{O}(1) \oplus \mathcal{O}(1) \to \mathbb{CP}^1 \,.$$

Points in complexified space-time $x \in \mathbb{C}^4$ parametrise holomorphic lines $\mathbb{CP}^1_x \to \mathbb{PT}$.

Those lines corresponding to points in Euclidean signature are fixed under an antiholomorphic involution $\sigma : \mathbb{PT} \to \mathbb{PT}$ acting as the antipodal map on $\mathbb{CP}^1$.

σ has no fixed points; therefore, given a point in $\mathbb{PT}$ we can construct the $\mathbb{CP}^1$ passing through both it and it's image under σ. This defines a smooth diffeomorphism

$$\mathbb{PT} \cong \mathbb{R}^4 \times S^2 \,.$$

Let's be explicit about this in coordinates.

We write z for the inhomogeneous holomorphic coordinate on the $\mathbb{CP}^1$ base (away from the south pole). On the $\mathcal{O}(1) \oplus \mathcal{O}(1)$ fibres we use the coordinates $v^{\dot\alpha} = (v^{\dot 0}, v^{\dot 1})$.

In the other patch (away from the north pole) have

$$\tilde z = -1/z\,, \qquad \tilde v^{\dot\alpha} = v^{\dot\alpha}/z\,.$$

Points $x^\mu = (u^{\dot\alpha}, \tilde u^{\dot\alpha}) \in \mathbb{C}^4$ in complexified space-time are recovered as holomorphic lines $\mathcal{L}_x : \mathbb{CP}^1 \to \mathbb{PT}$.

$$v^{\dot\alpha} = u^{\dot\alpha} + z\tilde u^{\dot\alpha}\,.$$

Since the $v^{\dot\alpha}$ are sections of $\mathcal{O}(1)$, this is holomorphic in both patches.

Points in space-time are null separated if their corresponding twistor lines intersected; in this way the complex geometry of twistor space encodes the conformal structure of space-time.

Example

The twistor line of the point $x^\mu = 0$ is the zero section. It intersects the twistor line of the point $x^\mu = (u^{\dot\alpha}, \tilde{u}^{\dot\alpha})$ when

$$u^{\dot 0} + z\tilde{u}^{\dot 0} = u^{\dot 1} + z\tilde{u}^{\dot 1} = 0\,.$$

For both of these to be satisfied we must have

$$u^{\dot 0}\tilde{u}^{\dot 1} - u^{\dot 1}\tilde{u}^{\dot 0} = 0\,.$$

This is the complexified light cone through the origin.

Penrose subsequently showed that any four dimensional conformal manifold $\mathcal{M}$ with sd Weyl tensor admits a three complex dimensional twistor space $\mathcal{PT}$. **[Penrose, 76]** When the conformal class contains a Ricci flat representative the corresponding twistor space has extra structure.

Ward showed that a similar correspondence exists between solutions of the sd Yang-Mills equations and holomorphic vector bundles on twistor space which are trivial on twistor lines. **[Ward, 77; Atiyah, Ward, 77]**

Remark

This is the basis of the ADHM construction of instantons. **[Atiyah et al., 78]**

Self-dual Yang-Mills

Let A be a connection on a principal G-bundle P over a 4d Riemannian manifold $\mathcal{M}$. (For G a complex semisimple Lie group with Lie algebra $\mathfrak{g}$.)

The self-dual Yang-Mills (sdYM) equations for a connection $A \in \Omega^1(\mathcal{M};\mathfrak{g})$

$$F(A) = \mathrm{d}A + \frac{1}{2}[A, A] = *F(A)\,.$$

Introducing a Lagrange multiplier field $B \in \Omega^2_-(\mathcal{M};\mathfrak{g})$ can define sdYM as both as a classical and perturbative quantum field theory using the action

$$S_{\mathrm{sdYM}}[B, A] = \int_{\mathcal{M}} \mathrm{tr}\big(B\, F(A)\big)\,.$$

As a classical field theory sdYM lifts to a holomorphic theory on twistor space with dynamical fields

$$a \in \Omega^{0,1}(\mathbb{PT};\mathfrak{g})\,, \qquad b \in \Omega^{3,1}(\mathbb{PT};\mathfrak{g})\,.$$

The Dolbeault operator $\bar{\partial} + a$ determines an almost complex structure on a G-bundle over twistor space. It defines an integrable complex structure, and therefore a holomorphic G-bundle, if

$$f(a) = \bar{\partial} a + \frac{1}{2}[a,a] = 0\,.$$

A suitable action principle is **[Witten, 03; Mason; 05]**

$$S_{\mathrm{hBF}}[b,a] = \frac{1}{(2\pi \mathrm{i})^3}\int_{\mathcal{PT}} \mathrm{tr}\big(b\, f(a)\big)\,.$$

To get back to space-time we first gauge fix along twistor lines, setting

$$\mathcal{L}_x^* a = 0\,.$$

Then can integrate out the components of b whose $(0,1)$-form part points in the space-time directions to learn that the remaining components of a are holomorphic.

$$a = \pi^{0,1}_{\mathcal{PT}}(A)$$

for $A \in \Omega^1(\mathcal{M};\mathfrak{g})$ and $\pi^{0,1}_{\mathcal{PT}}$ the projection operator onto $(0,1)$-form part of the complex structure on $\mathcal{PT}$. The Lagrange multiplier field B is

$$B \propto \int_{\mathcal{L}_x} b\,.$$

4d Wess-Zumino-Witten model

A closely related model is the 4d Wess-Zumino-Witten model (WZW_4).

It's equation of motion are a partial gauge fixing of the sdYM equations. First we choose a Kähler structure on $\mathcal{M}$ compatible with the metric. The sdYM equations then decompose into

$$F^{2,0}(A) = F^{0,2}(A) = \omega\, F^{1,1}(A) = 0\,.$$

The first equation allows us to attain the gauge $A^{1,0} = 0$, the second tells us that $A^{0,1} = \sigma^{-1}\bar{\partial}_{\mathcal{M}}\sigma$ is pure gauge. Yang's equation is what's left

$$\omega\, \partial(\sigma^{-1}\bar{\partial}_{\mathcal{M}}\sigma) = 0\,.$$

A suitable action for WZW_4 is **[Losev et al., 95]**

$$S_{\text{WZW}_4}[\sigma] = \int_{\mathcal{M}} \text{tr}(J \wedge *J) + \frac{1}{6}\int_{\mathcal{M}\times[0,1]} \omega \wedge \text{tr}\big(\tilde{J}^3\big)$$

where

$$J = \sigma^{-1}\text{d}_{\mathcal{M}}\sigma \,, \qquad \tilde{J} = \tilde{\sigma}^{-1}\text{d}_{[0,1]\times\mathcal{M}}\tilde{\sigma} \,,$$

for

$$\tilde{\sigma} : \mathcal{M} \times [0,1] \to G$$

is a smooth homotopy from σ to the identity map.

Note that this theory is not Lorentz invariant. In flat space $\text{SO}(4)$ is broken to $\text{U}(2)$ by the choice of Kähler structure.

As a classical field theory WZW_4 lifts to a holomorphic theory on twistor space with dynamical field

$$a \in \Omega^{0,1}(\mathbb{PT};\mathfrak{g})\,.$$

As before, the Dolbeault operator $\bar{\partial} + a$ determines an integrable complex structure on a G-bundle over twistor space if

$$f(a) = \bar{\partial}a + \frac{1}{2}[a,a] = 0\,.$$

We now use the alternative action principle

$$S_{\mathrm{hCS}}[a] = \frac{1}{(2\pi\mathrm{i})^3}\int_{\mathcal{PT}} \Omega\,\mathrm{tr}\left(\frac{1}{2}a\,\bar{\partial}a + \frac{1}{6}a\,[a,a]\right).$$

This is holomorphic Chern-Simons theory.

Twistor space is not Calabi-Yau; however, it does inherit a global section of $\Omega^{3,0}(\mathbb{PT}, \mathcal{O}(4))$ from its embedding into $\mathbb{CP}^3$.

Introducing divisors at $z = 0, \infty$ can choose

$$\Omega = \frac{\mathrm{d}z\, \mathrm{d}v^{\dot{0}}\, \mathrm{d}v^{\dot{1}}}{z^2}\,.$$

Varying the action now generates boundary terms, which can be eliminated by requiring that a vanishes to first order at $z = 0, \infty$.

By distinguishing the points $z = 0, \infty$, we pick a preferred Kähler structure on $\mathcal{M}$. This determines the one used in writing down WZW$_4$.

To get back to space-time we must fix the gauge. We proceed as before, and try to fix the gauge $\mathcal{L}_x^* a = 0$, but this is obstructed by our boundary conditions. The best we can do is get

$$\mathcal{L}_x^* a = \tilde{\sigma}^{-1} \bar{\partial}_{\mathcal{L}_x} \tilde{\sigma} \,.$$

The gauge invariant data stored in $\tilde{\sigma}$ is the holomorphic Wilson line from $z = 0$ to ∞

$$\mathcal{W}_{0\to\infty} = \tilde{\sigma}^{-1}|_{z=\infty} \tilde{\sigma}|_{z=0} = \sigma \,.$$

Integrating out the remaining component of a we learn that

$$a = \tilde{\sigma}^{-1} (\bar{\partial}_{\mathcal{PT}} - \pi^{0,1}_{\mathcal{PT}} \bar{\partial}_{\mathcal{M}} \sigma \sigma^{-1}) \tilde{\sigma} \,.$$

Can now directly integrate over twistor fibres to get WZW_4.

Integrability

The sdYM equations are integrable.

Their solutions admit the action of an infinite dimensional hidden symmetry algebra. **[Ward, 77; Chau et al, 83; Chakravarty, Mason, Newman, 88]**

This is closely related to the chiral algebra of positive-helicity asymptotic symmetries arising from soft theorems in YM. **[Guevara et al., 21; Strominger, 21; Adamo, 21]**

It naturally arises as the universal holomorphic surface defect on twistor space.

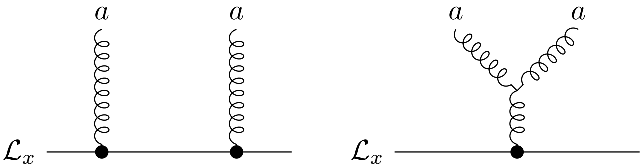
Consider the most general coupling between holomorphic Chern-Simons theory, and a surface defect supported on the twistor line $\mathcal{L}_x$

$$\sum_{m,n\in\mathbb{Z}_{\geq 0}} \frac{1}{m!n!} \int_{\mathcal{L}_x} \frac{\mathrm{d}z}{2\pi\mathrm{i}} \left\langle j[m,n](z), \partial^m_{v^{\dot{0}}} \partial^n_{v^{\dot{1}}} a \right\rangle .$$

Only holomorphic derivatives are allowed, as the antiholomorphic derivatives are exact in BRST cohomology.

BRST invariance of this defect imposes conditions on the operator products of the $j_{\mathsf{a}}[m,n](z)$.

At tree level the BRST variation of the defect receives contributions from these diagrams.



For these to cancel the generators must have operator products

$$j_{\mathsf{a}}[p,q](z) j_{\mathsf{b}}[r,s](0) \sim \frac{f_{\mathsf{ab}}{}^{\mathsf{c}}}{z} j_{\mathsf{c}}[p+r,q+s](0)\,.$$

This is the hidden symmetry algebra, or celestial chiral algebra (CCA), of sdYM.

The mode algebra is the universal enveloping algebra (UEA) of the Lie algebra of regular maps $\mathbb{C}^* \times \mathbb{C}^2 \to \mathfrak{g}$. The CCA is also equipped with distinguished vacuum modules at $z = 0, \infty$ determined by the boundary conditions on a.

Remark

sdYM is very similar; however, we can now couple to the b fields on twistor space, and so the CCA is extended by a copy of the adjoint.

Since the fields a, b are not required to vanish at $z = 0, \infty$, the vacuum module differs.

Celestial chiral algebras

This role of the CCA in governing gauge theory amplitudes has been expounded as part of the celestial holography program.

The operator product captures the holomorphic collinear singularities of tree gluon amplitudes in full Yang-Mills theory. **[Guevara et al., 21; Strominger, 21]**

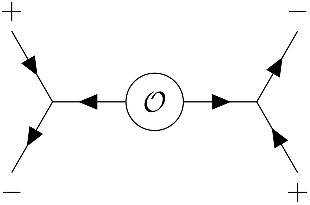
This remains true for the 1-loop all-plus gluon amplitudes. **[Ball et al., 21]**

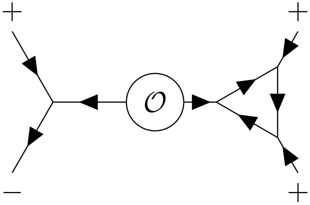
This was refined by Costello and Paquette, who showed that the CCA governs the collinear singularities of form factors. **[Costello, Paquette, 23]**

Form factors are amplitudes in the presence of a local operator. A particularly interesting local operator is

$$\mathcal{O} = \frac{1}{2}\mathrm{tr}\left(B^2\right).$$

Corresponding tree and 1-loop form factors receive contributions from the following two diagrams respectively.





There is then a correspondence:

local operators	$\leftrightarrow$	conformal blocks
form factors	$\leftrightarrow$	chiral algebra correlators

Remark

Once quantum effects have been taken into account this can be leveraged to compute Yang-Mills amplitudes as chiral algebra correlators.

Indeed, Costello exploited this to bootstrap the single trace term of the 2-loop all-plus amplitude in certain theories of QCD. **[Costello, 23]**

Questions

- Can this approach be leveraged to compute amplitudes beyond tree MHV, 1-loop mostly-plus, and 2-loop all-plus?
- How does this work in gravity, where there are no local operators? (See tomorrow's talk.)

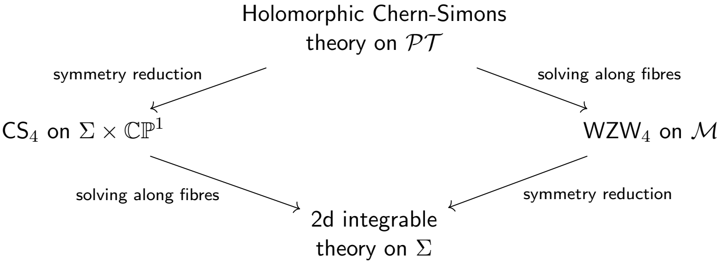
4d Chern-Simons

Ward conjectured that many, and perhaps all, integrable systems arise as symmetry reductions of the sdYM equations. **[Ward, 85]**

On the other hand, 4d Chern-Simons theory (CS_4) has been proposed as an organizing principle for 2d integrable systems. **[Nekrasov, 96; Costello, 13; Costello et al., 17, . . .]**

These two perspectives are connected via the twistor correspondence. **[RB, Skinner, 20]**

This is most easily understood by considering the following diagram:



Example

The principal chiral model arises as a symmetry reduction of the sdYM equations by translations in a spacelike plane.

Example

Lifting this group of spacetime translations $\mathbb{PT}$ and performing the reduction there (or compactifying on a torus and taking the small volume limit) gives

$$S_{\text{CS}_4}[\tilde{a}] = \frac{1}{2\pi \mathrm{i}} \int_{\mathbb{R}^2 \times \mathbb{CP}^1} \omega \, \mathrm{tr}\left(\frac{1}{2} \tilde{a} \, (\mathrm{d}_{\mathbb{R}^2} + \bar{\partial}_{\mathbb{CP}^1}) \tilde{a} + \frac{1}{6} \tilde{a} \, [\tilde{a}, \tilde{a}] \right),$$

where

$$\omega = \frac{(z - z_0)(z\bar{z}_0 + 1)}{(1 + |z_0|)^2} \frac{\mathrm{d}z}{z^2}.$$

The field $\tilde{a}$ acquires simple poles in its $(1,0)$- and $(0,1)$-form parts at $z = z_0$ and $-1/\bar{z}_0$ respectively.

This is the CS_4 description of the principal chiral model on $\mathbb{R}^2$.

Questions

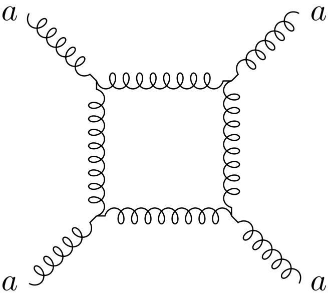
- Can this be leveraged to obtain CS_4 descriptions of affine Toda theories, including sine-Gordon? Can we connect it to known CS_4 descriptions of KdV and non-linear Schrödinger?
- Can this be used to obtain relations between the hidden symmetry algebras. For example, how is the CCA of sdYM related to the Yangian?

Quantizing holomorphic theories on twistor space

Why should we seek to quantize theories on twistor space?
Twistorial theories have remarkable properties. **[Costello, 21]**

- Correlation functions of a twistorial theory are meromorphic with poles on the complexified light cone.
- RG flow is periodic with period $2\pi\mathrm{i}$.
- They admit infinite dimensional hidden symmetry algebras playing a role analogous to the Yangian for the principal chiral model. Can be leveraged to compute form factors.
- Can be used to quantize apparently non-renormalizable theories.

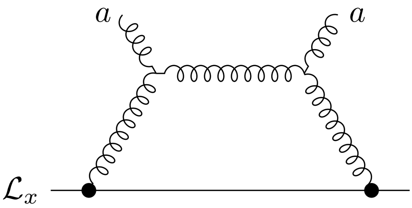
There is a catch: both holomorphic BF and Chern-Simons theory suffer from gauge anomalies.



Since twistor space is 6 real dimensional, these can be attributed to the failure of the above box diagram to be gauge invariant.

On space-time this does not represent a gauge anomaly. Instead, it can be identified with the 4-point 1-loop amplitude.

Instead should be interpreted as a global anomaly in the infinite hidden symmetry. The operators products of the CCA are corrected by the following diagram.



The deformed OPEs do not form a consistent vertex algebra: associativity of the operator product is violated.

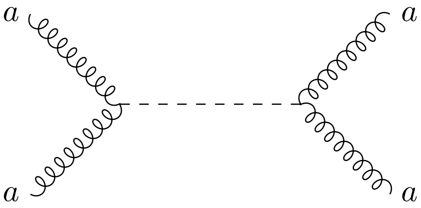
To restore associativity of the CCA we must cancel the gauge anomaly on twistor space. Equivalently, the 1-loop amplitudes on space-time must be eliminated.

Can couple to an appropriate number of fermions to get quantum integrable sdQCD.

Example

- $\mathrm{SU}(N)$ sdYM coupled to 8 fundamental and 1 antisymmetric Dirac fermions.
- $\mathrm{Sp}(2N)$ sdYM coupled to 16 fundamental and 1 antisymmetric Weyl fermions.

Alternatively can couple to an exotic axion with 4[th]-order kinetic term. This cancels the amplitudes on spacetime, or equivalently the anomaly on twistor space, by tree level exchange.



Requires $\mathrm{tr}_{\mathrm{ad.}}(X^4) \propto \mathrm{tr}(X^2)^2$, which holds for $G = \mathrm{SU}(2)$, $\mathrm{SU}(3)$, $\mathrm{SO}(8)$ and all exceptional Lie groups.

In both cases can check that associativity is restored.

Twistor strings

Cancelling the anomalies in holomorphic Chern-Simons theory is a little trickier. It's not 1-loop exact, and so can suffer from higher loop anomalies.

Fortunately holomorphic Chern-Simons theory arises as the open string sector of the string field theory of the B-model topological string. **[Witten, 92]**

Remark

One approach to getting an anomaly free theory is to take the Calabi-Yau supermanifold $\mathbb{CP}^{3|4}$ as the target (strictly $\mathbb{PT}^{3|4}$ subspace), and introduce a stack of (almost) space-filling branes.

This gives $\mathcal{N} = 4$ super Yang-Mills coupled to superconformal gravity. **[Witten, 03; Witten, Berkovitz, 04]**

Costello & Li showed that the open-closed B-model can be consistently quantized in a suitable background of space-filling branes. **[Costello, Li, 12; 15; 20]**

There a two options for the closed string sector: type I or type II Kodaira-Spencer gravity.

Costello & Li show that the anomalies vanish to all loop orders if the gauge supergroup of the holomorphic Chern-Simons theory is $\mathrm{OSp}(8+2N|N)$ for type I or $\mathrm{GL}(N|N)$ for type II.

Only in the type I case is it possible to have an ordinary Lie group as the gauge symmetry, in which case it is $\mathrm{SO}(8)$.

This construction must be modified to overcome the fact that twistor space is not Calabi-Yau. This leads to an anomaly free theory of type I Kodaira-Spencer gravity coupled to $\mathrm{SO}(8)$ holomorphic Chern-Simons theory on twistor space. **[Costello, 21]**

Kodaira-Spencer gravity descends to Mabuchi gravity on space-time. The dynamical field is a Kähler potential, and its equations of motion imply that the resulting Kähler metric is scalar flat. This can be viewed as a partial gauge fixing of a subsector of conformal gravity. **[Costello et al., 23]**

We've already seen that holomorphic Chern-Simons theory descends to WZW_4.

Costello & Li introduced a notion of twisted holography describing twisted subsectors of physical string dualities. Costello & Gaiotto showed that this could be applied to the duality between type II superstring theory on $\mathrm{AdS}_5 \times S_5$ and $\mathcal{N} = 4$ super Yang-Mills. **[Costello, Gaiotto, 21]**

The twisted duality relates the B-model topological string on the deformed conifold to a 2d holomorphic theory with target the ADHM quiver.

Applying this duality to the B-model on twistor space gives a duality between Mabuchi gravity coupled to $\mathrm{SO}(8)$ WZW_4 on Burns space and a 2d holomorphic theory with target the $\mathrm{SO}(8)$ ADHM quiver. **[Costello et al., 22, 23]**

Questions

- What are the defining relations of CCAs to all orders in $\hbar$? Can they be defined non-perturbatively, or through a compact set of relations?
- Are there other examples of such dualities, e.g., for Einstein gravity and on Einstein backgrounds?
- Can the duality be extended beyond the self-dual sector?
- In twisted holography giant gravitons in the bulk can be identified with determinant operators in the dual holomorphic theory. **[Budzik, Gaiotto, 21]** What is their role in the twistor story?

Thank you for listening.

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