Bulk Holographic Global Vortices

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Outline

Background

Bulk Theory and Vacuum

String Vortex Analysis

Numerical Results

Conclusion

Introduction to AdS/CFT

- **AdS/CFT correspondence:** A powerful tool for studying strongly coupled systems.
- Originated from Maldacena's work (Maldacena 1998), $(4+1)$D gravity about AdS dual to conformal field theory the boundary of AdS.
- Useful in analyzing Bose-Einstein Condensates (BEC) at strong coupling and finite temperatures (i.e. a strongly coupled $U(1)$ condensates).

GKP-Witten Relation

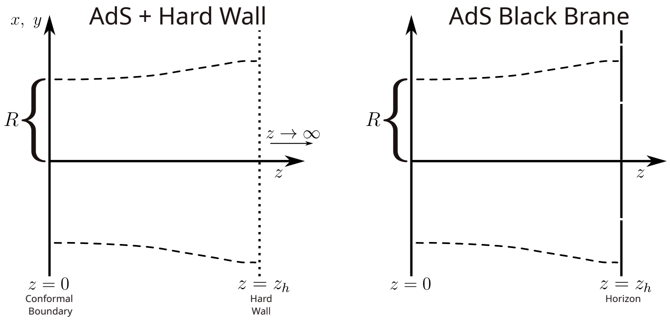
$$Z_{\mathrm{CFT}} = Z_{\mathrm{AdS}_5}$$

AdS/CFT and U(1) Condensates

- GP equations simulate low-temperature condensates in weakly coupled mean-field theory.
- The connection to AdS/CFT in the context of BECs is an area of ongoing research.
    - AdS/CFT has been applied to study various dynamics of BECs, including rotation and temperature effects.
    - It provides a framework for deriving higher order n-point functions for a dual conformal field theory.
    - Investigates a new approach to induce scalar condensation at the boundary of AdS space.
- Utilizes negative mass squared scalar fields and a stable quartic potential.
- Leads to a symmetry-breaking vacuum state without conformal scaling at the boundary.

Research Objectives

- $(3+1)$D AAdS Gravity + Scalar $\longleftrightarrow$ $(2+1)$D Conformal Field Theory
- analyze the stability of such a vacuum
- analyze the line vortex pairs
- analyze near boundary expansion of bulk vortices



Overview of the Bulk Theory

- Our focus: "ϕ^4" global U(1) scalar field coupled with Einstein Gravity.
- Negative cosmological constant in a $3+1$D asymptotically AdS spacetime.
- The action is a sum of gravity and matter actions.

Action

$$S = S_{\text{gravity}} + S_{\text{matter}} = \int \sqrt{-g}(R - 2\Lambda) - \int \sqrt{-g}\left(g^{\mu\nu}(\partial_\mu \Phi)(\partial_\nu \Phi)^* + V(|\Phi|^2)\right)$$

Potential

$$V(|\Phi|^2) = \frac{\lambda}{2}(|\Phi|^2)^2 + m^2|\Phi|^2$$

Asymptotically AdS Geometry

Metric (AdS Radius = L)

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + \frac{1}{f(z)}dz^2 + dx^2 + dy^2\right)$$

Blackening Factor

- AdS: $f(z) = 1$
- AdS Black Brane: $f(z) = 1 - z^3/r_h^3$

Important Spacetime Regions

- Conformal boundary: $z = 0$.
- Horizon/Hard wall: $z = z_{\mathrm{h}}$.

Near Boundary Expansion

To find near boundary homogeneous solutions $\phi \equiv \phi(z)$

Scalar Equations of Motion

$$-\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) + V'(|\Phi|^2)\Phi = 0$$

Indicial Equation and Solutions ($\phi \propto z^\beta$)

$$-\beta\,(\beta - 3)\,L^{-\beta}z^\beta + m^2L^{-\beta+2}z^\beta + L^{-3\beta}\lambda z^{3\beta} = 0$$

- For $\beta > 0$, $\beta(\beta - 3) = m^2L^2$.
- For $\beta = 0$, $L^2m^2 = -\lambda$.

Bulk $U(1)$ broken vacuum

Minimum Potential

Equations of motion allow for constant solution if...

- $L^2 m^2 = -\lambda$ allowing for z^0 near boundary
- V has a local minimum $\left(V_{\text{min}} = V\left(|\phi|^2 = -\frac{m^2}{\lambda}\right)\right)$

Units

From here on, $L = 1$ units will be used.

Linear Stability and Perturbations

- to test stability
- perturbation is a massive scalar field with $m^2 = 2\lambda$
- the perturbation must not source any current on the boundary

String Vortex Approximation

Vortex String Conditions

- large separations
- end on a horizon or hard wall and boundary with Neumann Boundary condition

Vortex string

is a scalar field that approximates a vortex solution parameterized with $R(z)$.

$$\phi_R/|\phi_{\mathrm{vac}}| = e^{is\Theta_R} = e^{is\tan^{-1}(y/(x-R(z)))}$$

Vortex String Pair

$$\phi_P/|\phi_{\mathrm{vac}}| = e^{i(s\Theta_{-R}+\Theta_R)}$$

Radial Profile Analysis

Finding Radial Profile

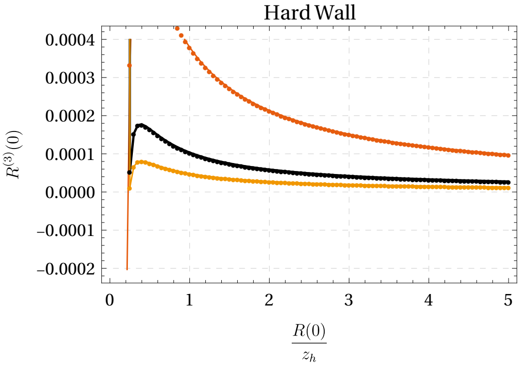
- Interaction Energy: $-\int dzdx^2\left(\mathcal{L}(\phi_P)-\mathcal{L}(\phi_R)-\mathcal{L}(\phi_{-R})\right),$
- Find R that minimizes the interaction energy
- and satisfies Neumann boundary conditions as $z=0$ and $z=z_{\mathrm{h}}$ for a set of $R^{(0)}(0)$ and $R^{(3)}(0)$ pairs.
- IR cutoff in the transverse radial direction Λ is required

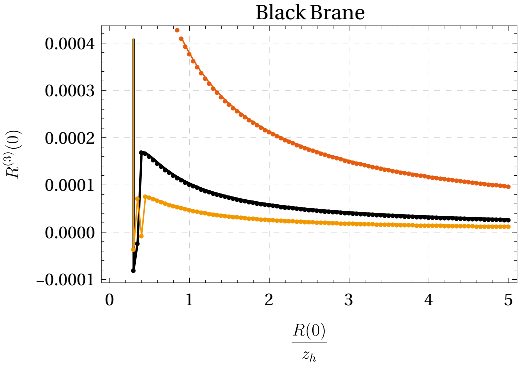
Strictly Large Λ - Analytical Radial profiles

- $R_{\text{AdS}}=R(0)+\frac{R^{(3)}(0)}{6}z^3$
- $R_{\text{Black Brane}}=R(0)-\frac{1}{6}z_{\mathrm{h}}^3R^{(3)}(0)\ln\left(1-z^3/z_{\mathrm{h}}^3\right)$

Numerical Radial Profiles: $R^{(3)}(0)$ vs $R(0)$

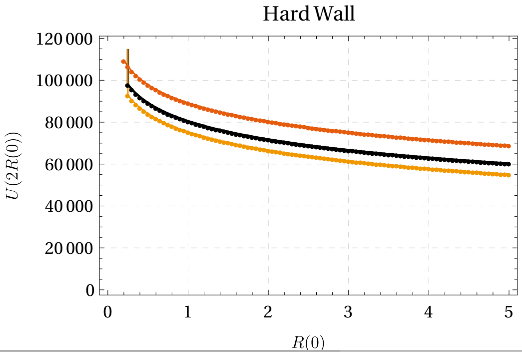
- $R^{(3)}(0) > 0$
- AdS Black Brane $\approx$ AdS + Hard Wall

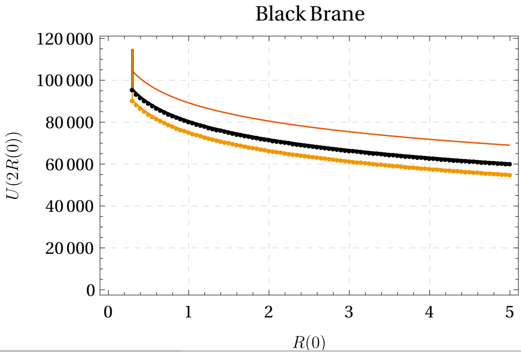




Numerical Radial Profiles: Interaction Energies

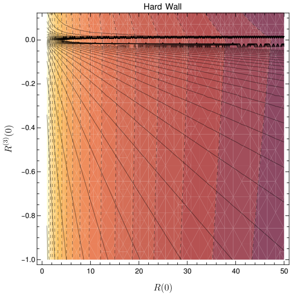
- Repulsive for vortex-vortex pair (implies attraction for vortex-antivortex)
- AdS Black Brane ≈ AdS + Hard Wall
- Holographic UV used $z_{\mathrm{UV}} \sim 0$

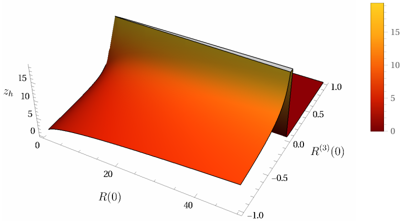




Contour Analysis of Radial Profiles

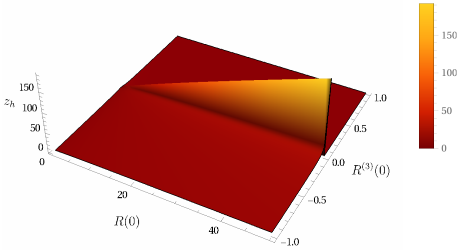
- Setting $R(0)$ and $R^{(3)}(0)$, solving for z_{h}
- solid lines = equidistant z_{h} and dashed lines = equipotential
- Different regions show distinct behaviors based on $R^{(3)}(0)$ values.

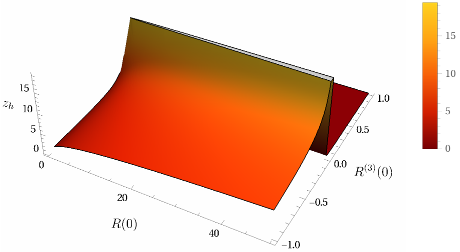




Numerical Results and Temperature Implications

- A critical temperature exists where the vortex approximation breaks down.
- Minimum temperature inversely proportional to $R(0)$.





Conclusion and Going Forward

- Analyzed a $(3+1)$D bulk $U(1)$ breaking vacuum in AdS and AdS Blackbrane spacetimes.
- The scalar field vacuum exhibits a constant behavior near the conformal boundary.
- Vortex solutions behave as string-like objects terminating on the boundary, requiring Neumann conditions at endpoints.
- Unique solutions for vortex profiles determined by specifying $R(0)$ and $R^{(3)}(0)$.
- Possible expansion on this research is to find gauge vortices

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