ERRATA AND ADDENDA 2004/5/11

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This is a supplement to [ST] and [W]. I may sometimes update this note.

(1) In [W, Example 2.4],

$$X := \{(x, y, z, w) \in \mathbb{C}^4 \mid xy + zw + z^3 + w^3 = 0\}.$$

Add "$= 0$".

(2) In page 6, line 9 in [ST], we add the following new lemma after "... the index i".

Lemma 1. *By* Remark 2.9, *we have* $a(E, S_i, B_{S_i}) \le a(E, S_{i+1}, B_{S_{i+1}})$ *for every valuation* E*. By* [FA, 7.4.4 Lemma] *and shifting the index* i*, we can assume that* $a(E, S_i, B_{S_i}) = a(E, S_{i+1}, B_{S_{i+1}})$ *for every* i *if* E *is a divisor on both* S_i *and* S_{i+1}*.*

(3) It is better to replace Remark 2.9 in [ST] with the following lemma.

Lemma 2. *By adjunction, we have*

$$a(E, S_i, B_{S_i}) \le a(E, S_{i+1}, B_{S_{i+1}}),$$

for every valuation E*. In particular,*

$$\mathrm{totaldiscrep}(S_i, B_{S_i}) \le \mathrm{totaldiscrep}(S_{i+1}, B_{S_{i+1}})$$

for every i*.*

Sketch of the proof. We can take a common log resolution

$$\begin{array}{ccc} & Y & \\ \swarrow & & \searrow \\ X_i & \dashrightarrow & X_{i+1} \end{array}$$

such that $Y \longrightarrow X_i$ and $Y \longrightarrow X_{i+1}$ are isomorphisms over the generic points of all CLC's by the resolution lemma (see [W, Section 5]). We note that $X_i \dashrightarrow X_{i+1}$ is an isomorphism at every generic point of CLC's. Apply the negativity lemma to

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This note may create new errors.

the flipping diagram $X_i \longrightarrow Z_i \longleftarrow X_{i+1}$ and compare discrepancies. Then, by restricting them to S_i and S_{i+1}, we obtain the desired inequalities of discrepancies. □

(4) Remark 1.1 in [W] should be replaced by the following remark.

Remark 3. In [Ma, Chapter 4], Matsuki explains various kinds of singularities in details. Unfortunately he made a mistake of using Theorem 5.1 with normal crossing divisors where it is only valid with simple normal crossing divisors. Accordingly, when we read [Ma] we have to replace *normal crossings* with *simple normal crossings* in the definition of dlt and so forth. See Definitions 2.8, 7.1, Remarks 7.6, 10.4, and [Ma, Definition 4-3-2 (2")].

(5) Alexeev pointed out that [FA, (4.12.2.1)] is wrong. The following example contradicts [FA, (4.12.1.3), (4.12.2.1)].

Example 4. Let $X = \mathbb{P}^2$, $B = \frac{2}{3}L$, where L is a line on X. Let P be any point on L. First, blow up X at P. Then we obtain an exceptional divisor E_P such that $a(E_P, X, B) = \frac{1}{3}$. Let L' be the strict transform of L. Next, take a blow-up at $L' \cap E_P$. Then we obtain an exceptional divisor F_P whose discrepancy $a(F_P, X, B) = \frac{2}{3}$. On the other hand, it is easy to see that $\mathrm{discrep}(X, B) = \frac{1}{3}$. Thus, $\min\{1, 1 + \mathrm{discrep}(X, B)\} = 1$.

Remark 5. By this example, Lemma 2.1, which is the same as [FA, (4.12.2.1)], in my paper: "Termination of 4-fold canonical flips" is incorrect. So, the arguments in my paper become nonsense. Note that [FA, 4.12.1 Lemma] originates from Corollary 3.2 in Kollár's paper: Flops. Lemma 2.2 in Matsuki's paper (Termination of flops for 4-folds) is a copy of Corollary 3.2 in Flops. We think that a right formulation is [KM, Proposition 2.36 (2)].

(6) It is better to mention *log discrepancies* in [W].

Remark 6. We put $a_\ell(E, X, D) = 1 + a(E, X, D)$ and call it a *log discrepancy*. We define

$$\mathrm{logdiscrep}(X, D) = 1 + \mathrm{discrep}(X, D).$$

In some formulas, log discrepancies behave much better than discrepancies.

(7) We add one remark on [KMM].

Remark 7. We note the following Matsuki's comment in [Ma, Remark 14-2-7]. In [KMM] at the end of Example 5-2-5 there is a slightly misleading statement: "The morphisms given in Example 5-2-4 and 5-2-5 are the only contractions of flipping type from $\mathbb{Q}$-factorial terminal toric varieties of dimension 3 by the theorem of White-Frumkin." This is, however, true only under the assumption that the extremal rational curve passes only one singular point.

(8) We add one remark on [KMM] and [Ma, Chapter 14].

Remark 8. In [KMM, §5-2] and [Ma, Chapter 14], toric varieties are investigated from the Mori theoretic viewpoint. The toric Mori theory originates from Reid's beautiful paper [R]. Chapter 14 in [Ma] corrects some minor errors in [R]. In [KMM] and [Ma], the toric Mori theory is formulated for toric projective morphism $f : X \longrightarrow S$. We note that X is always assumed to be *complete*. So, the statement at the end of [Ma, Proposition 14-1-5] is nonsense. Matuski wrote: "In the relative setting for statement (ii), such a vector v'_i may not exist at all. If that is the case, then the two $(n-1)$-dimensional cones $w_{i,n}$ and $w_{i,n+1}$ are on the boundary of Δ." However, Δ has no boundary since Δ is a complete fan in [Ma]. For the details of the toric Mori theory for the case when X is not complete, see [FS] (Introduction to the toric Mori theory), [F1] (Equivariant completions of toric contraction morphisms), and Sato's paper: Combinatorial descriptions of toric extremal contractions.

(9) One more remark on [KMM].

Remark 9. Theorem 6-1-6 in [KMM] is [Kawatama, Theorem 4.3] (Pluricanonical systems on minimal algebraic varieties). We use the same notation as in the proof of Theorem 4.3. By Theorem 3.2, ${}'E_1^{p,q} \longrightarrow {}''E_1^{p,q}$ are zero for all p and q. This just implies that

$$\mathrm{Gr}^p H^{p+q}(X, \mathcal{O}_X(-\ulcorner L \urcorner)) \longrightarrow \mathrm{Gr}^p H^{p+q}(D, \mathcal{O}_D(-\ulcorner L \urcorner))$$

are zero for all p and q. Kawamata said that we need one more Hodge theoretic argument to prove

$$H^i(X, \mathcal{O}_X(-\ulcorner L \urcorner)) \longrightarrow H^i(D, \mathcal{O}_D(-\ulcorner L \urcorner))$$

are zero for all i.

References

[ST] O. Fujino, Special termination and reduction theorem, 2004/4/21.
[W] O. Fujino, What is log terminal ?, 2004/4/23.

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