

SEMINAR A (MATHEMATICS) - SCHEDULE AND GRADING G30 PROGRAM - 1ST YEAR

Schedule. The sessions will be on October 12, 19, 26, November 2, 9, 16, 23, 30, December 7, 14, 21, January 11, 18, 25.

Sessions. Each session will be organized as follows:

- 1) 15 to 20 minutes: presentation 1;
- 2) 20 to 25 minutes: questions and comments about presentation 1;
- 3) 5 to 10 minutes: break;
- 4) 15 to 20 minutes: presentation 2;
- 5) 20 to 25 minutes: questions and comments about presentation 2.

Presentations / talk. Sources: any source can be accepted, as long as it is scientifically recognized. Usually, it is recommended to have a main source (short article for example). All topics proposed have good references in wikipedia.

The presentations should present clear ideas as precisely as possible. Any classical kind of support is authorized (detail in first class). In case you use the video projector, send me the electronic presentation (**original format + PDF**) at least one day in advance. See also the evaluation sheet.

Grading.

- **30 %** : class participation
- **70 %** : presentations

Minimal requirement for passing : 60 %.

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TOPICS

Note that the descriptions are not limiting. They are a guide to start the research process.

1. **Pythagorean theorem.** History and proofs of the Pythagorean theorem.
2. **Zeno's paradox.** This paradox concerns infinite sums of real numbers. The talk will explain the solutions this paradox as well as the notion of series.
3. **Mathematical induction.** Explanation of proofs by induction. Should include some examples.
4. **Proofs by contradiction.** What is a proof by contradiction? Should include examples.
5. **RSA cryptosystem.** This cryptosystem is nowadays widely used in industry. It is for example used for certain financial transactions (credit cards...). It is based on basic arithmetic. The concept of public-private key should be explained, as well as the mathematical arguments which makes the RSA system effective.
6. **Graphs.** A lot of mathematical problems concern mathematical graphs. The notion of Eulerian path, their existence, is one of the problems which could be discussed via the famous problem "Seven Bridges of Königsberg".
7. **Rational and irrational numbers.** What is a rational number? Are all real numbers rational? Some example of irrational numbers and the proofs they are irrational should appear.
8. **Koch snowflake.** Koch snowflake is a fractal. How can it be drawn? What is its length? What is its area? are examples of questions which could be studied during this talk.
9. **Mathematics and music.** How are mathematics and music related? What are the different musical scales used in the world and how do they differ from a mathematical point of view?
10. **Stereographic projection.** How to draw a map of the earth on a two dimensional plane? What are the advantage and the disadvantage of the usual method? Are there other methods?
11. **Monty hall problem (probability).** In this game, we are asked to choose a door among three to find and win a car. After choosing, the referee of the game opens a door which turns out not to contain a car and ask us if we want to change door. It seems it does not matter. Is it really the case?
12. **Ptolemy's theorem.** How can we check that a quadrilateral can be inscribed in a circle? This is the object of Ptolemy's theorem. This talk should contain a proof of the theorem.
13. **Prime numbers.** What is their definition? What are they? How many are they (and how to prove it)? are several questions this talk could try to answer.
14. **Opinion polls.** What is an opinion poll? Why does it works or when doesn't it work? On which mathematical tools do it rely? How many people should be sampled?
15. **Voting paradox.** Depending on countries there are several ways to organize elections. Is there a way which is better than others? This talk should present several protocol and explain which are the advantages of each of them with mathematical arguments.
16. **Computing π .** This talk will explain methods to get good approximations of π . It will also explain why it is impossible to hope for an exact value. It should contain explicit examples.
17. **Minimax algorithm.** How does a computer play chess? (or go, or shogi...) Minimax is the basic elementary method which should be explained here. What is missing to actually play chess should be also explained. Improvements could also be explained.
18. **Spherical geometry.** Let us replace the blackboard by a ball and try to do geometry on it. What will change? What is the shortest point between two points?

19. **Linear dynamical systems – Predator-prey model.** Suppose that we put a certain quantity of rabbits and foxes on an isolated island. What will happen after a while? Can it be predicted? Only rabbits? Only foxes? Some balanced situation?
20. **Crystal structure in daily life.** We all know how a pile of oranges as a natural tendency to get arranged. Why is that? The aim of this topic is to understand what geometrical properties of a crystal structure make it more or less likely to happen. It is possible to relate this to chemistry.
21. **Pythagorean triples.** We consider a right-angled triangle with sides of integer lengths. What can be these lengths? The aim is to show the formula giving these length (to prove how to find such triples of integers but also to prove that there are no other than the one given by the formula).
22. **Famous computable sums.** The aim is to prove several formulas for computing sums of numbers. For example, what is the value of $1+2+\dots+n$, or $1^2+2^2+\dots+n^2$, or $2^1+2^2+\dots+2^n$? There are several possible proofs. It includes elegant geometrical argument and inductive proofs.
23. **About magic squares.** A magic square is a 3×3 matrix that has the property that the sums of its entries on each row, column and diagonal are equal. Can we find all these magic squares? What are the possible variants?
24. **Platonic solids.** This is the other name of regular polyhedrons. How many are they? How to prove that these are the only ones? These will be some of the questions that will be answered.
25. **The problem of shortest path.** How does a famous website choose for us the best possible travel between two locations? The main aim will be to introduce Dijkstra's algorithm which permits to solve this problem. The talk should contain some examples of computations.