Linear Algebra II - Worksheet 8

Exercise 1 : We consider the map $T : \mathbb{R}^n \to \mathbb{R}$ defined by

$$T(\vec{v}) = \det \begin{bmatrix} | & | & | & & | \\ \vec{x} & \vec{v}_2 & \vec{v}_3 & \cdots & \vec{v}_n \\ | & | & | & & | \end{bmatrix}$$

1. Justify that T is linear.

2. What is $\operatorname{Im} T$?

3. What is $\operatorname{Ker} T$?

Exercise 2 : We consider a square matrix M with integer coefficients. In which case M admits an inverse with integer coefficients?

Exercise 3 : Compute
$$\det \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ a_0 & a_1 & a_2 & \cdots & a_n \\ a_0^2 & a_1^2 & a_2^2 & \cdots & a_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_0^n & a_1^n & a_2^n & \cdots & a_n^n \end{bmatrix}.$$