Linear Algebra II - Worksheet 5

Exercise 1 : Which of the following matrices are orthogonal?

$$A = \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \qquad B = \frac{1}{3} \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}$$

Exercise 2 : We suppose that A and B are orthogonal $n \times n$ matrices. Which ones of the following matrices are *necessarily* orthogonal?

$$C = 3A; \quad D = -B; \quad E = AB; \quad F = A + B; \quad G = B^{-1}; \quad H = B^{-1}AB; \quad K = A^{\mathrm{T}}$$

Exercise 3 : In each case, tell if an orthogonal transformation with the given property can exist. If it is the case, give an example of such a transformation.

1. $T \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ and $T \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$.

2. $T \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ and $T \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$.