Linear Algebra II - Worksheet 1

Exercise 1: Page 163, Exercise 32. We consider the set V of 2×2 real matrices S such that
$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} S = S \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}.$$

1. Prove that V is a subspace of $M_2(\mathbb{R})$.

2. Find a basis of V.

Exercise 2: Page 163, Exercise 14. We consider the set V of real sequences $(x_n)_{n \in \mathbb{N}}$ that satisfy $\lim_{n \to \infty} x_n = 0$.

1. Prove that V is a subspace of the space of real sequences.

2. What is the dimension of V?

Exercise 3: Page 170, exercise 6 and 8. For each of the following maps from $M_2(\mathbb{R})$ to $M_2(\mathbb{R})$, determine if it is linear and if it is an isomorphism.

6. $T(M) = M \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$;

8. $T(M) = M \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} M$.