Linear Algebra I - Homework 7

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 20. The number of points for each exercise is specified between parenthesis. To hand in November 30 at the beginning of the lecture.

Exercise 1 : (8) Page 110 of the textbook, Exercises 2, 4, 6. For each of these matrices, find the kernel and the image of the associated linear map. Find also as few vectors as possible which span the image and as few vectors as possible which span the kernel.

$$2)\ \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \quad 4)\ \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad 6)\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Exercise 2 : (6) Page 111 of the textbook, Exercise 38 (modified). Prove that, for $T : \mathbb{R}^m \to \mathbb{R}^n$ and $S : \mathbb{R}^n \to \mathbb{R}^p$ linear, we have

a) $\mathrm{Im}(S \circ T) \subset \mathrm{Im}(S)$;

b) $\mathrm{Ker}(S \circ T) \subset \mathrm{Ker}(T)$.

Exercise 3 : (6) Page 44 of the textbook, Exercise 44. Let A be a matrix and we denote as usual $\mathrm{rref}(A)$ its reduced row-echelon form.

a) Prove that $\mathrm{Ker}(A) = \mathrm{Ker}(\mathrm{rref}(A))$ (where $\mathrm{Ker}(A)$ is the kernel of the linear map associated to A).

b) Is that true that $\mathrm{Im}(A) = \mathrm{Im}(\mathrm{rref}(A))$?