Linear Algebra I - Homework 11

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 20. The number of points for each exercise is specified between parenthesis. To hand in January 19 at the beginning of the tutorial.

Exercise 1 : Page 163, Exercises 2 and 4. We consider the vector space P_2 which consists of polynomials of degree at most 2. Prove that the following subset of P_2 are subspaces and compute a basis:

2) $\{p \in P_2 \mid p(2) = 0\}$;

4) $\{p \in P_2 \mid \int_0^1 p(t)\,\mathrm{d}t = 0\}$.

Exercise 2 : Page 163, Exercises 7 and 9. We consider the vector space $M_3(\mathbb{R})$ which consists of 3×3 matrices. Tell which of these subsets are subspaces and in this case give a basis.

7) the set of 3×3 diagonal matrices (matrices whose all coefficients outside of the main diagonal are zeros);

9) the set of 3×3 matrices with non-negative entries.

Exercise 3 : We consider the following maps. For each of them tell if it is linear, if it is an isomorphism:

1) $T(M) = M + I_2$ from $M_2(\mathbb{R})$ to $M_2(\mathbb{R})$;

2) $T(M) = 7M$ from $M_3(\mathbb{R})$ to $M_3(\mathbb{R})$;

3) $T(M)$ is the sum of diagonal elements of M, from $M_3(\mathbb{R})$ to $\mathbb{R}$.

Exercise 4 : We fix a function $f : \mathbb{R} \to \mathbb{R}$ and we consider the following map:

$$\begin{aligned} T : \mathscr{F}(\mathbb{R},\mathbb{R}) &\to \mathscr{F}(\mathbb{R},\mathbb{R}) \\ g &\mapsto g \times f \end{aligned}$$

where $\mathscr{F}(\mathbb{R},\mathbb{R})$ is the space of functions from $\mathbb{R}$ to $\mathbb{R}$ ($(g \times f)(t) = g(t)f(t)$).

1) Prove that T is linear.

2) For which f is T an isomorphism? What is the inverse of T in this case? (justify)