Complex Analysis - Quiz 6
Solution

Evaluate $\displaystyle\int_\gamma \frac{1}{z}\,\mathrm{d}z$ where γ is the curve $\gamma(\theta) = \theta\exp(\theta i)$ for $\theta \in [\pi/2, 3\pi]$.

By definition, we have

$$\begin{aligned}\int_\gamma \frac{1}{z}\,\mathrm{d}z &= \int_{\pi/2}^{3\pi} \frac{1}{\gamma(\theta)}\gamma'(\theta)\,\mathrm{d}\theta = \int_{\pi/2}^{3\pi} \frac{1}{\theta\exp(\theta i)}(1+\theta i)\exp(\theta i)\,\mathrm{d}\theta \\ &= \int_{\pi/2}^{3\pi} \left(\frac{1}{\theta} + i\right)\,\mathrm{d}\theta = [\log\theta + \theta i]_{\pi/2}^{3\pi} \\ &= \log(3\pi) - \log(\pi/2) + \frac{5\pi}{2}i = \log 6 + \frac{5\pi}{2}i,\end{aligned}$$

where $\log = \ln$ is the usual natural logarithm over $\mathbb{R}$.

Warning: $1/z$ does not have an antiderivative on whole $\mathbb{C}$ so it would be tricky to use the Fundamental Theorem of Calculus.