Basic Mathematics - Quiz 8
Solution

Give the equations of the vertical, horizontal and slant asymptotes of the following functions :

1. $f(x) = \dfrac{x+1}{3x-2}$

   The vertical asymptotes correspond to the x where the denominator vanishes and the numerator does not. So the only vertical asymptote has equation $x = 2/3$ (the numerator does not vanish at $2/3$).

   For the other asymptotes, we perform the long division :

$$\begin{array}{r|l} & 1/3 \\ \hline 3x-2 & x+1 \\ & -(\ x-2/3\ ) \\ \hline & 5/3 \end{array}$$

   So we get

$$x+1 = \frac{1}{3}(3x-2) + \frac{5}{3}$$

   or in other terms

$$f(x) = \frac{x+1}{3x-2} = \frac{1}{3} + \frac{5}{3(3x-2)}.$$

   When x is "very positive" or "very negative", the second term is "very small" so f admits an horizontal asymptote of equation $y = 1/3$.

2. $g(x) = \dfrac{2x^2+x-3}{x+2}$

   The vertical asymptotes correspond to the x where the denominator vanishes and the numerator does not. So the only vertical asymptote has equation $x = -2$ (the numerator does not vanish at -2).

   For the other asymptotes, we perform the long division :

$$\begin{array}{r|l} & 2x-3 \\ \hline x+2 & 2x^2+x-3 \\ & -(\ 2x^2+4x\ ) \\ \hline & -3x-3 \\ & -(\ -3x-6\ ) \\ \hline & 3 \end{array}$$

   So we get

$$2x^2+x-3 = (2x-3)(x+2)+3$$

   or in other terms

$$g(x) = \frac{2x^2+x-3}{x+2} = (2x-3) + \frac{3}{x+2}$$

   when x is "very positive" or "very negative", the second term is "very small" so f admits a slant asymptote of equation $y = 2x-3$.