

Basic Mathematics - Quiz 5 Solution

We consider the function f defined by

$$f(x) = -x^2 + x + 6.$$

1. Find the solutions of the equation $f(x) = 0$ (justify).

First, we complete the square. In other terms, we search a such that $-(x+a)^2 = -x^2 + x + \dots$ so $a = -1/2$. So we have

$$f(x) = - \left[\left(x - \frac{1}{2} \right)^2 - \left(\frac{1}{2} \right)^2 \right] + 6 = - \left(x - \frac{1}{2} \right)^2 + \frac{1}{4} + 6 = - \left(x - \frac{1}{2} \right)^2 + \frac{25}{4}.$$

So $f(x) = 0$ if and only if

$$\left(x - \frac{1}{2} \right)^2 = \frac{25}{4}$$

which is satisfied when and only when

$$x - \frac{1}{2} = \frac{5}{2} \quad \text{or} \quad x - \frac{1}{2} = -\frac{5}{2}$$

so whenever $x = 3$ or $x = -2$.

2. Find the vertex of f (justify).

As we have

$$f(x) = - \left(x - \frac{1}{2} \right)^2 + \frac{25}{4},$$

then always $f(x) \leq 25/4$, with $f(x) = 25/4$ exactly when $x = 1/2$. So the vertex of f is $(1/2, 25/4)$.