

Basic Mathematics - Quiz 11

Solution

Solve the following system of equations (justify by writing intermediate steps) :

$$\begin{cases} x & +y & -z & = & 4 \\ x & -y & +2z & = & 3 \\ -x & +2y & +z & = & 2 \end{cases}$$

We use the elimination technique. As the coefficient of x in first equation is 1, we use it as a pivot : we subtract equation 1 from equation 2 and we add it to equation 3. The system is equivalent to

$$\begin{cases} x & +y & -z & = & 4 \\ & -2y & +3z & = & -1 \\ & 3y & & = & 6 \end{cases}$$

We exchange equations 2 and 3 to simplify. The system is equivalent to

$$\begin{cases} x & +y & -z & = & 4 \\ & 3y & & = & 6 \\ & -2y & +3z & = & -1 \end{cases}$$

We divide equation 2 by 3 to obtain a pivot. The system is equivalent to

$$\begin{cases} x & +y & -z & = & 4 \\ & y & & = & 2 \\ & -2y & +3z & = & -1 \end{cases}$$

We use the pivot (y with coefficient 1 in equation 2). We subtract equation 2 to equation 1 and we add twice equation 2 to equation 3. The system is equivalent to

$$\begin{cases} x & & -z & = & 2 \\ & y & & = & 2 \\ & & 3z & = & 3 \end{cases}$$

We divide equation 3 by 3 to obtain a pivot. The system is equivalent to

$$\begin{cases} x & & -z & = & 2 \\ & y & & = & 2 \\ & & z & = & 1 \end{cases}$$

We use the last pivot. We add equation 3 to equation 1. The system is equivalent to

$$\begin{cases} x & & & = & 3 \\ & y & & = & 2 \\ & & z & = & 1 \end{cases}$$

So the first system has the same solution(s) as the last one. The only solution of the system is $(x, y, z) = (3, 2, 1)$.