Basic Mathematics - Quiz 10
Solution

Solve the following equation (justify by writing intermediate steps) :

$$\ln(2x) - \ln(x+2) = \ln(6) - \ln(x+4).$$

Notice that, as exp is a bijection from $\mathbb{R}$ to $(0, +\infty)$ and ln is the inverse bijection, the equation is satisfied if and only if

$$\begin{cases} 2x > 0 \\ x+2 > 0 \\ x+4 > 0 \\ \exp(\ln(2x) - \ln(x+2)) = \exp(\ln(6) - \ln(x+4)). \end{cases}$$

We simplify. It is true if and only if

$$\begin{cases} x > 0 \\ x > -2 \\ x > -4 \\ \dfrac{2x}{x+2} = \dfrac{6}{x+4}. \end{cases}$$

if and only if

$$\begin{cases} x > 0 \\ x(x+4) = 3x + 6. \end{cases}$$

We start by solving the second degree equation : by expending, it becomes

$$x^2 + x - 6 = 0.$$

So we factorize a square :

$$x^2 + x - 6 = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 6$$

and we obtain that the equation is satisfied if and only if

$$\left(x + \frac{1}{2}\right)^2 = \frac{25}{4}$$

if and only if

$$x + \frac{1}{2} = \frac{5}{2} \quad \text{or} \quad x + \frac{1}{2} = -\frac{5}{2}$$

if and only if

$$x = 2 \quad \text{or} \quad x = -3.$$

Combining with the inequality $x > 0$, we obtain that the only solution of the initial equation is $x = 2$.