Name:

Basic Mathematics - Midterm examination

Duration: 90 minutes.

Documents and electronic devices are forbidden. According to Nagoya University Student Discipline Rules (article 5), cheating can lead, in addition to disciplinary action, to the loss of all credits earned in all subjects during the semester.

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 40. The number of points for each problem is specified between parenthesis. Each question will be graded independently: do not hesitate to skip some of them.

Problem 1 : (5)

1. Draw the line L passing through the points $(1, 3)$ and $(2, -1)$.
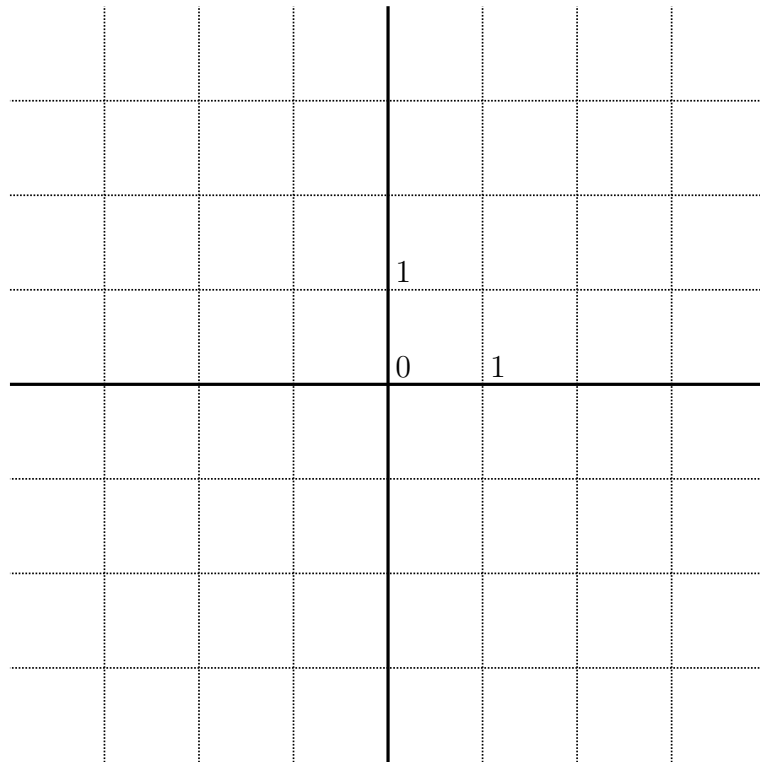
2. Compute an equation of L.

1

3. Compute an equation of the line parallel to L passing through $(2, 4)$.

4. Compute an equation of the line perpendicular to L passing through $(-1, 3)$.

Problem 2 : (8) We consider the function f defined by:
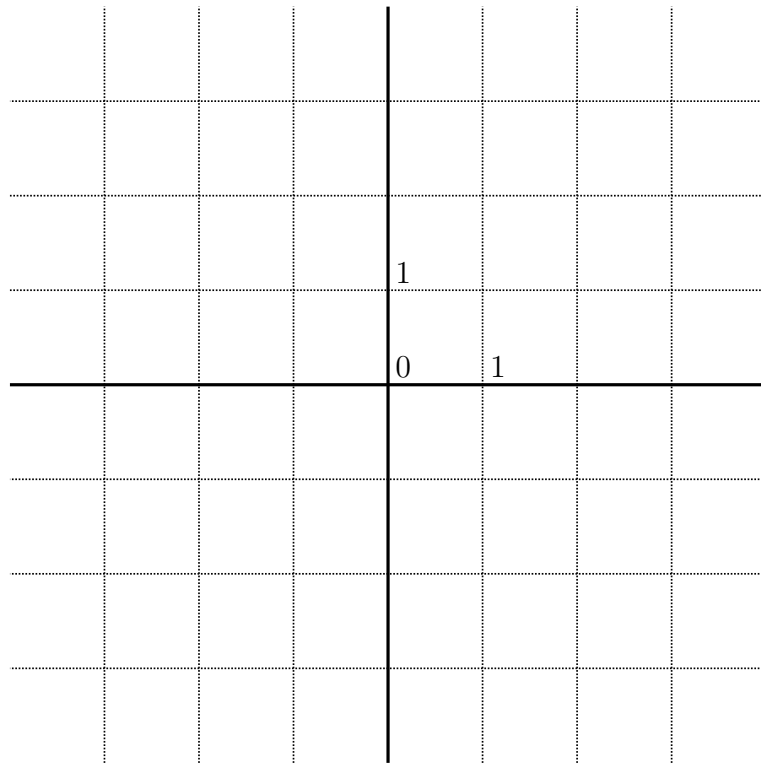
$$f(x) = \begin{cases} \dfrac{16}{(x+1)(x-2)} & \text{if } x < 1; \\ \\ -(x-2)^2 + 3 & \text{if } x \geqslant 1. \end{cases}$$

1. Compute:
   - $f(-4) =$
   - $f(-3) =$
   - $f(-2) =$
   - $f(-3/2) =$
   - $f(-1/2) =$
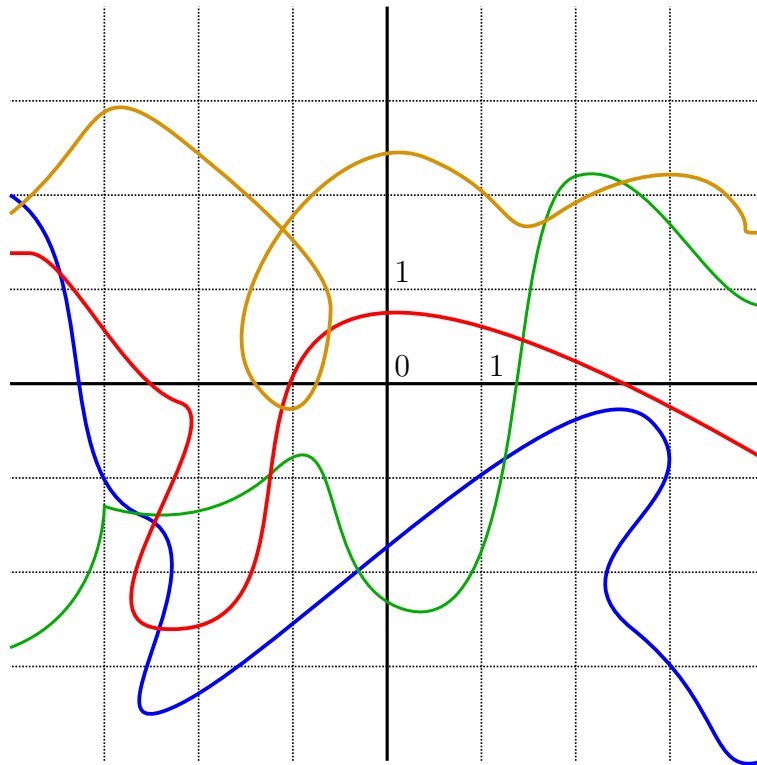   - $f(0) =$
   - $f(1) =$
   - $f(2) =$
   - $f(3) =$

2. What is the maximal domain of f?

3. Draw the graph of f.

4. What is the average rate of change of f between -3 and 2?

Problem 3 : **(4)** We consider the following curves:

For each of them, tell if it is the graph of a function and justify your answer:
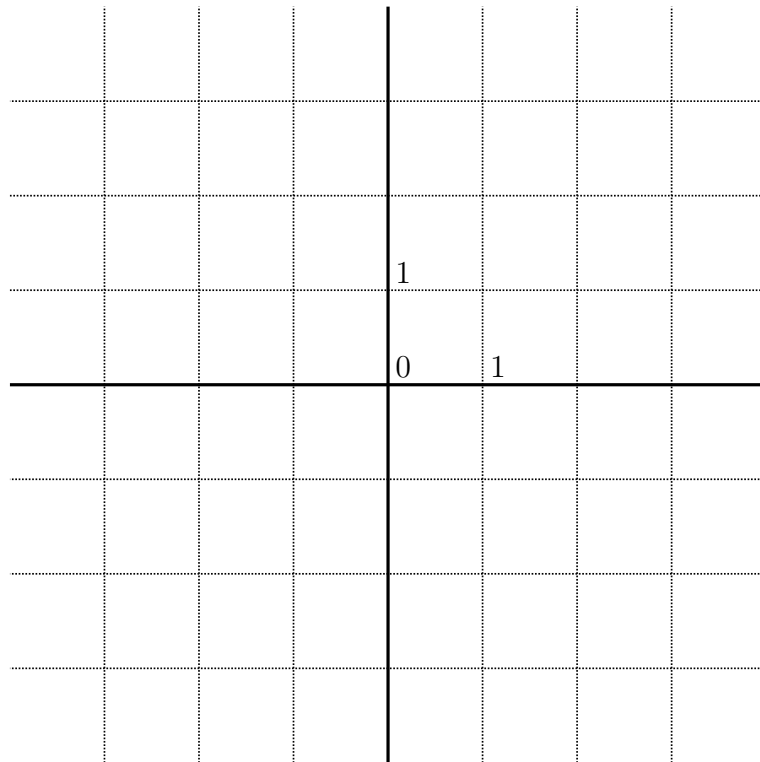
1. Red

2. Green

3. Blue

4. Orange

Problem 4: **(10)** We consider the functions from $\mathbb{R}$ to $\mathbb{R}$ defined by:
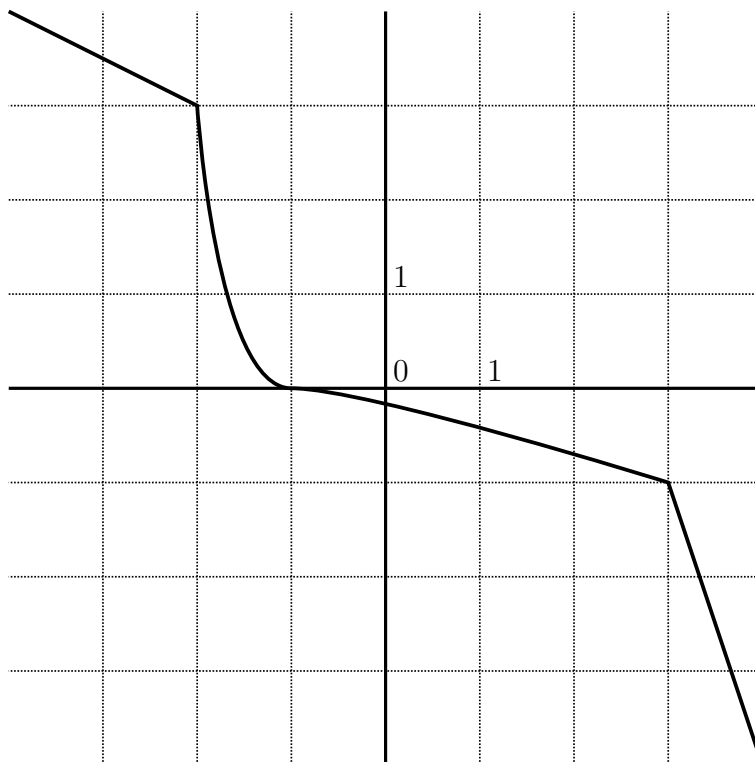
$$f(x) = \frac{-x^2 - x + 12}{5}$$

$$g(x) = x/2 - 5$$

1. Compute $f \circ g$.

2. Compute $g \circ f$.

3. Compute $2f - g$.

4. Compute g^{-1}.

5. Solve the equation $f(x) = 0$.

6. Solve the equation $g(x) = 0$.

7. What is the vertex of f?

8. Draw the graphs of f and g:

Problem 5: **(6)** We consider the function $f : [-4, 4] \to [-4, 4]$ with the following graph:

Draw the graphs of the functions defined by:

1. $g(x) = f(x + 1)$;

2. $h(x) = f(x)/2$;

3. $j(x) = f(-2x/3)$;

4. $k(x) = f^{-1}(x)$;

5. $\ell(x) = f(x) - 1$.

(don't forget to indicate their names clearly).

Problem 6 : **(7)** Mr. Yamamoto wants to enclose a (rectangular) patch of land to build his house. On north, east and west sides, he plans to use simple fence for 500 yens per meter. On the south side, he plans to use trees (to enjoy shadow) and it will cost 1500 yens per meter. He has got a budget of 100 000 yens to do that. We use the following notation:
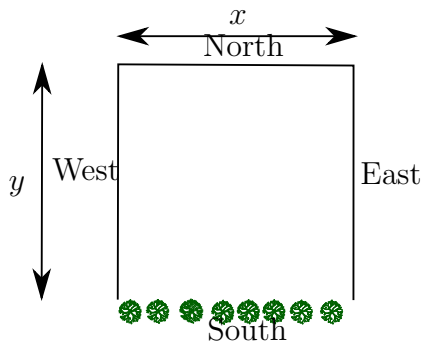
1. Compute the cost of the construction in function of x and y.

We suppose now that Mr. Yamamoto will spend all his budget.

2. Express y in function of x.

3. Express the area of the patch of land in function of x.

4. For which x will Mr. Yamamoto get the biggest possible area? What is this area?