Name:

Basic Mathematics - Final examination

Duration: 90 minutes.

Documents and electronic devices are forbidden. According to Nagoya University Student Discipline Rules (article 5), cheating can lead, in addition to disciplinary action, to the loss of all credits earned in all subjects during the semester.

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 40. The number of points for each problem is specified between parenthesis. Each question will be graded independently: do not hesitate to skip some of them.

Problem 1 : (13) The aim of this exercise is to draw the graph of the function f defined by

$$f(x) = \frac{2x^3 - 3x^2 - 4x - 1}{2x^2 - 2x - 4}$$

1. Check that $-1/2$ is a solution of $2x^3 - 3x^2 - 4x - 1 = 0$. Find the other solutions.

2. Solve the equation $2x^2 - 2x - 4 = 0$.

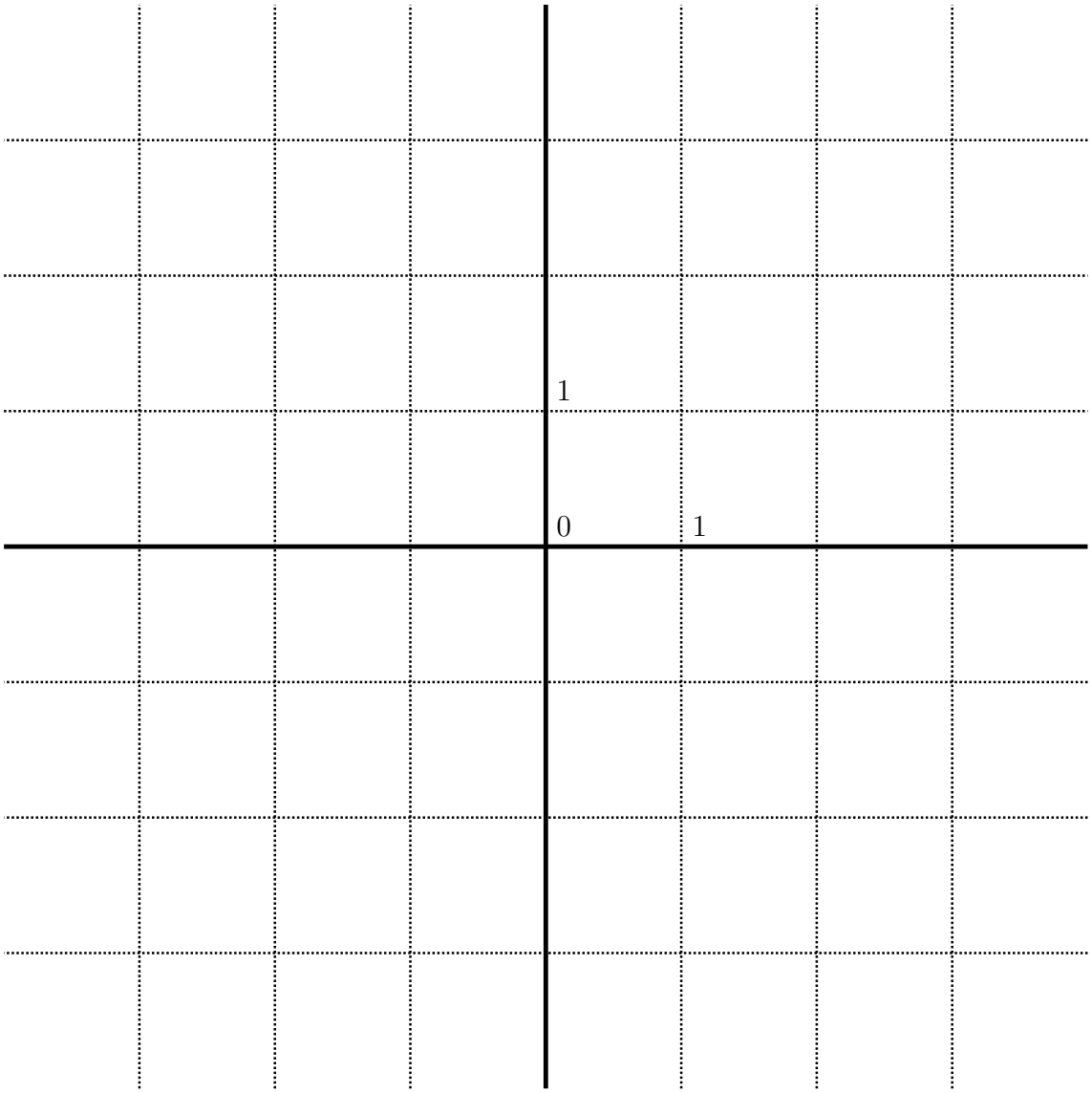
3. Give the x-intercepts and y-intercepts of f.

4. Give all vertical asymptotes of f. For each of them, tell if the graph of f goes up or down on the left of the asymptote and on the right of the asymptote (justify).

5. Perform the long division of $2x^3 - 3x^2 - 4x - 1$ by $2x^2 - 2x - 4$.

6. Give all horizontal and slant asymptotes of f. For each of them, tell if the graph of f is bellow or above the asymptote when x becomes very positive and when x becomes very negative (justify).

7. Draw the graph of f and its asymptotes.



Problem 2 : (3) Simplify as much as possible the following expressions:

1. $\exp(2\ln 4 - \ln 7) =$

   NB: $\exp(x) = e^x$.

2. $\log(4000) - 2\log(2) =$

Problem 3 : (3) Solve the equation $\exp(2x - 3) = 7$.

Problem 4 : (6) Solve the following system of equations:

$$\begin{cases} x - 2y + 5z = 30 \\ -x + y + 2z = 17 \\ x - 3y + 12z = 77 \end{cases}$$

Problem 5 : (6) Solve the equation $3x^3 + 8x^2 - 1 = 0$ (start by looking for rational solutions).

Problem 6 : (5) Mr. Tanaka has a bank account with 3% interest per year, compounded monthly. The first of January 2015, his account contained 1.000.000 yens. After how many months will his bank account contain 1.100.000 yens (He did not take or put any money from or on his account in between)? Note: you can give a formula.

Problem 7 : (4) To take off, a plane needs a certain quantity t of fuel (always the same quantity, in liters). It needs r liters of fuel per kilometers. Finally, it needs also t liters of fuel to land. To go from Nagoya to Beijing (2000 km), it burns 25000 liters. To go from Nagoya to Tokyo (300 km), it burns 4600 liters. How many liters does it need to go from Nagoya to Hanoi (3500 km)?